MMP I
Tutorial 12

HS 2017
Prof. M. Grazzini

S. Devoto, J. Yook, J. Mazzitelli
`http://www.physik.uzh.ch/en/teaching/PHY312/HS2017.html`

Issued: 05.12.2017
Due: 12.12.2017 15:00

Exercise 1: Linear operator in $\mathbb{C}^3$ (8 Pts.)
Consider the linear operator T that is represented by the hermitian matrix in the canonical basis:

$$T = \begin{pmatrix} 3 & i & -4 \\ -i & 3 & 4i \\ -4 & -4i & 18 \end{pmatrix}. \tag{1.1}$$

The matrix T can be diagonalised as $T = U^{-1} \cdot T_d \cdot U$, where

$$U = \begin{pmatrix} \frac{1}{3\sqrt{2}} & \frac{i}{3\sqrt{2}} & -\frac{2\sqrt{2}}{3} \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ \frac{2}{3} & \frac{2i}{3} & \frac{1}{3} \end{pmatrix} \tag{1.2}$$

and T_d is the diagonal matrix with entries $\{20, 2, 2\}$.

a) Compute the trace and the determinant of T.

b) Find the spectral representation of T in the canonical basis, i.e. find the matrices of projection operators P_i in the canonical basis, such that

$$T = \sum_i E_i P_i. \tag{1.3}$$

c) Compute T^{99}, e^T and T^{-1}.

d) Compute the matrix R_T representing the resolvent $R_T(z) = (T - z)^{-1}$ in the canonical basis. For which values of z does the resolvent exist?

– please turn over –

Exercise 2: Integral equations (5 Pts.)
Consider the compact and hermitian operator

$$|x\rangle \xrightarrow{T} T|x\rangle \sim (Tx)(t) = \int_0^1 \mathrm{d}s \left(\frac{1}{3} + st\right) x(s) \tag{2.1}$$

in the Hilbert space $L^2[0,1]$.

a) Find all eigenvalues λ_i and all eigenvectors $|\phi_i\rangle \sim \phi_i(t)$ of T.

b) Solve the inhomogeneous Fredholm integral equation

$$x(t) - 3\int_0^1 \mathrm{d}s \left(\frac{1}{3} + st\right) x(s) = 6t(2-t). \tag{2.2}$$