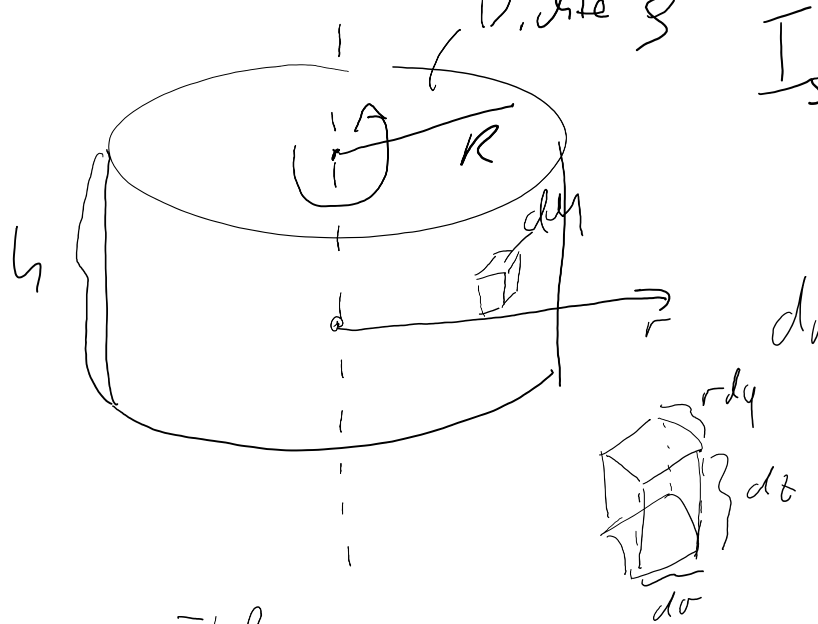
Trägheitsmomente

Zylinder



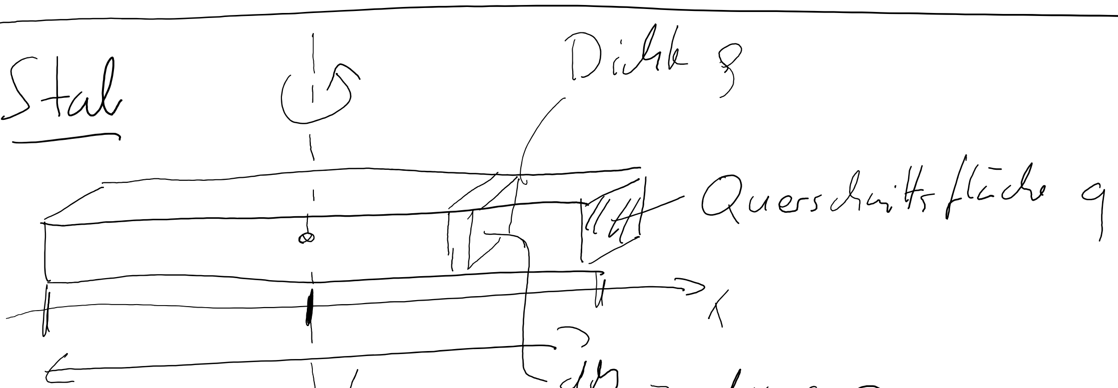
$$I_S = \int_Z r^2 \, dM$$

$$dM = \rho \, r \, d\varphi \, dz \, dr$$

$$I_S = \int_0^{2\pi}\int_0^{h}\int_0^{R} r^2 \rho \, r \, dr \, d\varphi \, dz = 2\pi h \rho \int_0^R r^3 \, dr = \frac{2\pi h \rho R^4}{4}$$

$$= \underbrace{\pi R^2 h \cdot \rho}_{M} \cdot \frac{R^2}{2} = M \frac{R^2}{2}$$

