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QFT methods for BSM theories
Problem Set 1.

HS 2019
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Exercise 1. *Power-counting of soft fields*

Analogously to the example of scalar fields in the lecture, determine the power-counting of a soft fermion and a soft gauge boson.

Exercise 2. *An Effective Field Theory example*

Consider the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{M^2}{2}\Phi^2 + \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{m^2}{2}\varphi^2 - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3 \Phi, \tag{1}$$

where $M \gg m$. We now want to consider processes at energies $E \ll \Lambda \sim M$.

(a) Compute the effective Lagrangian $\mathcal{L}_{\text{eff}}$ at tree-level both diagrammatically and with the aid of the equations of motion, up to $\mathcal{O}(1/\Lambda^2)$ in the power-counting of the effective theory.

(b) Perform the one-loop EFT matching diagrammatically using dimensional regularization with the $\overline{\text{MS}}$ subtraction scheme, without the method of regions.

(c) In the calculation of the one-loop diagrams of the full theory in part (ii) separate the integrals in hard $k \sim M$ and soft regions $k \sim m \ll M$. What do you observe with respect to the corresponding terms in the diagrams of the EFT?

(d) Compute the anomalous dimension matrix of the matching coefficients. Take $\alpha = 0$ at the matching scale $\mu = M$, where α is the coefficient of the φ^4 operator in the EFT and examine the RG evolution of the couplings. What do you observe at $\mu = m$? Why is a term of the type φ^3 not generated by the running?

(e) **Bonus:** Compute the effective Lagrangian at one-loop order using the background field method.

Exercise 3. *Redundant operators*

Consider the following effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4 + \frac{\mathcal{C}_1}{6!\Lambda^2}\varphi^6 + \frac{\mathcal{C}_2}{3!\Lambda^2}\varphi^3\partial^2\varphi + \frac{\mathcal{C}_3}{3!\Lambda^2}(\partial^\mu \varphi^3)\partial_\mu \varphi + \mathcal{O}\left(\frac{1}{\Lambda^3}\right). \tag{2}$$

(a) Use the equations of motion and integration-by-parts identities to reduce the operator basis to a non-redundant set.

(b) Show that the use of the equations of motion is equivalent to a field redefinition of the form $\varphi \to \varphi + \alpha\varphi^3$, with an appropriate choice of α.