The Higgs as a pseudo-Goldstone boson

Recall: the SM Higgs Lagrangian can be written as

$$\mathcal{L}_H = \frac{1}{4} \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger D^\mu \Sigma\right] + \frac{\mu^2}{2} \mathrm{Tr}\left[\Sigma^\dagger \Sigma\right] - \frac{\lambda}{4} \mathrm{Tr}\left[\Sigma^\dagger \Sigma\right]^2 ,$$

with $\Sigma = (h+v)\,\mathcal{U}$, $\mathcal{U} = \exp\left(i \frac{\sigma^a \pi^a}{v}\right)$

π^a are the Nambu-Goldstone bosons of the $SO(4) \longrightarrow SO(3)$ breaking ($\text{or } SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{L+R}$) $\rightsquigarrow$ longitudinal polarisations of the massive W and Z bosons.

$$\Sigma \longrightarrow g_L \cdot \Sigma \cdot g_R^\dagger \quad , \qquad g_{L,R} \equiv \exp\left(i \frac{\alpha^a_{L,R}\, \sigma^a}{2}\right)$$

the Goldstone fields transform non-linearly under the full $SU(2)_L \times SU(2)_R$:

$$\pi^a \longrightarrow \pi^a + \frac{v}{2}\left(\alpha^a_L - \alpha^a_R\right)$$

Higgs v.e.v. $\langle \Sigma \rangle = v \cdot \mathbb{1}$

EW symmetry breaking in the Higgs sector is a global $SO(4) \longrightarrow SO(3)$ ...

The "custodial" $SO(4)$ symmetry is explicitly violated by the gauge coupling g' and the Yukawa couplings $y_u^{ij} \neq y_d^{ij}$.

In the limit $g' \to 0$, $m_W = m_Z$, and the π^a transform as triplets of the unbroken $SO(3)$.

(an additional term $\mathrm{Tr}\left[\Sigma^\dagger D_\mu \Sigma\, \sigma_3\right]^2$ preserves the gauge $SU(2)_L \times U(1)_Y$ but violates $SU(2)_{L+R}$, and is excluded from the Lagrangian ... its presence would violate the relation $m_W = m_Z \sin\theta_W$ which is experimentally verified)

At energies $E \lesssim m_h$, the Higgs particle h can be removed from the effective Lagrangian. EW symmetry breaking can be described without introducing a Higgs boson: <u>non-linear sigma-model</u>

$$\mathcal{L}_{\cancel{Higgs}} = \frac{v^2}{4} \mathrm{Tr}\left[D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U}\right] + \bar{q}^i_L\, \mathcal{U} \begin{pmatrix} y^u_{ij} u^j_R \\ y^d_{ij} d^j_R \end{pmatrix}$$

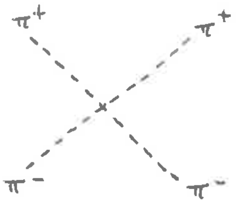
$\equiv$ chiral Lagrangian of QCD

$\hookrightarrow$ quark masses

(actually the QCD condensate breaks EW symmetry, and gives a mass $m_W = g f_\pi / 2 \simeq 29$ MeV to the vector bosons)

<u>Pion scattering</u> : $\mathcal{L}_{\cancel{Higgs}} \supset \frac{1}{v^2} (\pi \partial_\mu \pi)^2$ four-pion interactions

(non-derivative interactions are forbidden by the shift symmetry $\pi \to \pi + v\alpha$ of the Goldstone bosons)

$A(\pi^+\pi^- \to \pi^+\pi^-) =$ 

$= \frac{s+t}{v^2} \approx \frac{E^2}{v^2}$

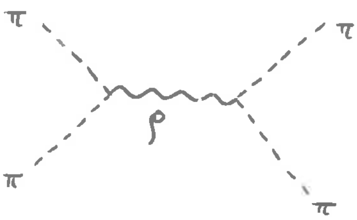
four-pion scattering amplitude grows with energy $\Rightarrow$ at some point the tree-level amplitude will violate the unitarity bound $|A|^2 \leq \mathrm{Im}\, A$

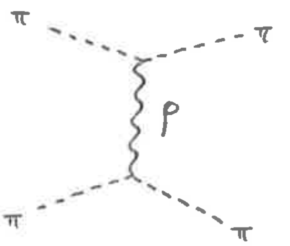
Unitarity can be restored by higher-order contributions

$\simeq \frac{s^2}{v^4} \frac{1}{16\pi^2} \log(s/\mu^2) \times N_\pi$ $\swarrow$ # of Goldstone bosons

$\Rightarrow$ perturbativity is lost when $A_{tree} \approx A_{loop}$, $\boxed{E \simeq 4\pi v / \sqrt{N_\pi}}$

This happens in QCD , where strongly coupled resonances appear at energies below $\Lambda_{QCD} \approx 4\pi f_\pi$.



