

Today:

torque - applications to NMR, MRI
angular momentum - applications to chemistry, NMR, MRI (electron orbits)
rotational inertia - applications to chemistry (heat capacity)
precession - applications to NMR, MRI

Also, new topic:

pressure
atmospheric pressure
fluids

PHY 117 HS2026

Week 4, Lecture 2

Oct. 7th, 2026

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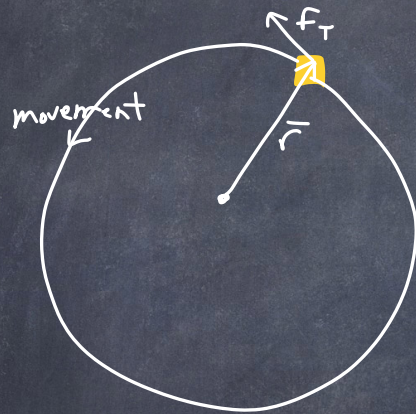
Remember to
do quizzes 1,2,3,4
on OLAT.

Stats

students:	700
quiz 1 :	267
quiz 2 :	238
quiz 3 :	76

Yesterday:

Consider a tangential force on an object, mass m , constrained to move in a circle.



The force is tangential, F_T

The force is unbalanced, creates a .

$$F_T = ma = m(r\alpha)$$

$a = r\alpha$
for circular motion.

we multiply both sides by r

$$r F_T = \underbrace{mr^2}_{\tau} \alpha$$

$$\tau = I \alpha$$

$$\tau = I \alpha$$

$I \equiv mr^2$: "moment of inertia": this is for a particle of mass m at a radius r from the center of rotation.

Yesterday:

Note: the parallels between linear + rotational motion:

linear motion

$$F = ma$$

rotational motion

$$\tau = I \alpha$$

I is like mass $\uparrow$ angular acceleration
($I = mr^2$)

Newton's
2nd law

$$\Sigma \vec{F} = m \vec{a}$$

Newton's second law

of rotation

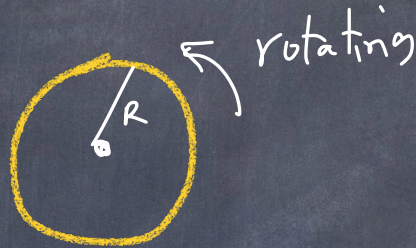
$$\Sigma \vec{\tau} = I \vec{\alpha}$$

I is a measure of how mass is distributed.

for discrete objects $I = \sum_i m_i r_i^2$

for objects with shape $I = \int r^2 dm$

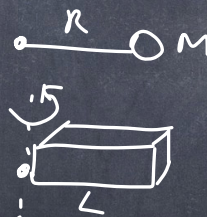
simple case: ring
total mass M , all at
a radius R



$$I_{\text{ring}} = MR^2$$

shape
* point
* ring
stick
sphere
disk

$$\begin{aligned} I &= MR^2 \\ I &= MR^2 \\ I &= \frac{1}{3} ML^2 \\ I &= \frac{2}{5} MR^2 \\ I &= \frac{1}{2} MR^2 \end{aligned}$$

