

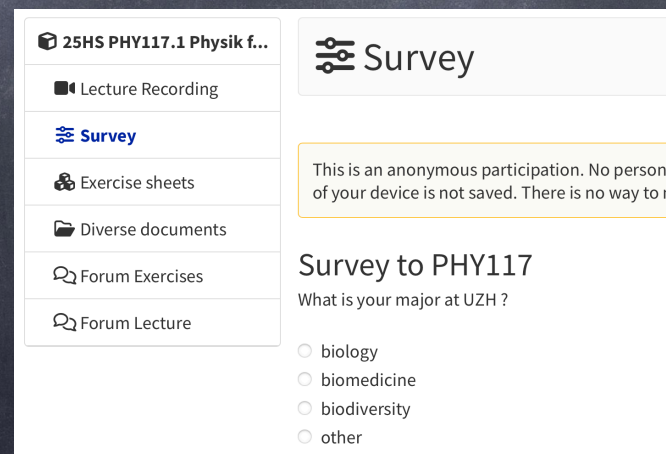
PHY117 HS2026

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Prof. Florencia Canelli
Sept. 16, 2026
Week 1, Lecture 2

Lecture notes here:

<https://www.physik.uzh.ch/de/lehre/PHY117/HS2026.html>

Make sure to complete survey:
[https://lms.uzh.ch/
26HS PHY117.1](https://lms.uzh.ch/26HS PHY117.1)
(takes just a few minutes)



The screenshot shows a user interface for a course. On the left is a sidebar menu with the title '25HS PHY117.1 Physik f...' and several items: 'Lecture Recording', 'Survey' (highlighted with a blue bar), 'Exercise sheets', 'Diverse documents', 'Forum Exercises', and 'Forum Lecture'. On the right, the 'Survey' section is active, displaying a warning: 'This is an anonymous participation. No personal data of your device is not saved. There is no way to m...'. Below this, the survey title 'Survey to PHY117' is shown, followed by the question 'What is your major at UZH ?' and a list of radio button options: 'biology', 'biomedicine', 'biodiversity', and 'other'.

Q: what are these: $\hat{x}$, $\hat{y}$, $\hat{z}$

A: They are special vectors with length 1 called unit vectors

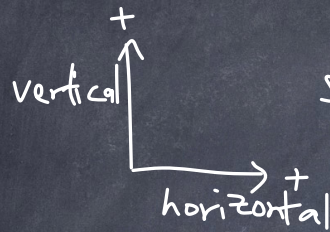


the "A" is called a "hat"

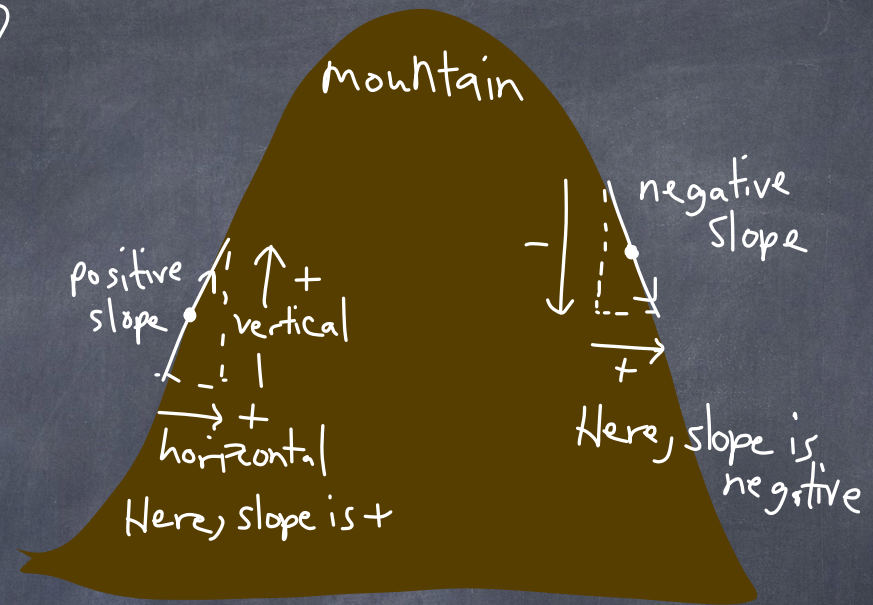
Unit vectors give us a coordinate system to define vectors.

Sometimes, people use $\hat{i}, \hat{j}, \hat{k}$ or $\hat{e}_1, \hat{e}_2, \hat{e}_3$.