$+$


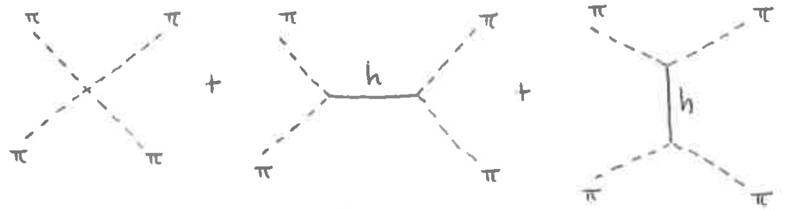
$+ \cdots$ restore unitarity of the scattering amplitude

ρ meson , $m_\rho \simeq 770$ MeV

In the SM, perturbative unitarity is mantained by the exchange of a Higgs boson.

$$\mathcal{L}_{int} = \frac{1}{4}(h^2 + 2vh)\,\mathrm{Tr}\left[D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U}\right]$$

$$\mathcal{A}(\pi\pi \to \pi\pi) =$$



$$= \frac{1}{v^2}\left[s + t - \frac{s^2}{s - m_h^2} - \frac{t^2}{t - m_h^2}\right] = \mathcal{O}\left(\frac{m_h^2}{E^2}\right) \xrightarrow[E\to\infty]{} 0$$

Consider the EW Chiral Lagrangian with the addition of a scalar degree of freedom, with generic couplings to the Goldstones:

$$\mathcal{L}_h = \frac{v^2}{4}\mathrm{Tr}\left[D_\mu \mathcal{U}^\dagger D^\mu \mathcal{U}\right]\left\{1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \cdots\right\} + \bar{q}_L \mathcal{U} Y q_R \left\{1 + c\frac{h}{v} + \cdots\right\}$$

$$\mathcal{A}(\pi\pi \to \pi\pi) = \frac{1}{v^2}\left[s + t - a^2\left(\frac{s^2}{s - m_h^2} + \frac{t^2}{t - m_h^2}\right)\right] \simeq \frac{s+t}{v^2}(1 - a^2) + \mathcal{O}\left(\frac{m_h^2}{E^2}\right)$$

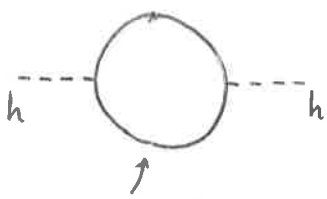
Perturbativity is lost at a scale $\boxed{\Lambda \simeq 4\pi v/\sqrt{1-a^2}}$

- $a = 0$: Higgs boson does not participate in EWSB $(\Lambda \simeq 4\pi v)$
- $a = 1$: Standard Model, (h, π^a) form a $SU(2)_L$ doublet, $\Lambda \to \infty$.
  (the theory is renormalisable!)

For generic $a \neq 1$, the Higgs is part of a sector which becomes strongly interacting at a scale $\Lambda > 4\pi v$: composite Higgs

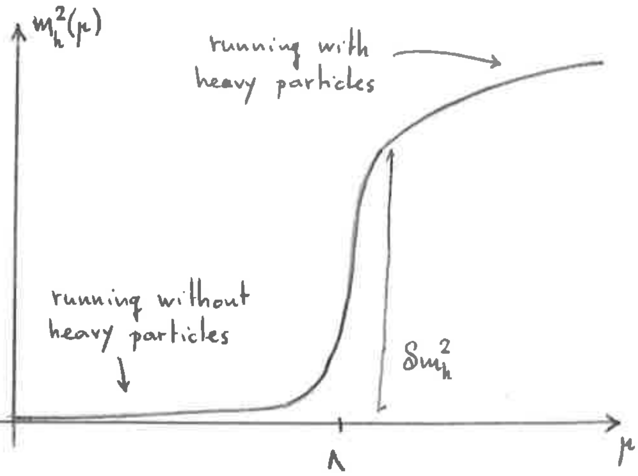
Other composite (ρ-like) resonances complete the unitarisation of the theory below Λ.