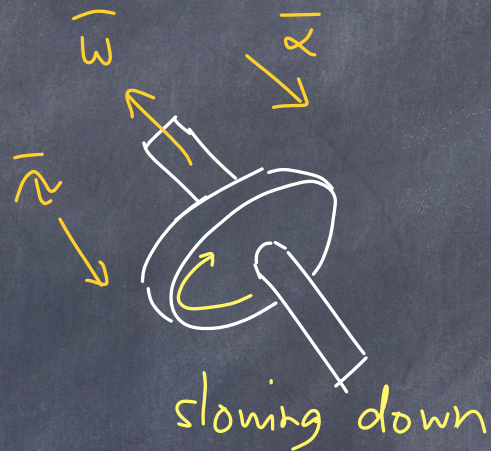
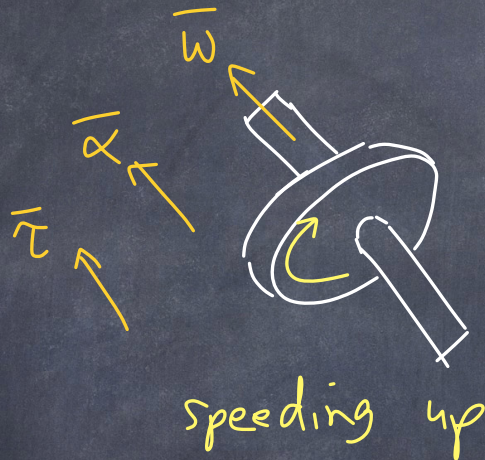


In the script, I derive some of these.
But, in this class, you only need to know these *
Formula for other ones would be provided on an exam..

$$\vec{\tau} = I \vec{\alpha}$$

what direction is $\vec{\alpha}$?

$\vec{\alpha}$ points in same direction as $\vec{\tau}$



For $\vec{\alpha}$, angular acceleration, it points in the same direction as $\vec{\omega}$ if $\vec{\omega}$ is getting bigger.

If $\vec{\omega}$ is slowing down, then $\vec{\alpha}$ points opposite to $\vec{\omega}$.

Derivation of angular momentum from linear momentum

In linear motion, we have $\vec{p} = m\vec{v}$

and $\sum \vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$ ①

If $\sum \vec{F} = 0$, no net force, then $\frac{d\vec{p}}{dt} = 0$,
and momentum is conserved.

$$\vec{p}_i = \vec{p}_f$$

Is there an equivalent momentum for rotations?

In a rotating system, $\vec{L} = \vec{r} \times \vec{p}$

start with
eg ①
we take " $\vec{r} \times$ "
of both
sides:

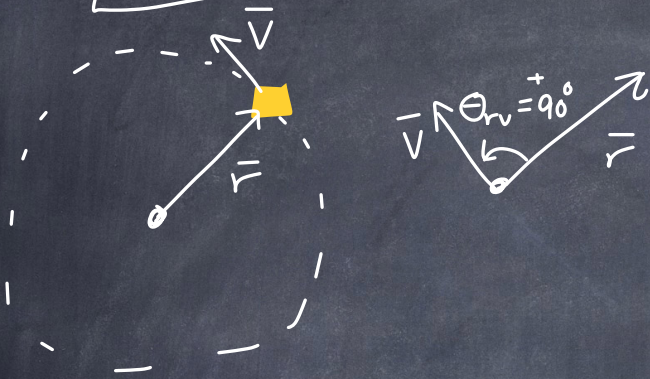
$$\sum (\vec{r} \times \vec{F}) = \frac{d(\vec{r} \times \vec{p})}{dt}$$

$$\sum \vec{L} = \frac{d\vec{L}}{dt}$$

where we define
 $\vec{L} = \vec{r} \times \vec{p}$

as the
angular
momentum.

Angular momentum $\vec{L} = \vec{r} \times \vec{p}$



$$\vec{L} = \vec{r} \times (m\vec{v}) = m(\vec{r} \times \vec{v})$$

In a circle, $\vec{v} \perp \vec{r}$

$$\vec{r} \times \vec{v} = rv \sin \theta_{rv} = rv \sin 90^\circ = rv$$

so here, angular momentum
for an object moving in a circle is

$$L = mvr$$

Now we also know that $v = \omega r$

Therefore, $L = m(r\omega)r = \underbrace{mr^2}_{I} \omega$

$$L = I\omega$$

so:

$$\vec{L} = I \vec{\omega}$$

$\swarrow$ momentum (angular) $\uparrow$ inertia (rotational) $\nwarrow$ Velocity (angular)



$$\vec{p} = m \vec{v}$$

$\swarrow$ momentum (linear) $\uparrow$ inertia (mass) $\nwarrow$ velocity (linear)



Direction of angular momentum
of the second hand of SBB
clock?