Q: what is a slope?



$$\text{slope} = \frac{\text{vertical}}{\text{horizontal}}$$



Q: what is the difference between velocity & speed?

A: velocity is a vector
speed is a scalar

$$\text{velocity} = \vec{v} \begin{pmatrix} \text{magnitude} \\ \text{direction} \end{pmatrix}$$

$$\text{speed} = v = |\vec{v}| \begin{pmatrix} \text{magnitude,} \\ \text{no direction} \end{pmatrix}$$

Example: $\begin{cases} \vec{v} = 3 \frac{\text{m}}{\text{s}} \hat{x} \\ v = 3 \frac{\text{m}}{\text{s}} \end{cases}$

Q: Reminder on derivatives: (useful for today)

start with function $f(x)$	$f(x) = \sin(x)$
derivative is $f'(x) = \frac{d}{dx} f(x)$	$\frac{df(x)}{dx} = f'(x) = \cos(x)$

suppose $f(x) = \sin(kx)$: what is the derivative?

chain rule: if $y = f(u)$ but $u = g(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Here, inside $u = kx$

$$\frac{du}{dx} = k$$

outside: $f(u) = \sin(u)$

$$\frac{df}{du} = \cos(u)$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \cos(u) \cdot k$$

$$\frac{df}{dx} = k \cos(kx)$$

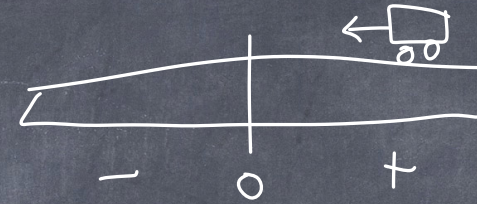
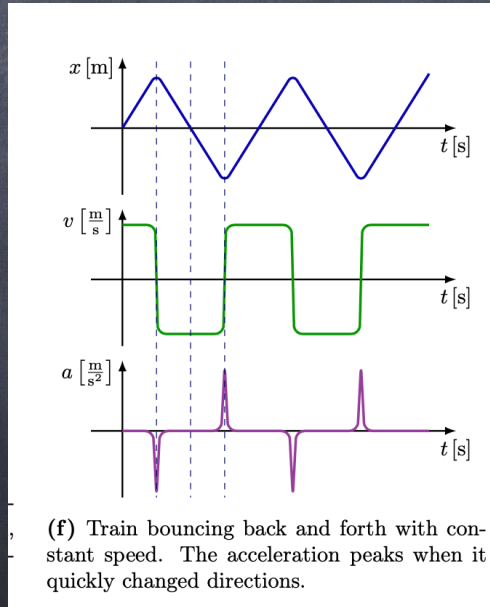
Explanation of yesterday's experiment:



Better diagram
in script

velocity is
the slope of
 x vs. t

acceleration
is the slope
of v vs. t



velocity is + : moving in
+ direction

- : moving in
- direction

$a = 0$: when velocity constant
 $a = (-)$: when velocity decreasing

Here? acceleration is not
constant. It has a peak.
we will discuss later
when we talk about "impulse".

Can we make our air-car accelerate more smoothly?

we can by making use of a "simple harmonic oscillator"
One example is a spring.

The position changes according to the formula

$$x(t) = x_0 \cos(\omega t)$$

↑
starting position

↑
 ω is the "angular frequency" or "angular velocity"

$$\omega = \frac{2\pi}{T}, \text{ where}$$

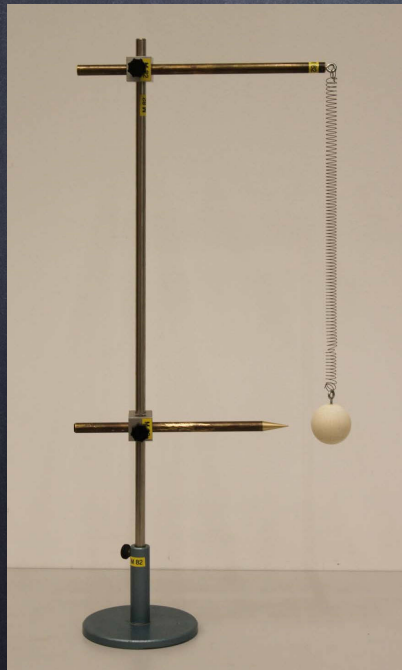
T is the period, the time it takes to do one cycle.