Hierarchy problem : the mass of a scalar field gets radiative corrections $\propto \Lambda^2$



$$\delta m_h^2 \simeq \frac{1}{16\pi^2} g_*^2 \Lambda^2$$

loop of particles with mass $M \simeq \Lambda$, and coupling g_*



$$m_h^2(\mu = m_h) = m_h^2(\mu = \Lambda) + \delta m_h^2$$

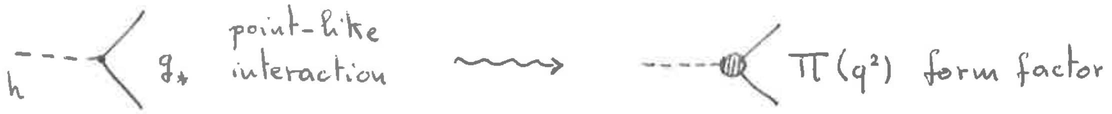
$\uparrow$ $(125\ \text{GeV})^2$ $\qquad$ $\nwarrow \propto \Lambda^2$

Quadratic sensitivity of m_h^2 on the scale of new physics: if Λ is large, a very precise cancellation has to take place between δm_h^2 and $m_h^2(\Lambda)$: fine-tuning

There is no symmetry that enforces $m_h \simeq 0$ $\Rightarrow$ renormalisation is not multiplicative, $\delta m_h^2 \not\propto m_h^2$
(fermions $\rightarrow$ chiral symmetry ; vectors $\rightarrow$ gauge symmetry)

In general: corrections for any threshold Λ coupled to the Higgs.

Composite Higgs: no contribution above the compositeness scale $\Lambda_{comp.}$
(the Higgs is not a physical degree of freedom at energies $E > \Lambda_{comp.}$)



$\Pi(q^2) \longrightarrow 0$ for $q^2 \gg \Lambda^2$ $\Rightarrow$ loop integral is cut-off at Λ

Hierarchy problem is solved if Λ is close to $4\pi v$.

Experimental constraints:

- other composite states expected below Λ (like in QCD)
- modification of Higgs couplings (experimentally $a \simeq 1 \pm 20\%$ ...)
- modification of other SM amplitudes (e.g. vector bosons, EW precision tests)

$\rightarrow$ Higgs boson is naturally lighter than other composite states if it is a Nambu-Goldstone boson of some extended symmetry breaking

$$\begin{array}{ccc} G & \longrightarrow & H \\ \cup\!\shortmid & & \cup\!\shortmid \\ SO(4) & & SO(3) \end{array} \quad : \quad \#\ \text{of Goldstones} = \dim(G) - \dim(H)$$

G/H coset should contain at least the EW $SO(4)/SO(3)$ Goldstone bosons + 1 broken generator $\rightsquigarrow$ Higgs.

Shift symmetry of the Goldstones $h \rightarrow h + \alpha$ forbids any potential $V(h)$ including the mass term $m_h = 0$: light Higgs!

Minimal Composite Higgs Model

$SO(5) \longrightarrow SO(4)$ $\quad$ $\dim(SO(N)) = \frac{N(N-1)}{2}$ $\Rightarrow$ $10-6=4$ Goldstones

(4 components of the Higgs field)

The Goldstone bosons transform as a 4-plet of the unbroken $SO(4)$ ✓

$(H_1, H_2, H_3, H_4) = H$

$$\Sigma = \exp\left(\frac{i t^a H^a}{f}\right) \Sigma_0 \quad , \quad \Sigma_0 \equiv (0,0,0,0,1)$$

(arbitrary, identifies the $SO(4) \subset SO(5)$)