If there are no external forces, then

$$\sum (\vec{r} \times \vec{F}) = 0 = \sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

then $\frac{d\vec{L}}{dt}$ is also 0, so then $\vec{L}$ is constant.

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

conservation of angular momentum
when there are no external
torques.

so $\bar{L}$ is conserved like $\bar{p}$

$$\bar{L} = I \bar{\omega}$$

this means that if we change I , then ω must also change ω for $\bar{L}$ to remain constant.

$$I = MR^2$$

For the weights



I is big



I is small

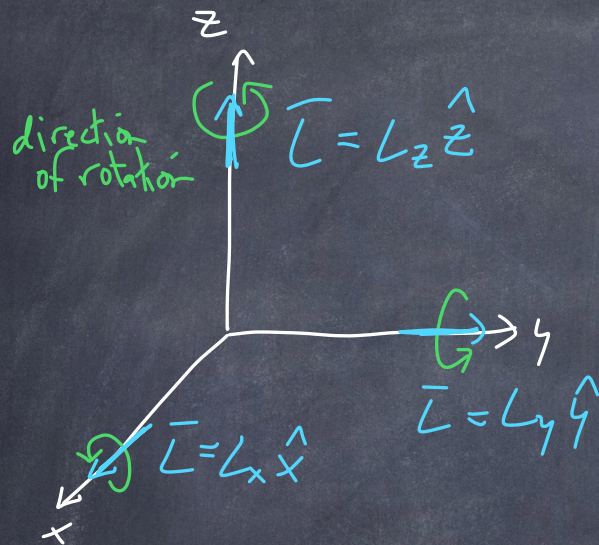


$$\underset{\substack{\uparrow \\ \text{same}}}{L} = \underset{\substack{\uparrow \\ \text{decreases}}}{I} \underset{\substack{\uparrow \\ \text{increases}}}{\omega}$$

Objects can spin around 3 axes.

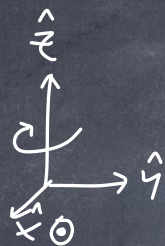
so $\vec{L}$ can be

$$\vec{L} = L_x \hat{x} + L_y \hat{y} + L_z \hat{z}$$



Angular momentum
must be conserved in all
3 directions, independently.

we have a chair that can spin only around z -axis



Now, I hold wheel with angular momentum in x -direction



then, I rotate wheel to have L_z component.



What happens?

At ①, $L_z = 0$

At ②, wheel has L_z

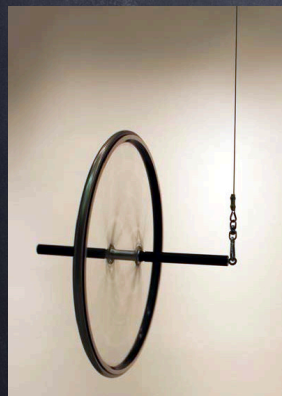
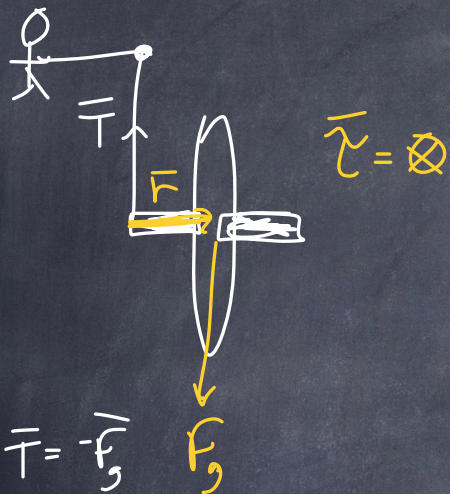
But between ① + ②, L_z must be conserved.