Then
$$v(t) = \frac{dx}{dt} = -\underbrace{x_0 \omega}_{v_0 = x_0 \omega} \sin(\omega t)$$
 v_0 is max speed

$$a(t) = \frac{dv}{dt} = -x_0 \omega^2 \cos(\omega t)$$

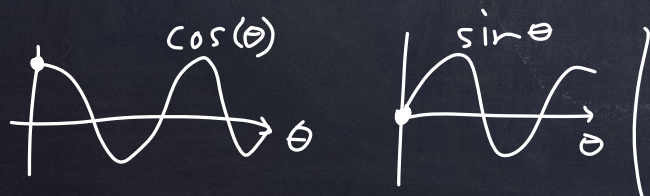
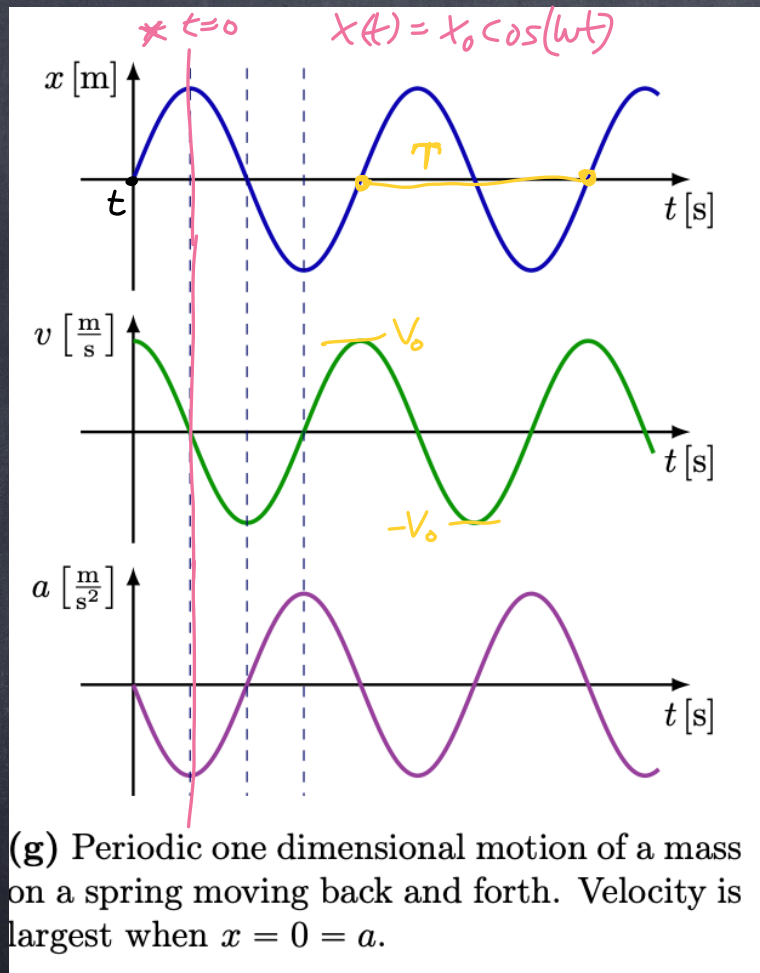
Notice: $a(t) = x(t)(-\omega^2)$

← This we come back to





Can we make our air-car accelerate more smoothly?



$\Rightarrow \phi = \frac{\pi}{2}$, $x(t) = x_0 \sin(\omega t + \frac{\pi}{2}) = x_0 \cos(\omega t)$



$$x(t) = x_0 \sin(\omega t)$$

$$\omega = \frac{2\pi}{T}$$

where x_0 = initial displacement

$$v(t) = V_0 \cos(\omega t)$$

where $V_0 = \omega x_0$ is the maximal speed

$$a(t) = -a_0 \sin(\omega t)$$

where $a_0 = \omega^2 x_0$ is the maximal acceleration.

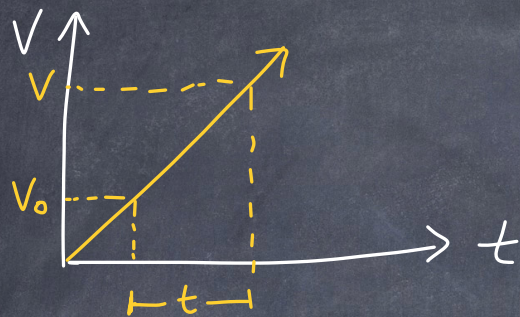
Notice, we could start $t=0$ at a different time. Then we get a phase shift.

$$x(t) = x_0 \sin(\omega t + \phi)$$

ϕ : arbitrary angle.