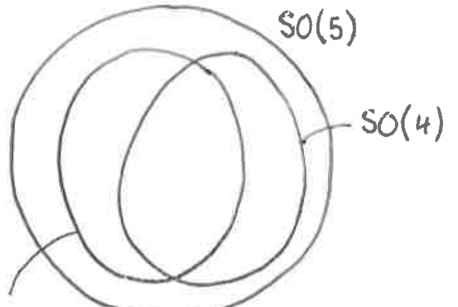
t^a generators of $SO(5)/SO(4)$

f scale of the $SO(5) \to SO(4)$ breaking

$$\Sigma = \Big(\underbrace{\sin\frac{H}{f} \cdot \hat{H}}_{\text{SM Higgs}}, \cos\frac{H}{f} \Big), \quad \text{with} \quad \hat{H} \equiv \frac{H_i}{H}, \quad H \equiv \sqrt{H_1^2 + H_2^2 + H_3^2 + H_4^2}$$

If $H = 0$ $\longrightarrow$ SM global $SO(4)$ $\equiv$ unbroken $SO(4) \subset SO(5)$.

If H has a v.e.v. $\longrightarrow$ misalignement between the unbroken $SO(4) \subset SO(5)$ and the SM $SO(4)$ ... rotation angle v/f.



$SO(4) \cong SU(2)_L \times SU(2)_R \subseteq SU(2)_L \times U(1)_Y$

One can easily estimate the Higgs couplings to vector bosons from the Chiral Lagr.

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma\right] \supseteq \frac{f^2}{4} g^2 \sum_{a,b=1}^{4} W_\mu^a W_\mu^b \,\mathrm{Tr}\left[\Sigma^\dagger t^a t^b \Sigma\right] = \frac{1}{2} \frac{g^2 f^2}{4} W_\mu^a W_\mu^a \sin^2\frac{H}{f}$$

H gets a v.e.v. : $\quad H = \langle H \rangle + h$

$$\sin^2\frac{\langle H\rangle + h}{f} = \left(\sin\frac{\langle H\rangle}{f} \cos\frac{h}{f} + \cos\frac{\langle H\rangle}{f} \sin\frac{h}{f} \right)^2$$

$$= \sin^2\frac{\langle H\rangle}{f} + 2 \sin\frac{\langle H\rangle}{f} \cos\frac{\langle H\rangle}{f} \times \frac{h}{f} + \left(1 - 2\sin^2\frac{\langle H\rangle}{f}\right) \times \frac{h^2}{f^2} + \dots$$

$\hookrightarrow$ mass of vector bosons $\quad m_W^2 = \frac{g^2 f^2}{4} \sin^2\frac{\langle H\rangle}{f} \Rightarrow \boxed{v^2 = f^2 \sin^2\frac{\langle H\rangle}{f}}$

Higgs couplings : $\quad v^2 \left\{ 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \cdots \right\}$

$f^2 \cdot 2 \sin \frac{\langle H \rangle}{f} \cos \frac{\langle H \rangle}{f} \cdot \frac{h}{f} = 2v \sqrt{1 - \frac{v^2}{f^2}}\, h \qquad$ define $\quad \xi \equiv \frac{v^2}{f^2} = \sin^2 \frac{\langle H \rangle}{f}$.

$a = \sqrt{1-\xi} \quad , \quad$ similarly $\quad b = 1 - 2\xi$

$\lim_{f \to \infty} \xi = 0$, and $a, b \xrightarrow[f \to \infty]{} 1$ (decoupling limit = SM) $\qquad \Lambda = 4\pi v / \sqrt{\xi} = 4\pi f$

ElectroWeak Precision Tests

Quadratic action for gauge fields (including effects from integrating out strongly interacting physics)

$$\mathcal{L} = \frac{1}{2} \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left\{ \Pi_{00}(q^2) B_\mu B_\nu + \Pi_{33}(q^2) W_\mu^3 W_\nu^3 + 2\Pi_{30}(q^2) W_\mu^3 B_\nu + 2\Pi_{WW}(q^2) W_\mu^+ W_\nu^- \right\}$$

expansion in q^2 : $\quad \Pi(q^2) = \Pi(0) + q^2 \Pi'(0) + \cdots$

- massless photon $\Rightarrow \Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0$ (2 conditions)
- $\Pi'_{WW}(0)$ and $\Pi'_{00}(0)$ fixed by gauge couplings ($\Pi'_{WW}(0) = \Pi'_{00}(0) = 1$ if gauge action normalised to $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$)
- $\Pi_{WW}(0) = m_W^2 \longrightarrow$ fixed by v.e.v., or G_F.

