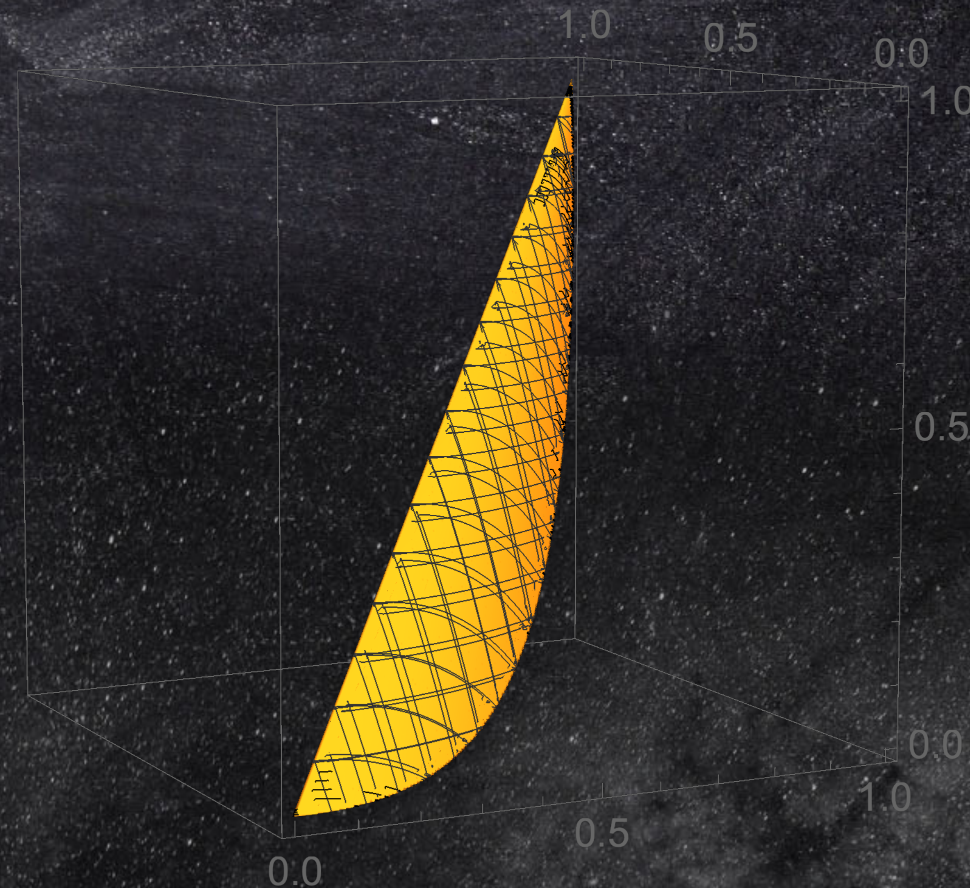
Positive Moments for Scattering Amplitudes