* Example:
start $t=0$ at $\frac{1}{4}$ of a period late:
 $\frac{1}{4} \cdot 2\pi = \frac{\pi}{2}$ (or 90°)
 $\Rightarrow \phi = \frac{\pi}{2}$, $x(t) = x_0 \sin(\omega t + \frac{\pi}{2}) = x_0 \cos(\omega t)$

Motion with constant acceleration:



$a \equiv$ slope of V vs. t
 $=$ constant (here)

$$a = \frac{dv}{dt} = \frac{\Delta V}{\Delta t}$$

Here: $a = \frac{V - V_0}{t}$ ①

we can determine V if we know V_0 and the slope, a

rewriting
①

$$V = V_0 + at$$

③

final velocity starting velocity

check unit

$$a \cdot t = \frac{m}{s^2} \cdot s = \frac{m}{s}$$

Average velocity is half-way between V_0 and V

$$V_{av} = \frac{1}{2} (V_0 + V)$$

③

Some other useful formulas for constant acceleration
from ③

$$\Delta X = \frac{1}{2}(V_0 + V) t \quad ④$$

$$X = X_0 + V_0 t + \frac{1}{2} a t^2 \quad ⑤$$

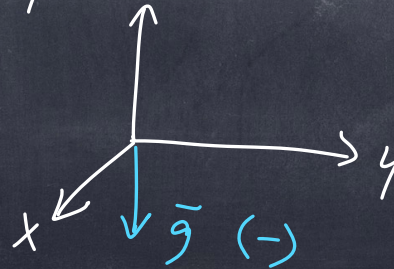
↑ ↑ ↑
final starting starting
position position velocity

we will derive
later once we
do "integrals".

$$V^2 = V_0^2 + 2a \Delta X \quad ⑥$$

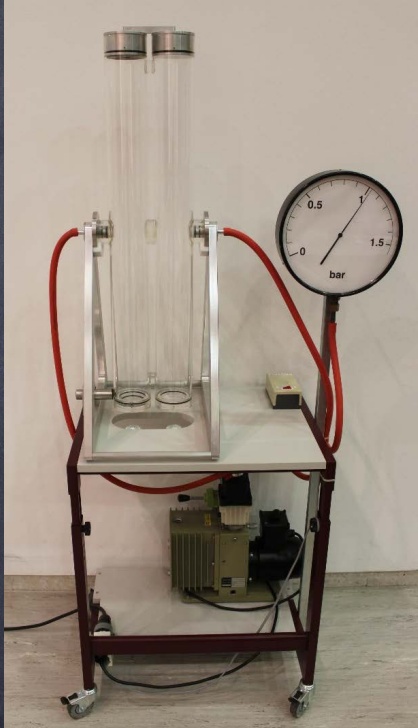
Most of the time, on earth, the acceleration
is due to gravity - $\hat{z}$

$$\vec{a} = -g \hat{z}$$



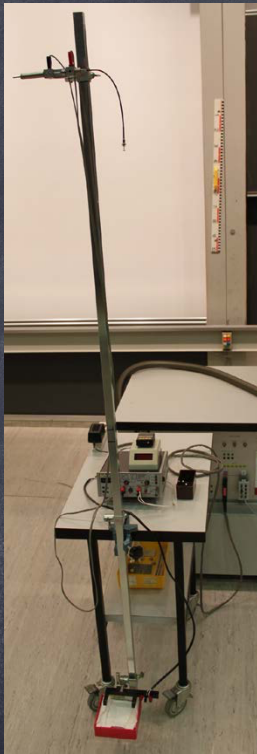
Does gravity accelerate everything the same?

yes (but sometimes other
forces like air
resistance)