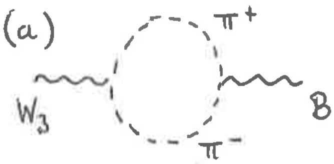
$\Rightarrow$ 3 independent quantities

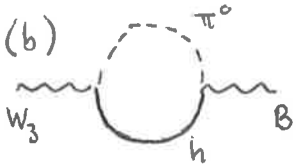
$$\hat{S} \equiv \frac{g}{g'} \Pi'_{30}(0) \qquad \hat{U} \equiv \Pi'_{33}(0) - \Pi'_{WW}(0) \qquad \hat{T} \equiv \frac{\Pi_{33}(0) - \Pi_{WW}(0)}{m_W^2}$$

In the SM no further counterterm is present $\Rightarrow$ <u>finite quantities</u>

<u>Example</u> : $\hat{T}$ measures the breaking of custodial $SU(2)_{L+R}$ (different vacuum polarisations for $W_\mu^\pm$ and W_μ^3) $\longrightarrow$ finite contributions $\propto y_t$ and g' ...

<u>Example</u>: $\hat{S}$ parameter in the SM

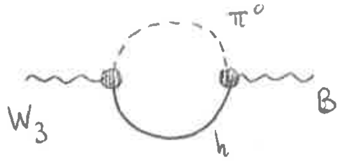
(a) 

$\simeq \frac{G_F m_W^2}{12\sqrt{2}\,\pi^2} \log \frac{\Lambda}{m_W^2}$, (b) 

$\simeq - \frac{G_F m_W^2}{12\sqrt{2}\,\pi^2} \log \frac{\Lambda}{m_h}$

$$\Rightarrow \hat{S}_{SM} = \frac{G_F m_W^2}{12\sqrt{2}\,\pi^2} \log (m_h / m_W)$$

$\hat{S}$ parameter in composite Higgs :



$\simeq (b) \times a^2$

modified Higgs coupling

$$\Rightarrow \quad \hat{S} = (a) + (b) \times a^2 = \hat{S}_{SM} + (a^2 - 1) \times (b) = \hat{S}_{SM} + \boxed{\frac{G_F m_W^2}{12\sqrt{2}\,\pi^2} (1 - a^2) \log \frac{\Lambda}{m_W^2}}$$

Dependence on the cut-off is reintroduced, proportional to $1 - a^2$ ...
(similarly for $\hat{T}$ ...)

Minimal Composite Higgs : vacuum polarisations can be written in $SO(5)$-invariant way

$$\mathcal{L} = \frac{1}{2}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Big\{ \hat{\Pi}_B(q^2) B_\mu B_\nu + \hat{\Pi}_W(q^2) W_\mu^a W_\nu^a + \tilde{\Pi}_1(q^2)\Big[W_\mu^a W_\nu^b \, Tr[\Sigma^\dagger \sigma^a \sigma^b \Sigma] +$$
$$+ 2 W_\mu^a B_\nu \, Tr[\Sigma^\dagger \sigma^a \sigma^3 \Sigma] + B_\mu B_\nu \, Tr[\Sigma^\dagger \Sigma] \Big] \Big\}$$

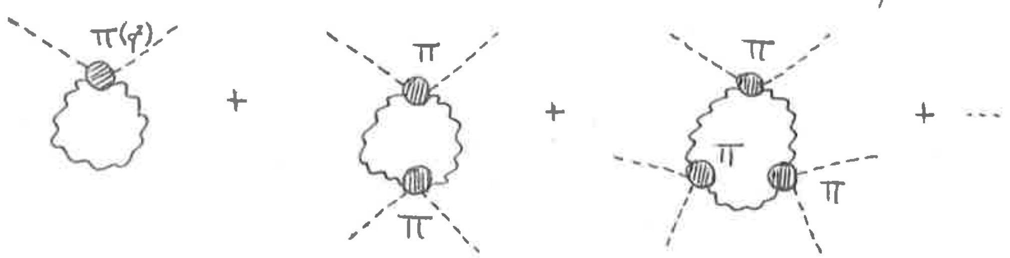
$$= \frac{1}{2}\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Big\{ \Big(\hat{\Pi}_B(q^2) + \tilde{\Pi}_1(q^2) \sin^2 \frac{H}{f}\Big) B_\mu B_\nu + \Big(\hat{\Pi}_W(q^2) + \tilde{\Pi}_1(q^2) \sin^2 \frac{H}{f}\Big) W_\mu^a W_\nu^a$$
$$+ 2 \tilde{\Pi}_1(q^2) \sin^2 \frac{H}{f} \left(\hat{H}^\dagger \sigma^a Y H\right) W_\mu^a B_\nu \Big\}$$