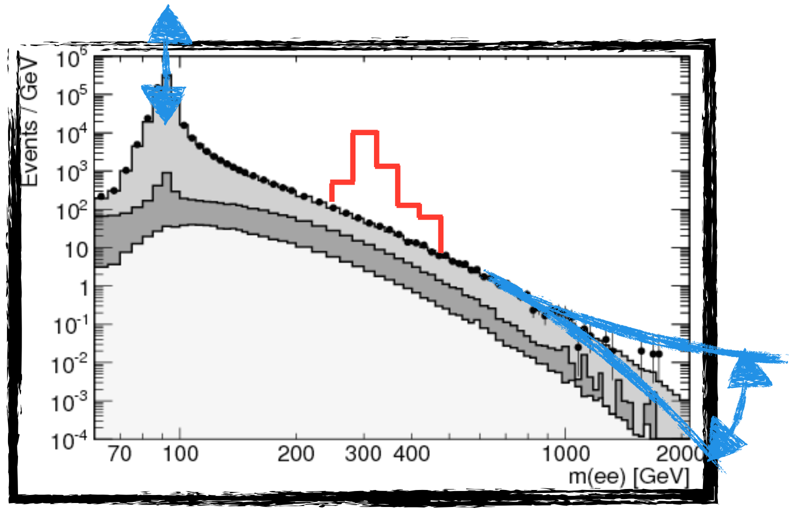


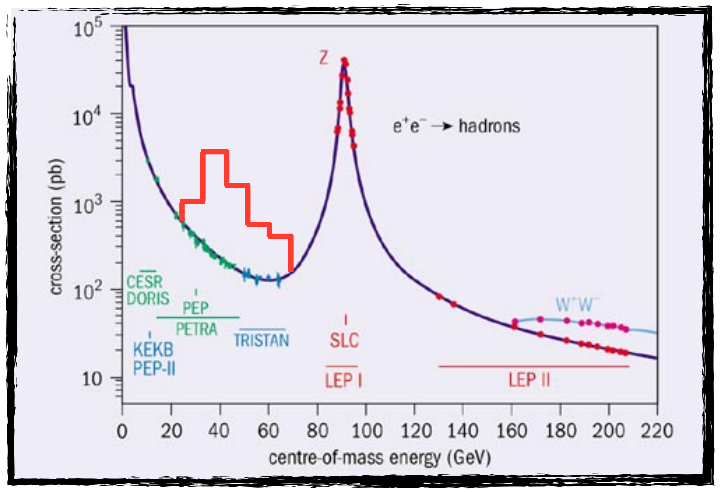
Francesco Riva
(Geneva University)

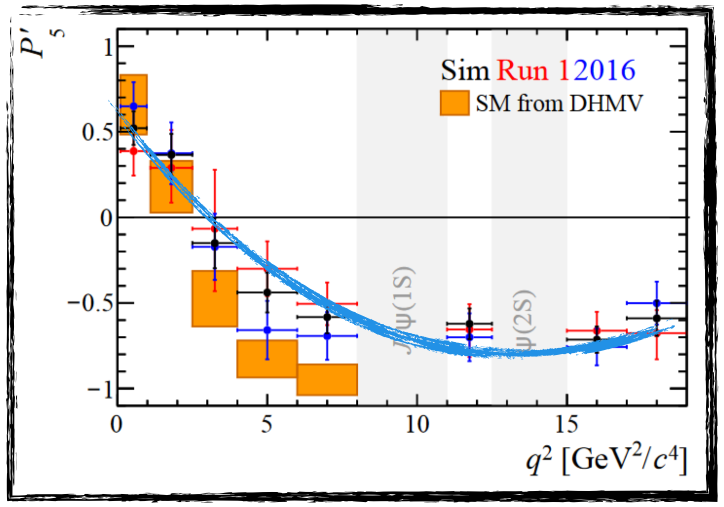
Zurich University 22.9.21

Precision Measurements

At the frontier of experimental capabilities:

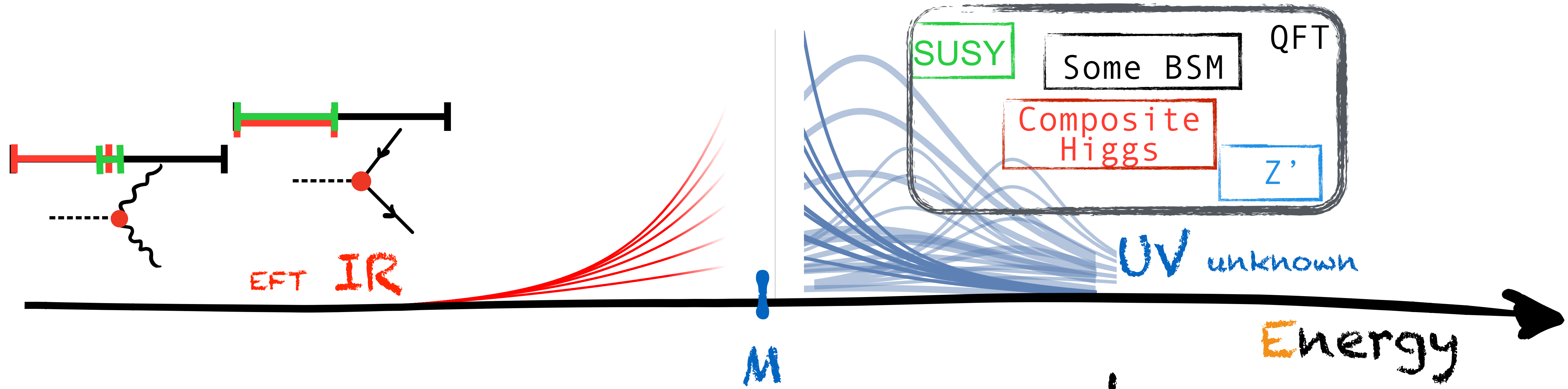






Effective Field Theories ↔ BSM Searches

Effective Field Theories



What can be ~~learned~~ measured?

IR Predictions ⟵ UV Hypotheses

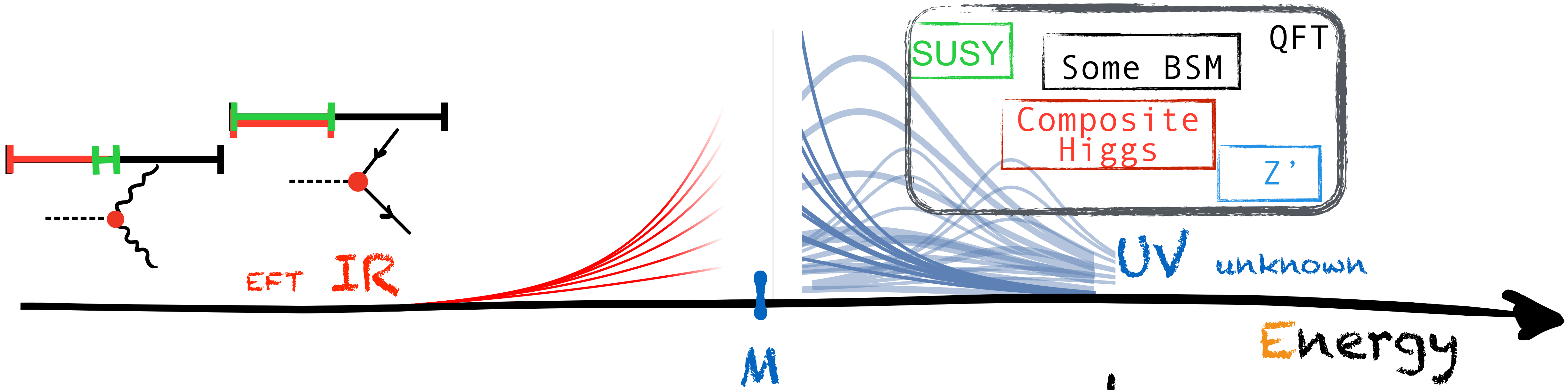
broader... broader... broader...

Lore: all E-behaviours IR consistent!

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

(e.g. tree-level)