what is g ? $\left[\frac{m}{s^2}\right]$

$+z$
↑



→ z_0 : initial

$$h = 2.00 \pm 0.005 \text{ m}$$

→ z : final

$$\bar{a} = -g \hat{z}$$

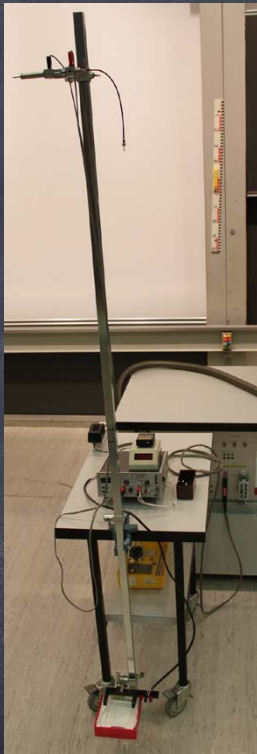
$$z = z_0 + v_{0z}t + \frac{1}{2}at^2$$

$$\underbrace{z - z_0}_{(-)} = \cancel{v_{0z}}t - \frac{1}{2}gt^2$$

$$\underbrace{z_0 - z}_{+} = +\frac{1}{2}gt^2$$

$$h = \frac{1}{2}gt^2 \quad g = \frac{2h}{t^2}$$

+z
↑



$h = 2.00 \pm 0.005 \text{ m}$

measure the time:

$$t = 0.6399 \text{ s}$$

today we estimate the uncertainty

$$t = 0.64 \pm 0.03 \text{ s}$$

calculate g : $g = \frac{2h}{t^2} = \frac{2(2.00 \text{ m})}{(0.64 \text{ s})^2} =$

$$g = 9.76 \frac{\text{m}}{\text{s}^2}$$

uncertainty in g

Example 2.4: For something more complicated like

$$f(x, y, a, b) = K \frac{xy^n}{ab^m} \quad (2.16)$$

with a constant K , we find after some algebra

$$\sigma_f = |f| \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{n\sigma_y}{y}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{m\sigma_b}{b}\right)^2}, \quad (2.17)$$

$$\sigma_g = |g| \sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(2\frac{\sigma_t}{t}\right)^2}$$

$$\sigma_g = 9.76 \frac{\text{m}}{\text{s}^2} \sqrt{\left(\frac{0.005 \text{ m}}{2.00 \text{ m}}\right)^2 + \left(\frac{2(0.03 \text{ s})}{0.64 \text{ s}}\right)^2}$$

$$g = 9.76 \pm 0.915 \frac{\text{m}}{\text{s}^2}$$

Actual value in Zurich: $g = 9.807 \frac{\text{m}}{\text{s}^2}$



Assume we throw a ball up at 20 m/s , what is its position and velocity in 4 s ?

$$z = z_0 + v_{0z}t + \frac{1}{2}at^2 \leftarrow (4\text{s})^2$$

$\uparrow$ $\uparrow$ $\uparrow$
 1m $+20 \frac{\text{m}}{\text{s}}$ 4s $a = g \sim 10 \frac{\text{m}}{\text{s}^2}$

$$z = 1\text{m} + 80 \frac{\text{m}}{\text{s}} \cdot \cancel{\text{s}} + \frac{1}{2} \left(-10 \frac{\text{m}}{\text{s}^2} \right) (4\text{s})^2$$

$$z = 1\text{m} \text{ (Same as initial position)}$$

How fast is it moving?

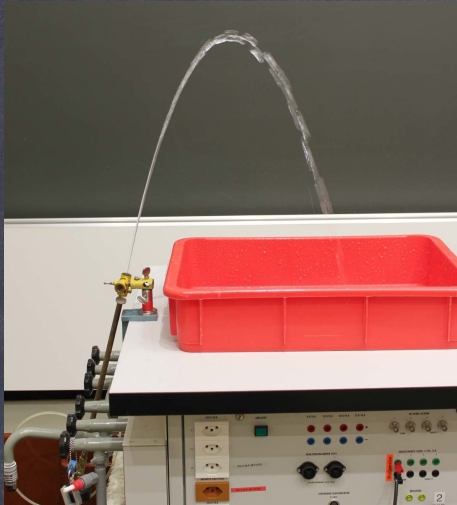
$$v_z = v_{0z} + at = 20 \frac{\text{m}}{\text{s}} + \left(-10 \frac{\text{m}}{\text{s}^2} \right) (4\text{s})$$

$$v_z = -20 \frac{\text{m}}{\text{s}}$$

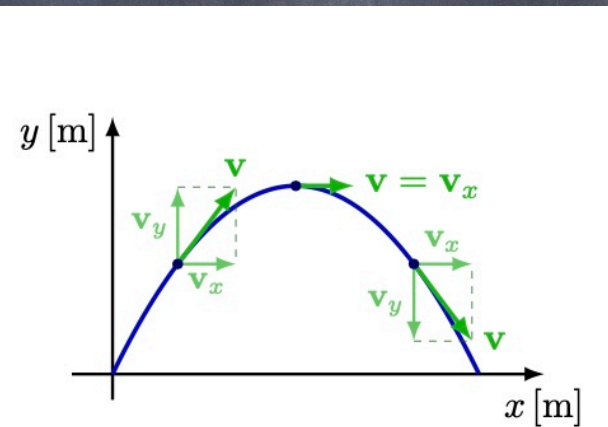
(opposite initial velocity)