This means at ②, I must start rotating in the opposite direction.



initial	final
①	②
$L_z = 0$	$L_z = L_{z_{\text{wheel}}} + L_{z_{\text{me}}}$
	so $L_{z_{\text{wheel}}} = -L_{z_{\text{me}}}$

precession

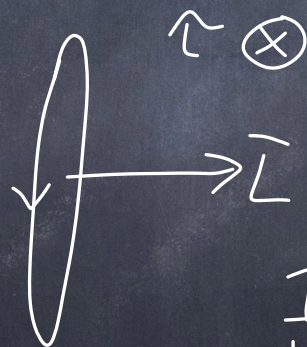


wheel does not fall when spinning

Remember that $\vec{\tau} = \frac{d\vec{L}}{dt}$

when not spinning, $\vec{L} = \vec{r} \times \vec{F} = rMg$
 Causes wheel to fall $\uparrow$
mass of wheel

But if we have a spinning wheel, and apply the same torque, the wheel precesses (rotates)



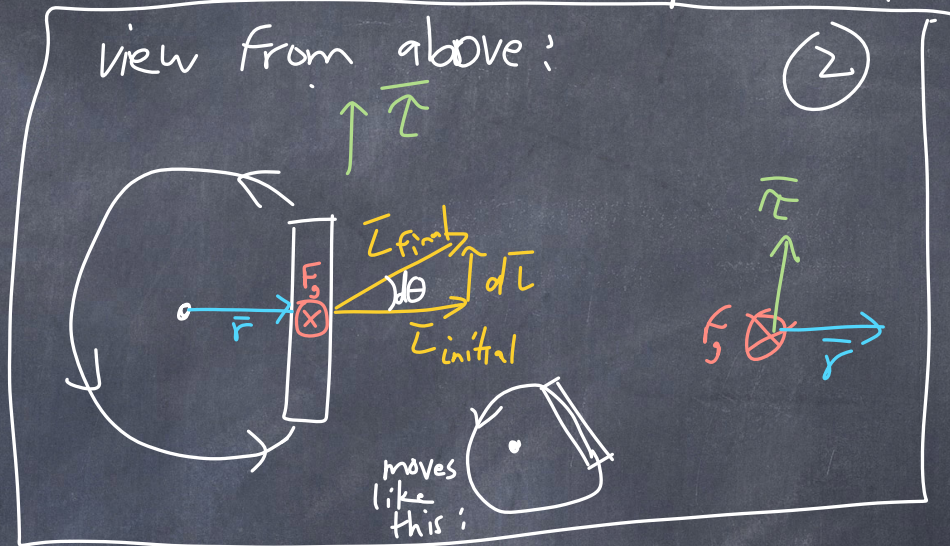
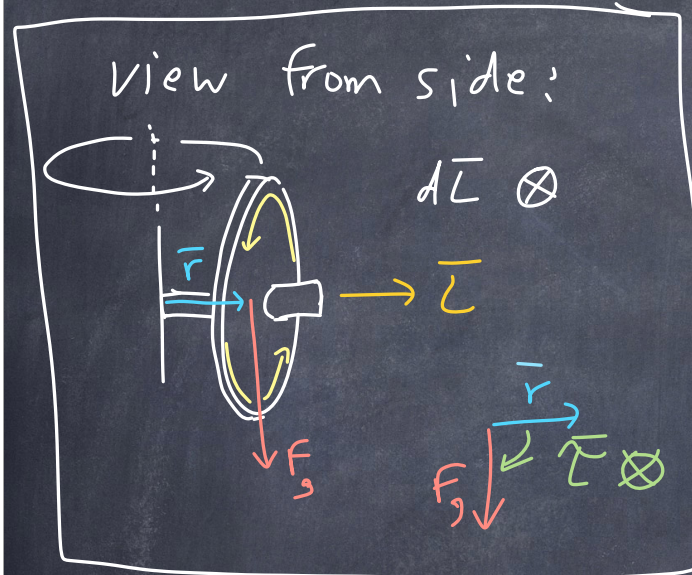
Since $\vec{\tau} = \frac{d\vec{L}}{dt}$

then $d\vec{L} = \vec{\tau} dt$

If we have a torque, then we have change the angular momentum.