$\hat{\Pi}_{B,W}(q^2)$ come from UV effects due to strong interactions (composite resonances)
($\hat{\Pi}_B(0) = \hat{\Pi}_W(0)$ at leading order, if custodial symmetry is preserved)
$\tilde{\Pi}_1(q^2)$ comes from the Nambu-Goldstone bosons (fluctuations around Σ_0)

Higgs potential

If $SO(5)$ symmetry is exact, $V(h) \equiv 0$, and $m_h = 0$.

The SM couplings explicitly violate the $SO(5)$ symmetry $\Rightarrow$ they induce a non-vanishing Higgs mass and potential @ loop level.



from loop of gauge fields

Coleman - Weinberg potential :

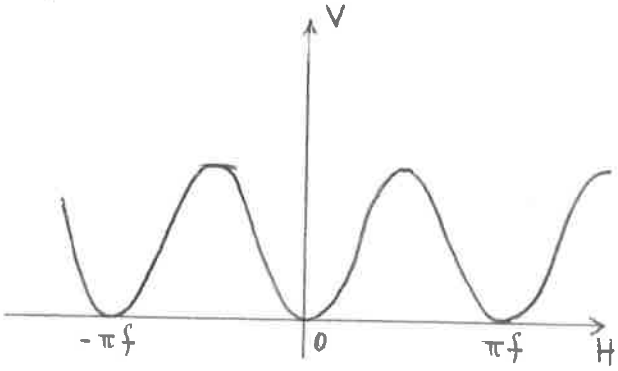
$$V(H) \propto \int \frac{d^4q}{(2\pi)^4} \log\left(\hat{\Pi}_w(q^2) + \tilde{\Pi}_1(q^2) \sin^2\frac{H}{f} \right)$$ resumming all the W- loops

$V(H)$ is <u>finite</u> because the form factors $\Pi(q^2) \xrightarrow[q^2\to\infty]{} 0$ "fast enough"

$$V(H) \propto \text{const.} + \int \frac{d^4q}{(2\pi)^4} \log\left(1 + \frac{\tilde{\Pi}_1(q^2)}{\hat{\Pi}_w(q^2)} \sin^2\frac{H}{f} \right)$$

$$\approx \text{const.} + \alpha_1 \sin^2\frac{H}{f} + \alpha_2 \sin^4\frac{H}{f} + \cdots$$

from form factors



V is a function of $\sin^2 H/f$

$\Rightarrow$ can not break EW symmetry with $v \ll f$

(problem of <u>vacuum misalignment</u>)

A <u>tuning</u> with a different contribution to the potential is needed

<u>Example</u> : loops of top quark can give the right contribution

$+ \cdots$

$$V(H) = \alpha \cos\frac{H}{f} - \beta \sin^2\frac{H}{f} \qquad (\alpha, \beta \text{ from strong dynamics})$$

$$\frac{\partial V}{\partial H} = \frac{1}{f}\left[-\alpha \sin\frac{H}{f} - 2\beta \sin\frac{H}{f}\cos\frac{H}{f} \right] = 0 \iff \alpha + 2\beta\cos\frac{\langle H\rangle}{f} = 0$$

$$\xi \equiv \frac{v^2}{f^2} = \sin^2\frac{\langle H\rangle}{f} = 1 - \cos^2\frac{\langle H\rangle}{f} = 1 - \left(\frac{\alpha}{2\beta}\right)^2 \ll 1$$

<u>tuning</u> $\alpha \approx 2\beta$ to have $v \ll f$

larger $f \longrightarrow$ larger tuning (SM is recovered for $f \to \infty$)

<u>References</u>

- Contino, arXiv: 1005.4269
- Barbieri, arXiv: 0706.0684 (Ch.4: EWPT)
- Panico, Wulzer, arXiv: 1506.01961
- Georgi, Kaplan, PLB 136 (1984) 183, and PLB 145 (1984) 216. (pseudo-Goldstone Higgs)
- Agashe, Contino, Pomarol, hep-ph/0412089 (Minimal Composite Higgs)