



Mechanics

Exercise Sheet 3

HS 2026
Prof. G. Isidori

Issued: 30.09.2026

Discussed: 07.10.2026

<https://www.physik.uzh.ch/en/teaching/PHY311/HS2026.html>

Exercise 1: The Morse potential

The Morse Potential can be used as a simplified model for the potential energy of a diatomic molecule. It takes the form

$$V(r) = A \left(1 - e^{-\alpha(r-r_e)}\right)^2, \quad (1.1)$$

where A , α and r_e are dimensionful constants and r is the distance between two atoms with mass m .

- Sketch the potential as a function of r and describe the motion qualitatively for various values of the molecule's energy E .
- Determine the turning distances for molecules with energies $E = A/2$ and $E = 2A$.
- Calculate the period of the motion for $E = A/2$.

Hint: You might find the following integral useful

$$\int \frac{du}{(1 - \frac{u}{\sqrt{2}})\sqrt{1-u^2}} = \sqrt{2} \arctan \left(\frac{\sqrt{2}u - 1}{\sqrt{1-u^2}} \right). \quad (1.2)$$

Exercise 2: Minimisation of travel time of a bead on a wire

- Consider a bead falling along a wire due to gravity. In this exercise you are asked to find the shape of the wire (the curve) which minimises the time taken for the bead to fall from the origin $(0,0)$ to an arbitrary point (x_0, y_0) . If the path is parameterised by $(x, y(x))$, use conservation of energy to show that the time taken is given by

$$t = \int_0^{x_0} \sqrt{\frac{1 + \dot{y}^2}{2gy}} dx, \quad (2.1)$$

where g is the acceleration due to gravity and y is measured down from the initial point.

- Using the minimisation condition (2.11) from the lecture notes, find the differential equation satisfied by $y(x)$ for the extremal path.
- Using the fact that

$$\frac{d}{dx}(y + y\dot{y}^2) = \dot{y} + 2y\dot{y}\ddot{y} + \dot{y}^3 \quad (2.2)$$

and the substitution

$$y = C \sin^2 \theta, \quad (2.3)$$

where θ parametrises the path and C is a constant of integration, show that

$$x = \frac{C}{2} (2\theta - \sin 2\theta) , \quad (2.4)$$

$$y = \frac{C}{2} (1 - \cos 2\theta) , \quad (2.5)$$

is the solution to the differential equation.

d) Sketch the curve.

e) From an arbitrary point on the curve (x_1, y_1) , the time taken to reach the bottom $(x_b, y_{\min})$ is given by

$$t = \int_{x_1}^{x_b} \sqrt{\frac{1 + \dot{y}^2}{2g(y - y_1)}} dx . \quad (2.6)$$

Find the time taken to reach $(x_b, y_{\min})$ from (x_1, y_1) . What do you notice?