The direction of $d\vec{L}$ is in the direction of $\vec{\tau}$.

The amount of $d\vec{L} = \vec{\tau} dt = rMg dt$ Eq. ①
 we expect $\vec{L}$ to change in the direction of $\vec{\tau}$. This causes a precession.



How Fast does it precess?

From picture ②: $dL = L d\theta \Rightarrow d\theta = \frac{dL}{L}$

From eq. ①: $dL = rMg dt$

so $d\theta = \frac{rMg dt}{L} \Rightarrow \frac{d\theta}{dt} = \frac{rMg}{L}$

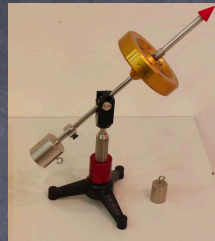
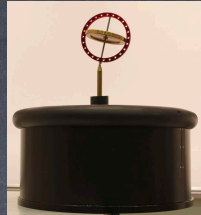
$\frac{d\theta}{dt} \equiv \omega_p$: angular velocity of precession

so $\omega_p = \frac{rMg}{L} = \frac{rMg}{I\omega}$

↑ angular velocity of precession

↑ angular velocity of the spinning wheel

Two angular velocities (or angular frequencies)



Angular momentum, torque, + precession
will come up in NMR + MRI
(we will discuss in PHY 127)

New topic: Fluids

we start by defining pressure
and doing some measurements.

what is pressure?

$$P = \text{pressure} = \frac{\text{force}}{\text{area}}$$

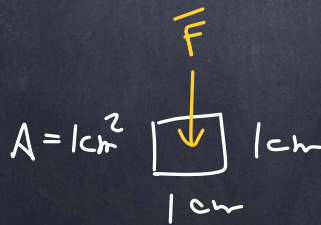
$$\text{units } \left[\frac{\text{N}}{\text{m}^2} \right] \equiv [\text{Pa}]$$

Pascal

$$1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2}$$

The atmospheric pressure at sea level is 101.325 kPa. This is coming from the weight of all the air above us on some area.

Now much weight does the atmosphere put on an area of 1 cm^2 ?

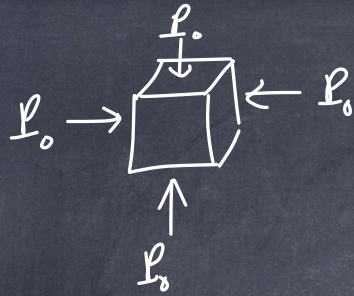


$$P_0 = 101325 \frac{\text{N}}{\text{m}^2}$$

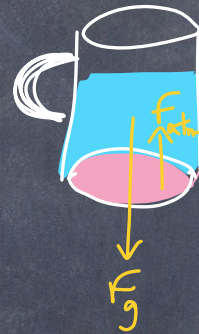
↑
atm.
pressure

$$F = P_0 A = \left(101325 \frac{\text{N}}{\text{m}^2} \right) (1 \text{ cm}^2) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^2$$

$$F = 10 \frac{\text{N}}{\text{cm}^2} \Rightarrow 1 \text{ kg of mass per cm}^2$$



Pressure is not a vector.
 Pressure pushes in all directions
 with the same value (at a given height)
 Atmospheric pressure even points up.



