MMP I
Tutorial 9

HS 2017
Prof. M. Grazzini

S. Devoto, J. Yook, J. Mazzitelli
http://www.physik.uzh.ch/en/teaching/PHY312/HS2017.html

Issued: 14.11.2017
Due: 21.11.2017 15:00

Exercise 1: Orthonormal bases (5 Pts.)
Consider the Hilbert space $H = L^2[0, +\infty)$. Define the vectors $x_0(t) = \exp\{-\frac{1}{2}t\}$ and $x_i(t) = tx_{i-1}(t)$.

Show that the x_i are linearly independent. Argue that $\text{Span}\{|x_i\rangle\}$ is dense in H. Use the Gram-Schmidt procedure to build an orthonormal basis. Explicitly construct the first four orthonormal basis vectors and show that they lead to the Laguerre polynomials.

Exercise 2: Dual Space (2 Pts.)
Show that the dual space of a normed space is a Banach space.

Exercise 3: Closed subspaces (2 Pts.)
Let H be a Hilbert space and $X \in H$ a closed subspace. Show that for every $h \in H$ the projection of h over X is unique.

Exercise 4: Bounded linear operators (2 Pts.)
If T is a bounded linear operator on a Hilbert space, prove that $||TT^\dagger|| = ||T^\dagger T|| = ||T||^2 = ||T^\dagger||^2$.

Exercise 5: Operators (3 Pts.)
Consider the vector space $X = C[0, 1]$ of the continuous functions $f : x \to f(x)$ over the interval $[0, 1]$, and the norm $||f|| = \sup_{t\in[0,1]} |f(t)|$.
Consider the operator on X defined as $(Af)(x) = xf(x)$. Show that it is bounded and compute its norm.