Motion in 2D:

An object with an initial velocity $\vec{v}_0 = v_{0x} \hat{x} + v_{0y} \hat{y}$, which feels gravity, will move in the shape of a parabola

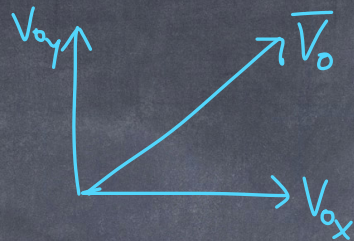
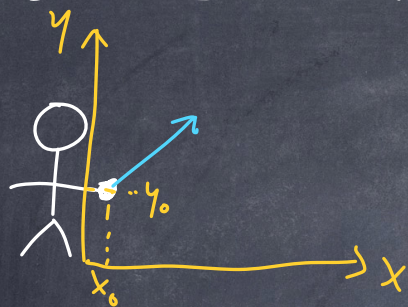


Its v_x does not change)
but its v_y changes
(like the ball thrown vertically)



(c) y - x diagram.

Let's calculate the formulas for parabolic motion:



Initial velocity:

$$\vec{V}_0 = V_{0x} \hat{x} + V_{0y} \hat{y}$$

Consider x and y separately using (2) + (5)

(Neglect
air
resistance)

x -direction: $V_x = V_{0x} + at$

$$x = x_0 + V_{0x}t + \frac{1}{2}at^2$$

velocity in x -direction is constant

y -direction: $V_y = V_{0y} + at \Rightarrow V_y = V_{0y} - gt$

$$y = y_0 + V_{0y}t + \frac{1}{2}at^2$$

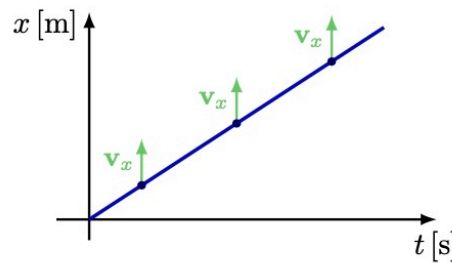
$$y = y_0 + V_{0y}t - \frac{1}{2}gt^2$$

$$V_x = V_{0x}$$

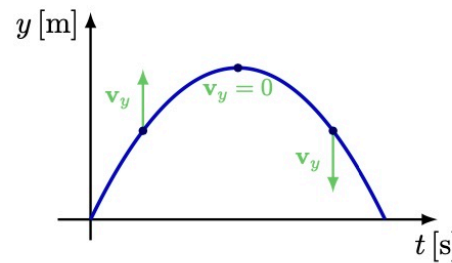
$$x = x_0 + V_{0x}t$$

$$V_y = V_{0y} - gt$$

$$y = y_0 + V_{0y}t - \frac{1}{2}gt^2$$



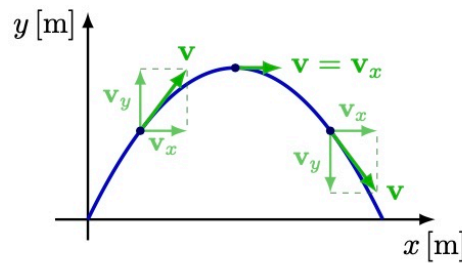
(a) x - t diagram.



(b) y - t diagram.

Diagram
assumes
 $t_0 = y_0 = 0$

what we
see
(with water)

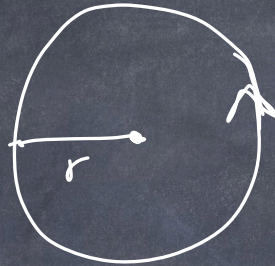


(c) y - x diagram.