Effective Field Theories



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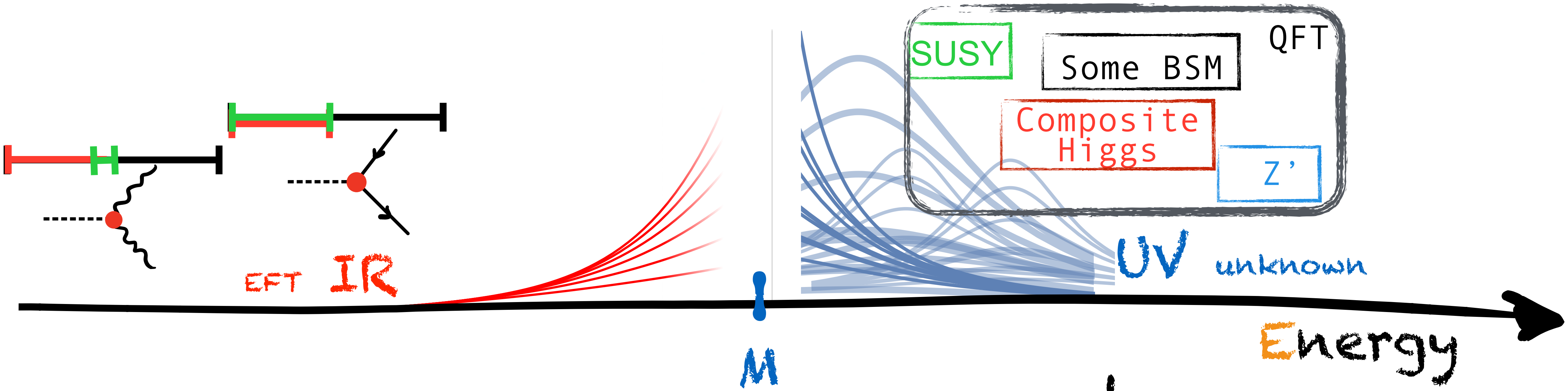

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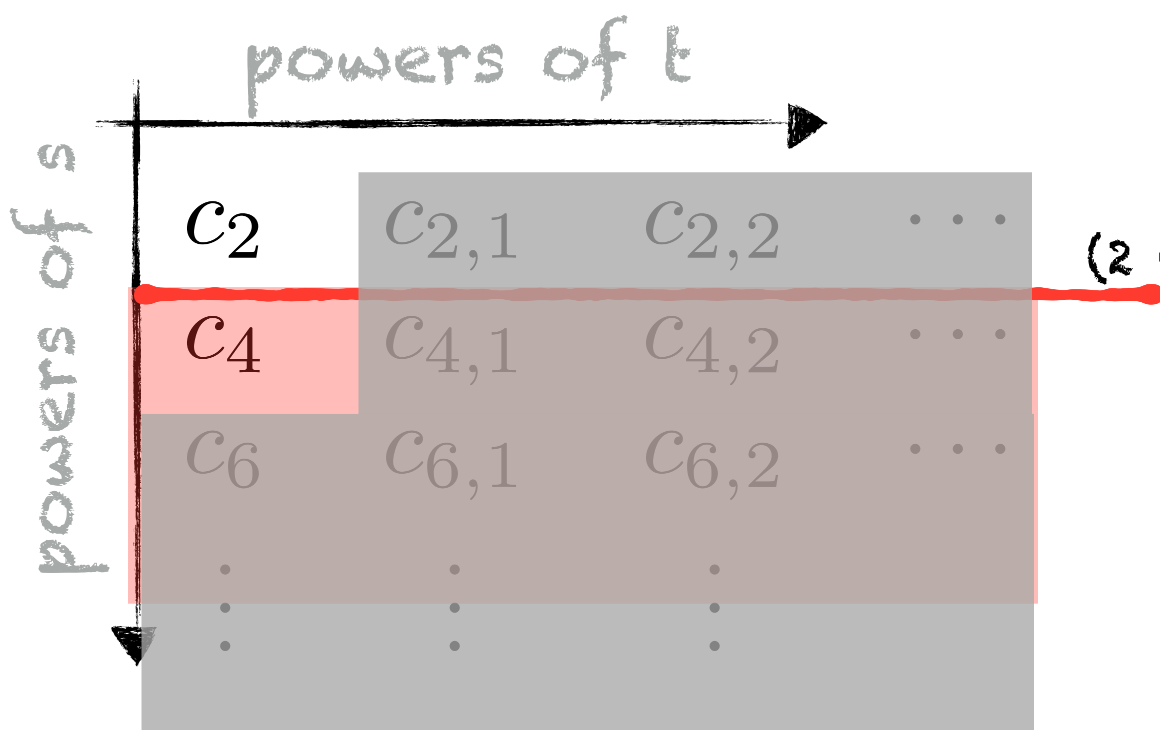
(e.g. tree-level)

Which IR theories are Causal and Unitary in UV?

