



Mechanics

Exercise Sheet 1

HS 2026
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Exercise 1: Point mass on wedge

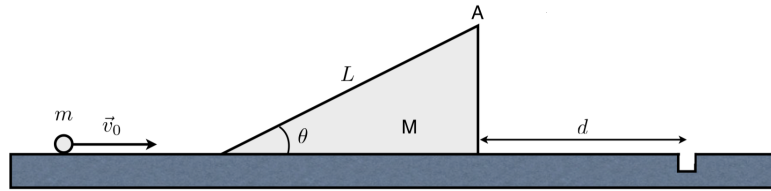


Figure 1: The system in exercise (1).

A wedge of mass M is placed on a smooth horizontal table. The table has a hole at a distance d from the wedge (see Fig. 1). A point-like body of mass m approaches the wedge with velocity $\vec{v}_0$.

- a) Consider the case where the wedge cannot slide on the table. Assuming the wedge itself to be frictionless, compute the minimal initial velocity v_{min} required for the body to reach the highest point of the wedge, A , as a function of L , θ .
How does the situation change if the wedge has a dynamical friction coefficient $\mu > 0$? Compute the corresponding v_{min} .
- b) Set $\mu = 0$. For a given $v_0 > v_{min}$, write the horizontal and vertical components of the velocity of m in A . For which value of v_0 does it fall in the hole? Compute it as a function of L , θ , d .
- c) Now suppose that the wedge is free to slide on the table. Compute again v_{min} as in (a) in absence of friction.

Solution:

- a) The body is acted upon by gravity and by the reaction force exerted by the plane/wedge. The former is conservative; the latter is non-conservative but it does not do any work because it is perpendicular to the motion. The mechanical energy of the particle is therefore conserved. Denoting by $\vec{v}$ the velocity of the particle in A and by E_{in} and E_{fin} its mechanical energy in the initial position and in A , respectively, we have

$$E_{in} = E_{fin} \leftrightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh, \quad (1.1)$$

with $h = L \sin \theta$. To find v_{min} we impose $v = 0$:

$$\frac{1}{2}mv_{min}^2 = mgh \rightarrow v_{min} = \sqrt{2gh} = \sqrt{2gL \sin \theta}. \quad (1.2)$$

In this case mechanical energy is not conserved, since we have a non-conservative force, friction, doing non-zero work along the trajectory. In particular, the variation $\Delta E = E_f - E_i$ of mechanical energy is given by

$$\Delta E = W, \quad (1.3)$$

where $\Delta E = mgh - \frac{1}{2}mv_{min}^2$ and W is the work of friction along the trajectory. Denoting by $\hat{u}$ the unit vector parallel to the wedge in the direction of motion, friction is given by $\vec{R} = -\mu N\hat{u}$, where $N = mg \cos \theta$, and its work is simply

$$W = \vec{R} \cdot \vec{L} = -\mu mgL \cos \theta. \quad (1.4)$$

Substituting this in 1.3 we get

$$mgh - \frac{1}{2}mv_{min}^2 = -\mu mgL \cos \theta, \quad (1.5)$$

hence

$$v_{min} = \sqrt{2gL(\sin \theta + \mu \cos \theta)}. \quad (1.6)$$

Note that this correctly reduces to the result in point (a) for $\mu = 0$.

b) As in (a), the conservation of energy yields the modulus of the velocity:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgh \rightarrow v = \sqrt{v_0^2 - 2gh}. \quad (1.7)$$

Taking axes x, y parallel and perpendicular to the plane (y upwards), the components of $\vec{v}$ in (A) read

$$\begin{aligned} v_x &= v \cos \theta = \sqrt{v_0^2 - 2gh} \cos \theta \\ v_y &= v \sin \theta = \sqrt{v_0^2 - 2gh} \sin \theta \end{aligned} \quad (1.8)$$

The particle is then subject only to gravity and follows a parabolic motion. Taking the origin of the axes to coincide with the right base of the wedge, the trajectory is described by

$$\begin{cases} x(t) = v_x t \end{cases} \quad (1.9)$$

$$\begin{cases} y(t) = h + v_y t - \frac{1}{2}gt^2 \end{cases} \quad (1.10)$$

For the body to follow in the hole we impose $x = d$ and $y = 0$:

$$\begin{cases} d = v_x t \end{cases} \quad (1.11)$$

$$\begin{cases} 0 = h + v_y t - \frac{1}{2}gt^2 \end{cases} \quad (1.12)$$

from which we obtain

$$0 = h + v_y \frac{d}{v_x} - \frac{1}{2}g \frac{d^2}{v_x^2} \quad (1.13)$$

substituting the expressions for v_x and v_y in Eq. 1.8 we obtain

$$v^2 = \frac{gd^2}{2 \cos^2 \theta (h + d \tan \theta)} \quad (1.14)$$

hence

$$v_0 = \sqrt{v^2 + 2gh} = \sqrt{\frac{gd^2}{2 \cos^2 \theta (L \sin \theta + d \tan \theta)} + 2gL \sin \theta} \quad (1.15)$$

- c) We are interested in the limit case where the mass m reaches A at rest with respect to the wedge; in this situation m and M move along x with the same velocity $\vec{V}$. Since we are back to the frictionless case, we can use again conservation laws: the mechanical energy E and the x component of the (linear) momentum $\vec{P}$ of the system body + wedge are conserved, i.e.

$$\begin{cases} P_x^{in} = P_x^{fin} \\ E_{in} = E_{fin} \end{cases} \quad (1.16)$$

$$(1.17)$$

hence

$$\begin{cases} mv_{\min} = (m + M)V \\ \frac{1}{2}mv_{\min}^2 = \frac{1}{2}(m + M)V^2 + mgh \end{cases} \quad (1.18)$$

$$(1.19)$$

Solving the first equation for V we get

$$V = \frac{m}{m + M}v_{\min} \quad (1.20)$$

and substituting the result in the second, we have

$$v_{\min} = \sqrt{2gh \left(\frac{m}{M} + 1 \right)} \quad (1.21)$$

Note that for $M \gg m$ we correctly recover the result obtained in point (a).