y vs. t

Motion in a circle:

radius, r

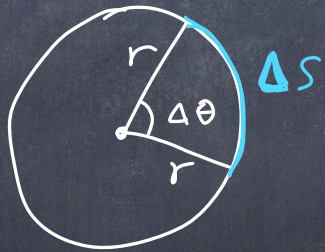


circle has $360^\circ = 2\pi$ radians

$$90^\circ = \frac{\pi}{2} \text{ radians}$$

calculator choice: "deg" / "rad"

circumference of a circle is $C = 2\pi r$



$$\Delta s = r \Delta \theta$$

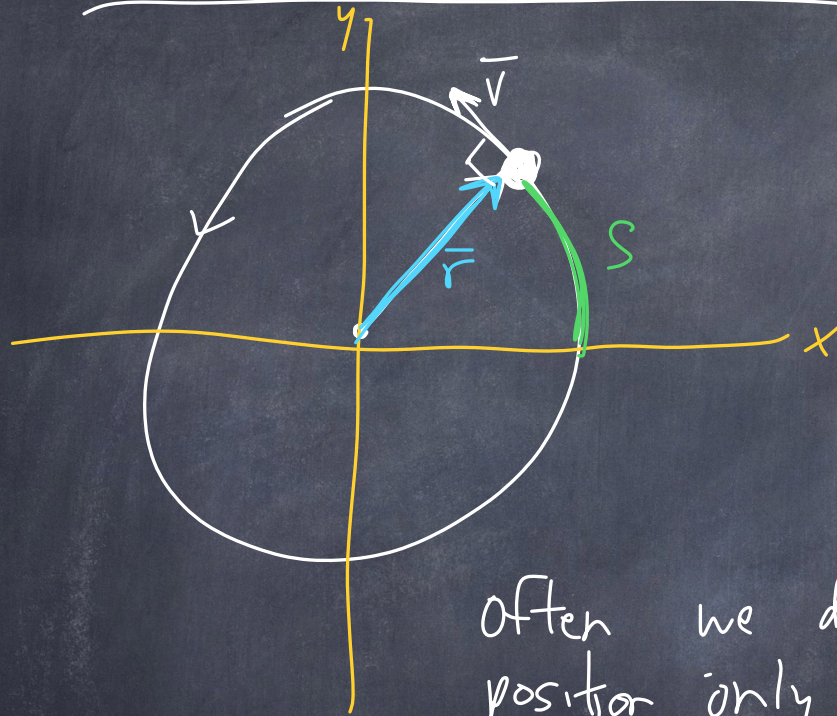
If $\Delta \theta = 2\pi$,
then $\Delta s = 2\pi r$
(circumference)

Half a circle

$$\Delta \theta = 180^\circ = \pi \text{ radians}$$

$$\Delta s = r\pi$$

Circular motion (in 2D):



position $\vec{r}$, velocity $\vec{v}$
 $\vec{r} \perp \vec{v}$

distance traveled = $S = r\theta$

speed = $|\vec{v}| = \frac{\text{circumference}}{\text{time to do a circle}}$

$$v = \frac{2\pi r}{T}$$

T : period of a cycle

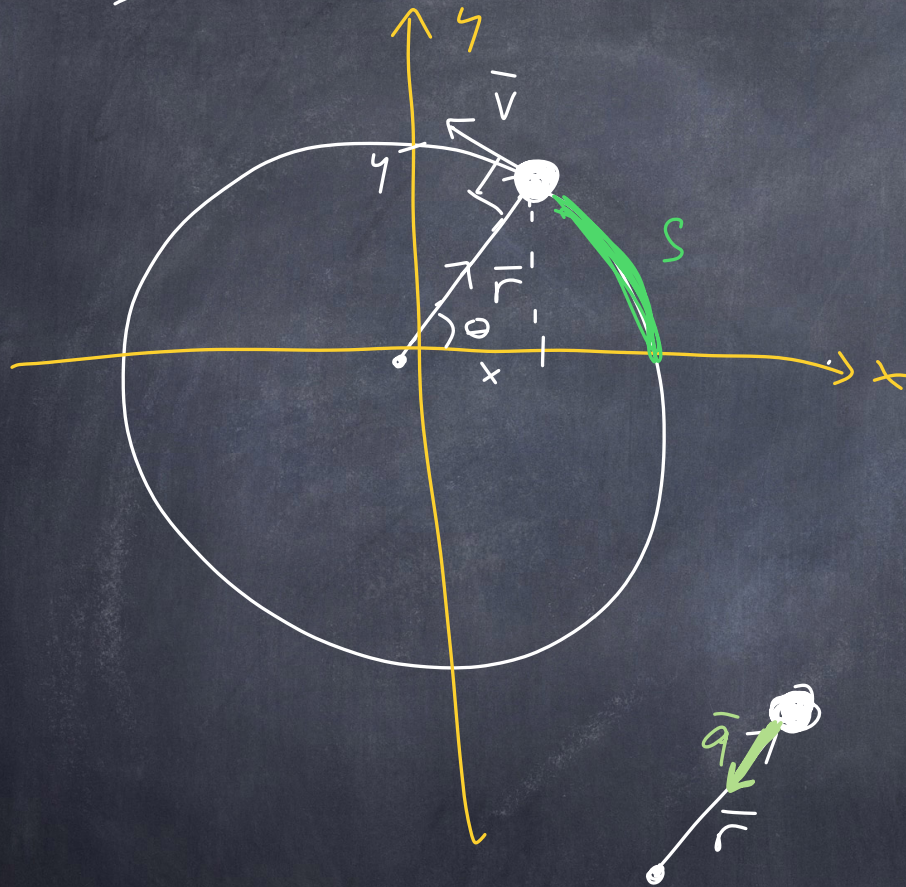
often we describe the position only with θ , since $|\vec{r}|$ is constant