Notation/Outline



1. UV, IR

(2 →2 amplitude, dispersion relation, arcs, moments)

2. Implications
   - weak coupling
   - strong coupling

3. Non forward & Gravity

$$A_{2\to2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

(e.g. tree-level)

1. UV → IR

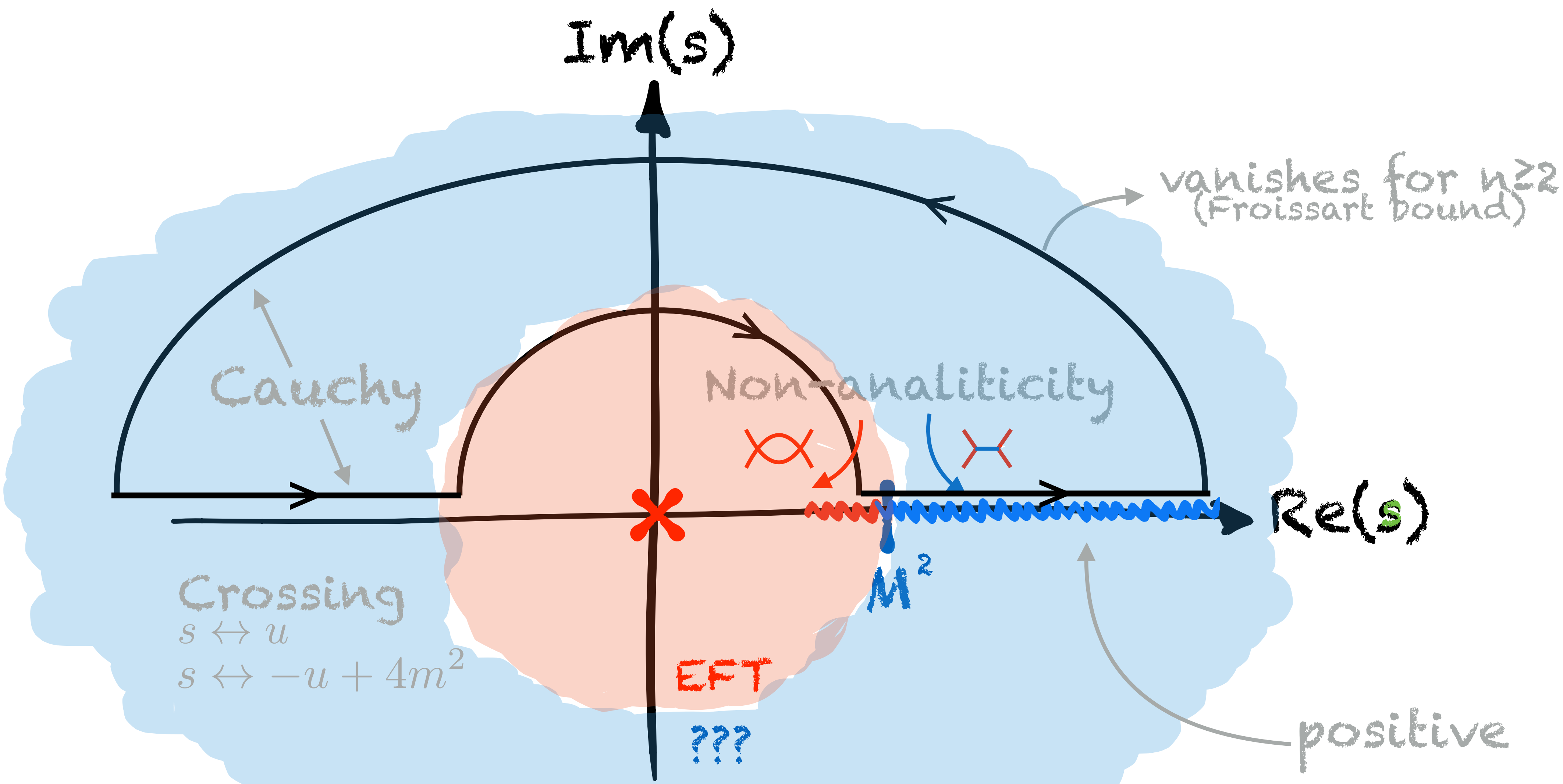
UV-IR Connection

Forward Scattering $t \sim \theta^2 = 0$ ⟶

Total energy $s = E^2$

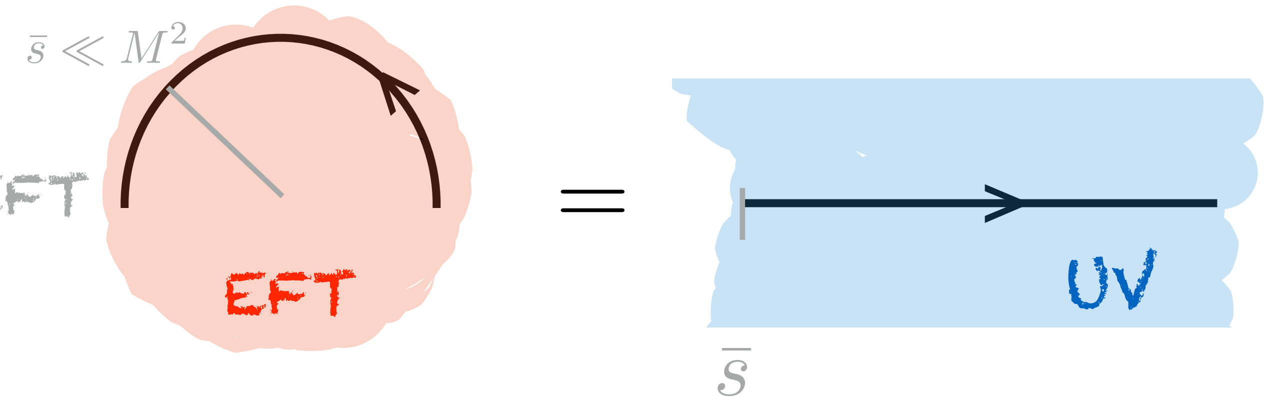
Froissart,Martin',...60s
Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,
...

Physical Properties		Mathematical Properties of 2→2 forward amplitude $A(s)/s^n$
Causality	▶	Analytic in $s \in \mathbb{C}/phys$
Unitarity	▶ (optical theorem)	Positive across $s \in \mathbb{R}$



Arcs: UV-IR Connection

$\bar{s}$ = any energy within EFT



positive