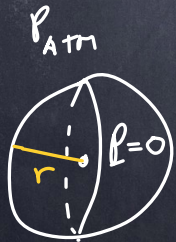
$$\text{Disk } A = \pi r^2 = 75 \text{ cm}^2$$

$$F_{\text{ATM}} = P_0 \cdot A = \frac{10 \text{ N}}{\text{cm}^2} \cdot 75 \text{ cm}^2 = 750 \text{ N}$$

$$F_g = (m_{\text{water}})(g) = (0.5 \text{ kg})\left(\frac{10 \text{ m}}{\text{s}^2}\right) = 5 \text{ N}$$

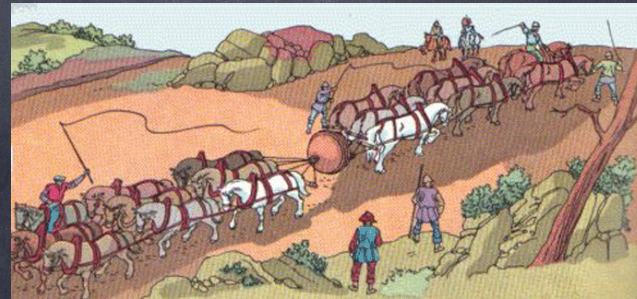
$$F_g \ll F_{\text{ATM}}$$

Area of sphere:



$$F = \Delta P A = (P_{\text{ATM}} - P_{\text{inside}}) \pi r^2$$

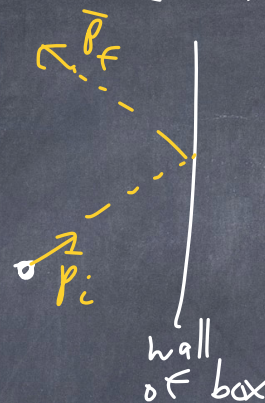
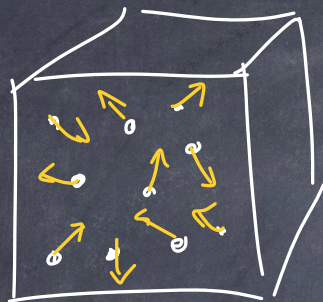
for $r = 5 \text{ cm}$, $F \approx P_{\text{ATM}} \cdot \pi \cdot (5 \text{ cm})^2$
 $F \approx 800 \text{ N}$ or 80 kg of weight



1654

Magdeburg
 hemisphere's
 experiment

where does pressure come from?



$\vec{p}_i$: initial momentum of molecule before it hits the box

$\vec{p}_f$: final momentum

There is a change in momentum:

$$\underbrace{\Delta \vec{p}}_{\text{impulse}} = \vec{p}_f - \vec{p}_i$$

remember

$$F = p \cdot A$$

$\Rightarrow$

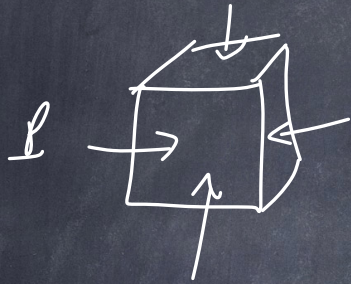
$$\vec{F} = \underbrace{\frac{\Delta \vec{p}}{\Delta t}}_{\text{impulse}} = \underbrace{p}_{\text{pressure}} \cdot A$$

Careful:
p: momentum
p: pressure

For N molecules, $F = N \frac{\Delta p}{\Delta t} = p \cdot A$

pressure depends on momentum (velocity) of molecules

If we apply a pressure to a substance of volume V , it gets compressed by some amount ΔV .



we can define the Bulk modulus, B , to describe how much a substance resists compression.

$$B \equiv \frac{-P}{\frac{\Delta V}{V}}$$

If ΔV is large, then B is small.

The $(-)$ sign keeps B positive. (when you compress, ΔV is $(-)$)

gases: highly compressible

$\frac{\Delta V}{V} \rightarrow B$ very small (depends on temp.)

liquids & solids: B is large because it is hard to compress them.

materials

iron
lead
water
air

B [GPa = 10^9 Pa]

100

8

2

~ 0.0001