linearized Gierer-Meinhardt equations:

$$\frac{\partial \delta a}{\partial t} = D\frac{\partial^2 \delta a}{\partial x^2} + \left(\frac{2}{a_0}-1\right)\delta a - \frac{1}{a_0^2}\delta h$$

$$\frac{\partial \delta h}{\partial t} = \frac{\partial^2 \delta h}{\partial x^2} + \mu 2 a_0 \delta a - \mu \delta h$$

$$\boxed{0 = \underbrace{\begin{pmatrix} -\partial_t + D\frac{\partial^2}{\partial x^2} + \left(\frac{2}{a_0}-1\right) & -\frac{1}{a_0^2} \\ 2\mu a_0 & -\partial_t + \frac{\partial^2}{\partial x^2} - \mu \end{pmatrix}}_{\mathcal{L}} \begin{pmatrix} \delta a \\ \delta h \end{pmatrix}}$$

$$\boxed{\begin{pmatrix} \delta a \\ \delta h \end{pmatrix} = \begin{pmatrix} \delta\tilde{a} \\ \delta\tilde{h} \end{pmatrix} e^{i(\omega t - kx)}} \Rightarrow \begin{aligned} \partial_t &\to i\omega \\ \partial_x^2 &\to (-ik)^2 = -k^2 \end{aligned}$$

$$\underbrace{\begin{pmatrix} +i\omega + k^2 D - \left(\frac{2}{a_0}-1\right) & +\frac{1}{a_0^2} \\ -2\mu a_0 & +i\omega + k^2 + \mu \end{pmatrix}} \begin{pmatrix} \delta\tilde{a} \\ \delta\tilde{h} \end{pmatrix} = 0$$

$$\det(\mathcal{L}) = 0$$

$$\left(i\omega + k^2 D - \frac{2}{a_0} + 1\right)\left(i\omega + k^2 + \mu\right) + 2\mu/a_0 = 0$$

$$\boxed{(i\omega)^2 + i\omega\underbrace{\left(k^2+\mu+k^2D-\frac{2}{a_s}+1\right)}_{\alpha} + \underbrace{\left(k^2D-\frac{2}{a_s}+1\right)(k^2+\mu) + \frac{2\mu}{a_s}}_{\beta} = 0}$$

$$\underbrace{k^4D + k^2\left(D\mu-\frac{2}{a}+1\right)+\mu}_{\beta}$$

$\Rightarrow$ Instability condition: if $\boxed{Re(i\omega) > 0}$ then perturbation grows

$$\boxed{i\omega = \frac{-\alpha \pm \sqrt{\alpha^2-4\beta}}{2}} \Rightarrow \omega(k)$$

if $\boxed{i\omega \text{ is complex}}$ $\Rightarrow$ $Re(i\omega) = -\alpha/2 > 0 \Rightarrow \alpha < 0$

$$k^2(D+1)+\mu+1-\frac{2}{a_s} < 0$$

$$\left(\frac{2\pi}{\lambda_{crit}}\right)^2 = k_c^2 < \left(\frac{2}{a_s}-\mu-1\right)/(D+1)$$

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if $i\omega$ is real $\Rightarrow$ $-\alpha+\sqrt{\alpha^2-4\beta} > 0$

$\Leftrightarrow$ $\alpha^2-4\beta > \alpha^2$ $\Rightarrow$ $\beta < 0$

$$k_c^4D + k_c^2\left(D\mu-\frac{2}{a_s}+1\right)+\mu = 0$$

$\Leftrightarrow$ $\boxed{k_c = \ldots}$

$e^{ikx} \underset{k=i\alpha}{\Rightarrow} e^{-\alpha x}$