Arcs: $$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{\bar{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\mathrm{Im}A(s)}{s^{n+1}}$$

Calculable in EFT

(e.g at tree-level $\mathcal{A}_n = c_n$, the energy coefficients in EFT)

$$\mathcal{A}_n > 0 \quad (n \geq 2) \qquad\qquad \mathcal{A}_n > 0 \quad (n \geq 2)$$

Adams,Arkani-Hamed,Dubovsky, Nicolis,Rattazzi'06,

- **Consistency condition for EFTs**

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

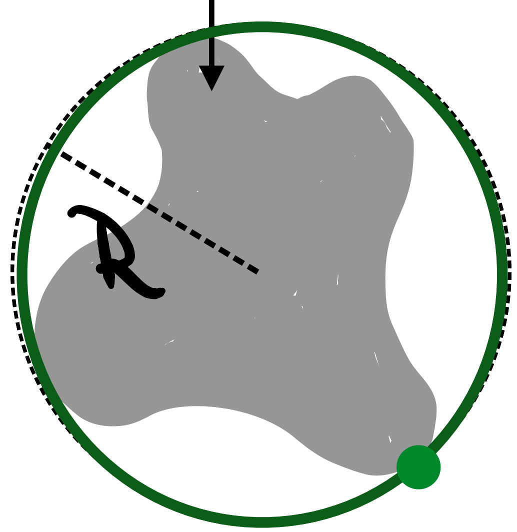
Moments

$$\int_0^1 d\mu(x) x^n \xleftarrow{\text{variables change}} \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\text{Im}A(s)}{s^{n+1}}$$

positive ($\text{Im}A(s)$)

Moments appear everywhere in physics...

e.g. stones $\quad d\mu(x)$ = mass distributions



n=0: total mass M (sets units)

n=1: centre of mass <RM

n=2: moment of inertia <R²M

Bounded

What bounds do moments satisfy?