Stab



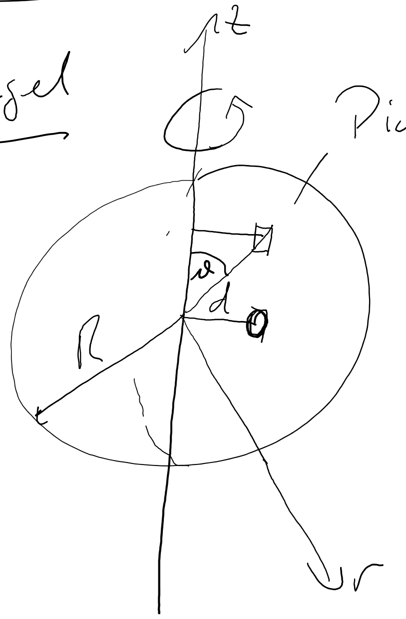
$$dM = dx \cdot q \, \rho$$

$$I_S = \int x^2 \cdot dM = \int^{L/2} x^2 \, dx \, q \rho = q\rho \left. \frac{x^3}{3} \right|^{L/2}$$

$$I_S = \int_S x^2 \cdot dM = \int_{-L/2} x^2\, dx\, q\rho = q\rho \left.\frac{x^3}{3}\right|_{-L/2}$$

$$= q\rho \cdot \frac{L^3/4}{3} = \underbrace{q\rho \cdot L}_{M} \cdot \frac{L^2}{12} = M \frac{L^2}{12}$$

Kugel



$$d = r \sin\vartheta$$

$$I_S = \int_K d^2 \cdot dM$$

$$dM = \rho \cdot dV = \rho\, \underline{r^2 \sin\vartheta\, dr\, d\varphi\, d\vartheta}$$

$$I_S = \int_0^R \int_0^{2\pi} \int_0^{\pi} \rho r^2 \sin\vartheta\, r^2 \underbrace{\sin^2\vartheta}_{1-\cos^2\vartheta}\, d\varphi\, d\vartheta\, dr \qquad -\sin\vartheta\, d\vartheta = d\cos\vartheta$$

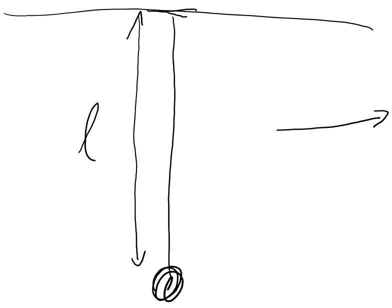
$$= \rho 2\pi \underbrace{\int_0^R r^4\, dr}\ \underbrace{\int_{-1}^{1} (1-u^2)\, du}$$

$$= \rho 2\pi \frac{R^5}{5} \cdot \left(2 - \frac{2}{3}\right) = \frac{\rho 4\pi}{3} R^3 \cdot \frac{2R^2}{5} = \frac{2MR^2}{5}$$

$1/3$ M

physikalisches Pendel

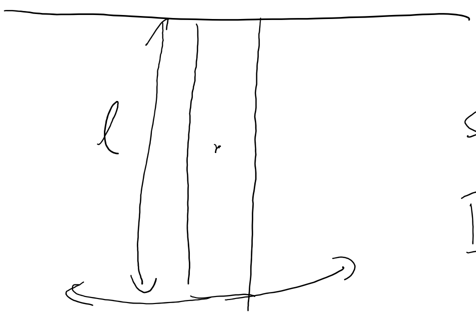
$$\frac{d^2\varphi}{dt^2} = -\frac{Mgs}{I_s}\varphi \qquad \omega = \sqrt{\frac{Mgs}{I_0}} = \sqrt{\frac{Mgl}{ml^2}} = \sqrt{g/l}$$



$\rightarrow$ $s = l$

$I_0 = ml^2$

Stab



$s = l/2$ $\Rightarrow$ $\omega = \sqrt{\frac{Mgl\,3}{2ml^2}}$

$I_0 = \frac{ml^2}{3}$ $= \sqrt{\frac{3}{2}} \cdot \sqrt{g/l}$

kinetische Energie der Rotation

$$E_{pot,Anf} = mgh = E_{rot,End} = \frac{I}{2}\omega^2$$

$\uparrow$

Zylinder; Radius R, Masse M

$I = MR^2/2$

$$mgh = \frac{MR^2}{4}\omega^2 \longrightarrow \omega^2 = \frac{4mgh}{MR^2} = \frac{4gh}{R^2}$$

$m = M = 10\,kg \qquad R = 10\,cm = 0.1\,m \qquad h = 2\,m$

$$\omega = \sqrt{\frac{4 \cdot 10\,m/s^2 \cdot 2\,m}{(0.1\,m)^2}} \cong 2\pi\,\sqrt{2} \cdot 10\,s^{-1}$$

$\nu \cong 14\,Hz$