MMP I
Tutorial 1

HS 2019
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Exercise 1: Fourier series I (6 Pts.)

Let $\tilde{f} : (-\pi, \pi] \to \mathbb{R}$ be the function defined as

$$\tilde{f}(x) = |\sin x|.$$

Consider its periodic extension $f : \mathbb{R} \to \mathbb{R}$.

a) Calculate the coefficients of its Fourier series

$$S_g \;=\; \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right) \;. \tag{1.1}$$

Find S_g.

b) Study the convergence properties of the series. Which is the convergence type of the series (pointwise, uniform, ...)? Justify your answer.

c) Use the previous result to compute:

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \;. \tag{1.2}$$

Exercise 2: Series expansion (4 Pts.)

Show that for $-\pi < x < \pi$ the following equation is valid

$$x \;=\; 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \; \sin nx \;. \tag{2.1}$$

Using the above result, derive the following

$$1 - \frac{1}{3} + \frac{1}{5} - \ldots \;=\; \frac{\pi}{4} \;. \tag{2.2}$$

Exercise 3: Fourier series II (5 Pts.)

Let $g :\; \mathbb{R} \to \mathbb{R}$ be the function obtained through the periodic extension of

$$f : (-\pi, \pi] \to \mathbb{R} \qquad \text{with} \qquad f(x) \;=\; \begin{cases} x \cos x & -\pi < x < \pi \\ 0 & x = \pi \;. \end{cases} \tag{3.1}$$

a) Determine the Fourier coefficients of g.

b) Study the convergence properties of the series.