Exercise 2: Elastic collision

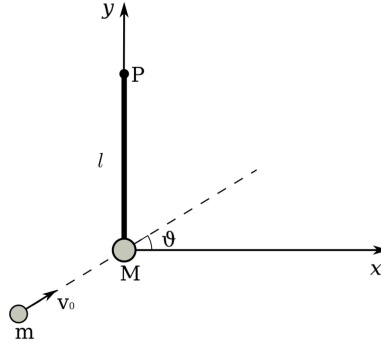


Figure 2: The system in exercise (2).

Consider a body of mass M attached to a rod of length l and negligible mass. The rod is pivoted in P and can rotate without friction in the horizontal xy plane. M is initially at rest. A second body of mass m , also moving in the xy plane, collides with M . The velocity of m before the collision has modulus v_0 and forms an angle θ with respect to the x axis, as shown in figure 2. Assume the angle θ to remain the same before and after the collision.

- Which quantities are conserved in the collision? Why? Write the corresponding conservation laws.
- Write the expressions for the velocity v of m and the angular velocity ω of M immediately after the collision. Discuss the limit cases $\theta = 0$ and $\theta = \pi/2$, and check that for $M > m$ the body of mass m always bounces back.

Solution:

- Since the pivot exerts an impulsive external force $\vec{F}$, linear momentum is not conserved in the collision. The conserved quantities are the kinetic energy (the collision is elastic) and the z component of angular momentum computed with respect to P , $(L_P)_z$ (the torque τ_P of $\vec{F}$ is 0).

The corresponding conservation laws are

$$mv_0 l \cos \theta = mvl \cos \theta + Ml^2 \omega, \quad (2.1)$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}Ml^2\omega^2. \quad (2.2)$$

- To solve the previous system we rewrite it as

$$(v_0 - v) \cos \theta = \frac{M}{m}l\omega, \quad (2.3)$$

$$v_0^2 - v^2 = \frac{M}{m}l^2\omega^2. \quad (2.4)$$

Taking the ratio of the two equations we find $\omega = \frac{v_0 - v}{l \cos \theta}$. Substituting this in the first equation we get an equation in v , with solution

$$v = v_0 \frac{\cos^2 \theta - \frac{M}{m}}{\cos^2 \theta + \frac{M}{m}}, \quad (2.5)$$

from which we derive

$$\omega = \frac{2v_0}{l} \frac{\cos \theta}{\cos^2 \theta + \frac{M}{m}}. \quad (2.6)$$

For $\theta = \pi/2$ the collision occurs parallel to the rigid rod, hence M cannot move. Since the collision is elastic, we expect m to bounce back with the same velocity it had at the beginning. This is indeed what happens. Substituting $\theta = \pi/2$ we get $v = -v_0$ and $\omega = 0$. For $\theta = 0$ instead, the collision occurs perpendicularly to the rod, hence the latter cannot exert any impulsive force. As a consequence linear momentum should also be conserved. Let's check this explicitly. By substituting $\theta = 0$ in the expression above we have

$$v = v_0 \frac{1 - \frac{M}{m}}{1 + \frac{M}{m}}, \quad (2.7)$$

$$\omega = \frac{2v_0}{l} \frac{1}{1 + \frac{M}{m}}. \quad (2.8)$$

from which we find

$$mv + Ml\omega = mv_0. \quad (2.9)$$

Note also that the sign of v is fixed by the sign of $\cos^2 \theta - \frac{M}{m}$; since $\cos^2 \theta$ is at most 1, for $M > m$ this sign is always negative.

Exercise 3: Transformation properties of the force.

Consider Newton's second law for a point like mass

$$\vec{F} := m \frac{d^2}{dt^2} \vec{x}. \quad (3.1)$$

- a) How does the force $\vec{F}$ transform under a general Galilei transformation?
- b) One example of a non Galilei transformation is a transformation to a rotating reference frame $\vec{x}' = \mathbf{U}(t)\vec{x}$. How does the force $\vec{F}$ transform in this case?

Solution:

- a) The position vector $\vec{x}$ has the following general Galilei transformation

$$\vec{x}'(t) = \mathbf{U}\vec{x}(t + t_0) + \vec{v}t + \vec{a} \quad (3.2)$$

for a time translation t_0 , constant vectors $\vec{v}, \vec{a}$ (boost and translation) and a constant orthogonal rotation matrix $\mathbf{U}$. Thus the force transforms as follows (neglecting the arguments),

$$\vec{F}' := m \frac{d^2}{dt^2} \vec{x}'(t) = m \mathbf{U} \frac{d^2}{dt^2} \vec{x}(t + t_0) = \mathbf{U} \vec{F}. \quad (3.3)$$

- b) In this case, the second derivative of the time also acts on the rotation matrix and one gets

$$\vec{F}' = m \ddot{\vec{x}}' = m \frac{d^2}{dt^2} \mathbf{U} \vec{x} = \mathbf{U} \vec{F} + 2m \dot{\mathbf{U}} \dot{\vec{x}} + m \ddot{\mathbf{U}} \vec{x}. \quad (3.4)$$