For the velocity, we often use $\frac{\Delta\theta}{\Delta t} = \omega$: angular velocity

$$\omega = \frac{v}{r}$$

$$\frac{\left[\frac{m}{s}\right]}{\left[m\right]} = \left[\frac{1}{s}\right] = \text{radians per second}$$

Circular motion (in 2D):



$$a = \frac{v^2}{r} = \text{centripetal acceleration}$$

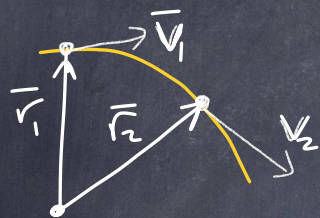
(Derivation of $a = \frac{v^2}{r}$
on the next page
if you are interested)

$\vec{a}$ always points to
the center of
the circle

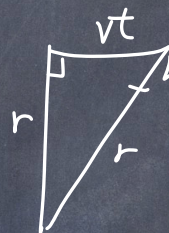
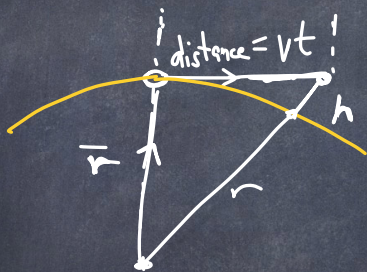
$$\vec{a} = -\frac{v^2}{r} \hat{r}$$

skip Motion in a circle, Ball on a string

assumption: The speed is the same vs. time



The velocity is changing (direction changes)
If the ball's velocity stayed the same, it would be at a height of $r+h$ after a time t .



Pythagorean theorem:

$$(vt)^2 + r^2 = (r+h)^2$$

$$(vt)^2 + \cancel{r^2} = \cancel{r^2} + 2rh + h^2$$

For small time t , h is small

h is very small compared to r ($h \ll r$)

$$\text{so } h^2 \ll rh$$

$$\text{Therefore, } (vt)^2 \approx 2rh + \cancel{h^2}$$

$$\text{so } h \approx \frac{1}{2} \left(\frac{v^2}{r} \right) t^2$$

Compare this to $s = \frac{1}{2} at^2$, we see that

$$a = \frac{v^2}{r}$$

This is the centripetal acceleration. Movement in circle.

A satellite is moving around the earth
at a height of 100 km above the ground.
How long does it take to go around
the earth?

$$C_{\text{earth}} = 40,000 \text{ km}$$

$$g \sim 9.5 \frac{\text{m}}{\text{s}^2} \text{ at } 100 \text{ km}$$



$$C = 2\pi r \Rightarrow r_{\text{earth}} = 6370 \text{ km}$$

$$r_{\text{satellite}} = 6470 \text{ km}$$

we know
2 things: $\left\{ \begin{array}{l} a = \frac{v^2}{r} \\ a = g \end{array} \right\}$

Therefore, $\frac{v^2}{r} = g \Rightarrow v = \sqrt{rg}$

This is just
to get m
instead of km

$$v = \sqrt{(6470 \text{ km}) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) (9.5 \frac{\text{m}}{\text{s}^2})} = 7840 \frac{\text{m}}{\text{s}}$$

$$T = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{v} = \frac{2\pi (6470 \text{ km})}{7.84 \frac{\text{km}}{\text{s}}} \cdot \left(\frac{1 \text{ minute}}{60 \text{ seconds}} \right) = 86 \text{ minutes}$$

$\Rightarrow$ Space station takes 92 minutes

we are
very close!

Experiments

