MMP I
Tutorial 2

HS 2017
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http://www.physik.uzh.ch/en/teaching/PHY312/HS2017.html

Issued: 26.09.2017
Due: 03.10.2017 15:00

Exercise 1: Fourier analysis and minimization (6 Pts.)

f is a 2π periodical function defined as

$$f(x) = \pi - |x|, \qquad -\pi \leq x \leq \pi \; . \tag{1.1}$$

a) Find the Fourier coefficients for f.

b) Find parameters a and b for which the integral

$$\int_{-\pi}^{\pi} \mathrm{d}x (f(x) - a\cos(3x) - b\sin(4x))^2$$

is minimal.
Hint: Use the analogy between the Fourier series expansion of the function $f(x)$ and its expansion over the orthonormal basis given by $\cos(nx)$ and $\sin(nx)$ for $n = 0, 1, \ldots$

Exercise 2: Convolution theorem (2 Pts.) Let f, $g : x \to \mathbb{R}$ be continuous 2π periodical functions which are integrable. The convolution of those two functions is defined as

$$(f * g)(x) = \frac{1}{2\pi} \int_0^{2\pi} f(y) g(x - y) \mathrm{d}y \; . \tag{2.1}$$

Show that the Fourier coefficients of $f * g$ are given by $c_n(f * g) = c_n(f) \cdot c_n(g)$.

Exercise 3: Fourier Series (4 Pts.) Consider an odd function f, periodic with 2π, with $f(x) > 0$ for $x \in (0, \pi)$. Show that for the Fourier coefficient b_n of f the following is valid:

$$|b_n| \leq n b_1 \; .$$

Hint: Show that

$$|\sin(nx)| \leq n|\sin(x)| \qquad \forall x \; .$$

Exercise 4: 3-dimensional Fourier transform (4 Pts.) The Yukawa potential is a spherically symmetric potential in 3 dimensions, defined as $V_Y = e^{-\alpha r}/r$, with $\alpha > 0$ and $r = |\vec{r}|$.

a) Compute the Fourier transform of the Yukawa potential, $\mathcal{F}[V_Y(r)] = V_Y(\vec{k})$;

b) Use your result of a) to compute the Fourier transform of the Coulomb potential.