# Summary Lecture 8

April 26, 2017

In the lecture 8 we looked at materials containing freely moving electrons. These electrons can be viewed as moving in a square potential well with length L. Their kinetic energy is quantized and can only have certain values. The solution to their movement is a plane wave with energies and momenta:

$$\Psi = a \cdot e^{ikx}$$
$$E = \frac{\hbar^2 k^2}{2m} \tag{1}$$
$$k_x = 0; \pm\frac{2\pi}{L}; \pm\frac{4\pi}{L}; ...$$

We use the Fermi Dirac distribution to model their kinetic energy. Since they are freely moving and can interact with one another inside their pocket, the Pauli exclusion principle dictates, that every electron in the quantum system has a unique energy, due to the antisymmetric properties of the electron's and (fermions in general) wave function. The distribution holds for every fermion:

$$f(E) = \frac{1}{\exp(\frac{E-\mu}{k_b T}) + 1}; \; ; \text{ where } \mu \text{ is the chemical potential.}$$
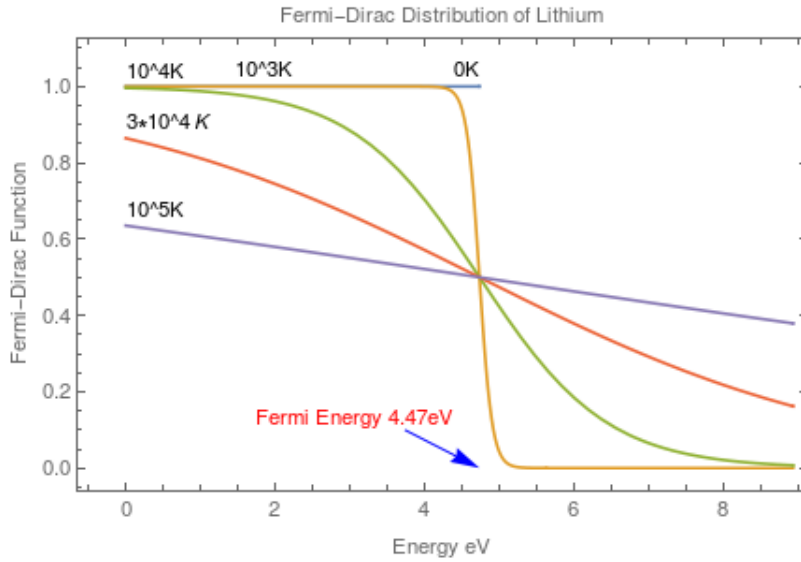
The function gives the average number of particles occupying a specific state in the system. If the temperature changes, we can observe a shift. At $T = 0$, the distribution is a pure step function, where no electron is found with energy above the Fermi energy E_F. This energy is more generally defined as:

$$E_F = k_B \cdot T_F = \frac{\hbar^2 k_F^2}{2m}$$

, where T_F is the Fermi temperature and k_F the Fermi momentum. Since the energy of the electrons can be written as:

$$E = \frac{\hbar^2}{2m}(k_x^2, k_y^2, k_z^2)$$

For different k_i the electrons span a sphere in k-space.
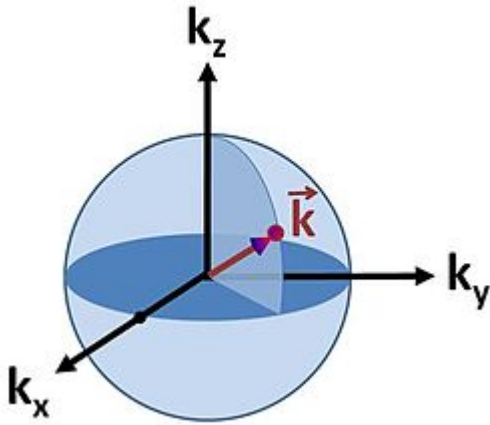
On the surface of the sphere we find the maximum kinetic energy, or the Fermi energy. From this and the fact that for every Energy we have a degencery due to $\pm$ half integer spin, we attain the expressions:

$$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}$$

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

For the Fermi momentum and energy. This means as a general rule that the more electrons we have in the volume of a unit cell, the higher the Fermi energy is. We now seek to attain the formula for the density of states from which we will compute the electronic heat capicty. It is found with:

$$D(E) = \frac{dN}{dE} = \frac{V}{2\pi^2}\cdot\left(\frac{2m}{\hbar^2}\right)^{3/2}\cdot\sqrt{E} \approx \frac{3N}{2E}$$
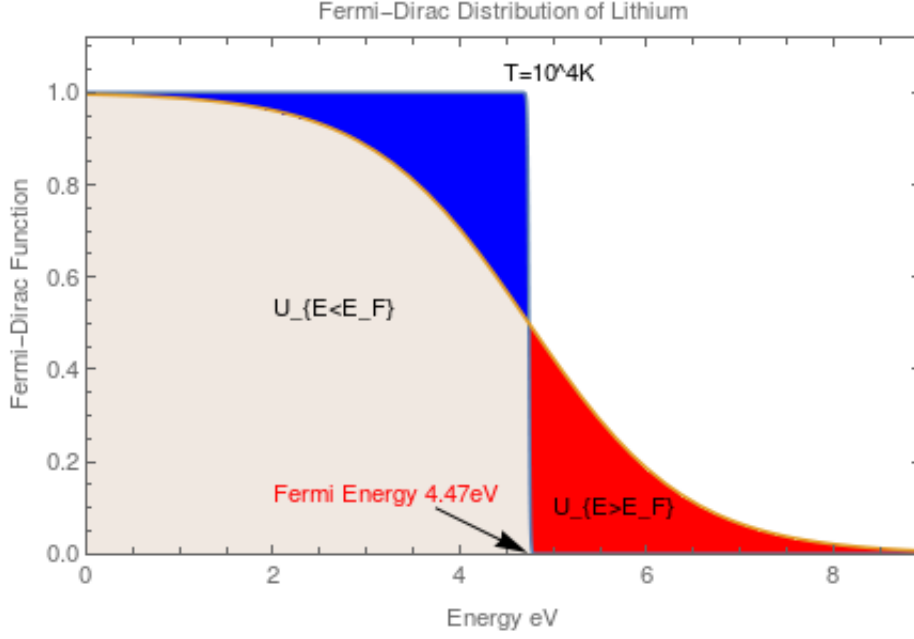
Integrating the Fermi-Dirac distribution we then get the number of Orbitals for the Fermi energy at infinity:

$$\int_0^\infty f(E)D(E) = \int_0^{E_F} D(E)dE = N$$

$$\int_0^{E_F} ED(E)dE = U_0$$

$$\int_0^\infty f(E)D(E)EdE = U(T)$$

We then look at the area under the curve:

Fermi–Dirac Distribution of Lithium

$$U(T) = E_{E<E_F} + U_{E>E_F} \; ; \; U(T) > U_0$$

$$\left(\int_0^{E_F} + \int_{E_F}^{\infty} \right) E_F F(E) D(E) = \int_0^{E_F} dE E_F D(E)$$

$$U(T) = \int_0^{\infty} f(E) D(E)(E - E_F) dE + C$$

To find the heat capacity of the electrons we then get:

$$C_{el} = \frac{dU}{dT} = \int_0^{\infty} \frac{df(E)}{dT} D(E)(E - E_F) dE$$

$$\approx D(E_F) - k_B^2 T \int dx$$

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Or the exact value, which is linearly dependent on the temperature.

$$C_{el} = \pi^2 N k_B \frac{T}{2T_F} = \gamma \cdot T$$

The accumulated heat capacity for both the phonon and electronic terms is then:

$$C_V = \gamma T + A T^3$$

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$$\frac{C_V}{T} = \gamma + AT^2$$

We therefore see, that for very low temperatures, the electronic heat capacity dominates, and inversely for higher temperatures, the phonon heat capacity. In the lecture we defined resistivity as:

$$\rho = \frac{A}{l} \cdot R$$

It is therefore material specific. Kittel defines it as:

$$\rho = \frac{m}{ne^2\tau}$$

From Physics II we vaguely remember that the mean collision time is quadratic anti-proportional to the temperature: $\tau \approx T^{-2}$. It follows that for the resistivity: $\rho \approx T^2$.