Bounds ⟷ Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT arcs $\sim$ Wilson coef. ⟷ **Bounds on Moments** ⟷ **Positive polynomials in [0,1]**

$$\mathcal{A}_n = \int_0^1 d\mu(x) x^n$$

$$\sum_{i=0}^{N} \alpha_i \mathcal{A}_i > 0 \longleftarrow \int_0^1 p_N(x) d\mu(x) > 0 \longleftarrow p_N(x) = \sum_{i=0}^{N} \alpha_i x^i > 0$$

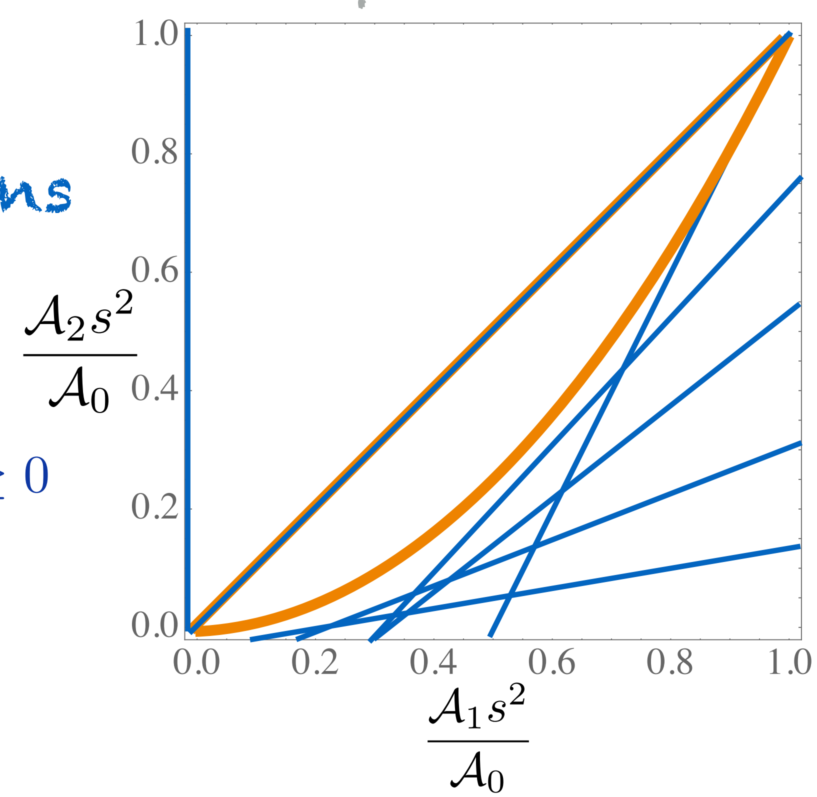
Characterisations of positive polynomials

$$p = x^n (1-x)^m$$

- linear
- Always ∞ conditions

$$p = x(1-x) \geq 0 \Rightarrow \mathcal{A}_1 - s^2 \mathcal{A}_2 \geq 0$$

...up to 3 arcs...



generic polynomials of fixed order

$$p = q_1^2 + x q_2^2 + (1-x) q_3^2 + x(1-x) q_4^2$$

- Non-Linear (semidefinite)
- Finite conditions for Finite number of arcs

$$q_1 = \sum^{N} \beta_i x^i \Rightarrow \sum^{N} \beta_i \mathcal{A}_{i+j} \beta_j > 0$$

Hankel Matrix Must be Positive Definite

Two-Sided Bounds

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

All conditions involving N arcs **only**, written as Hankel Matrices:

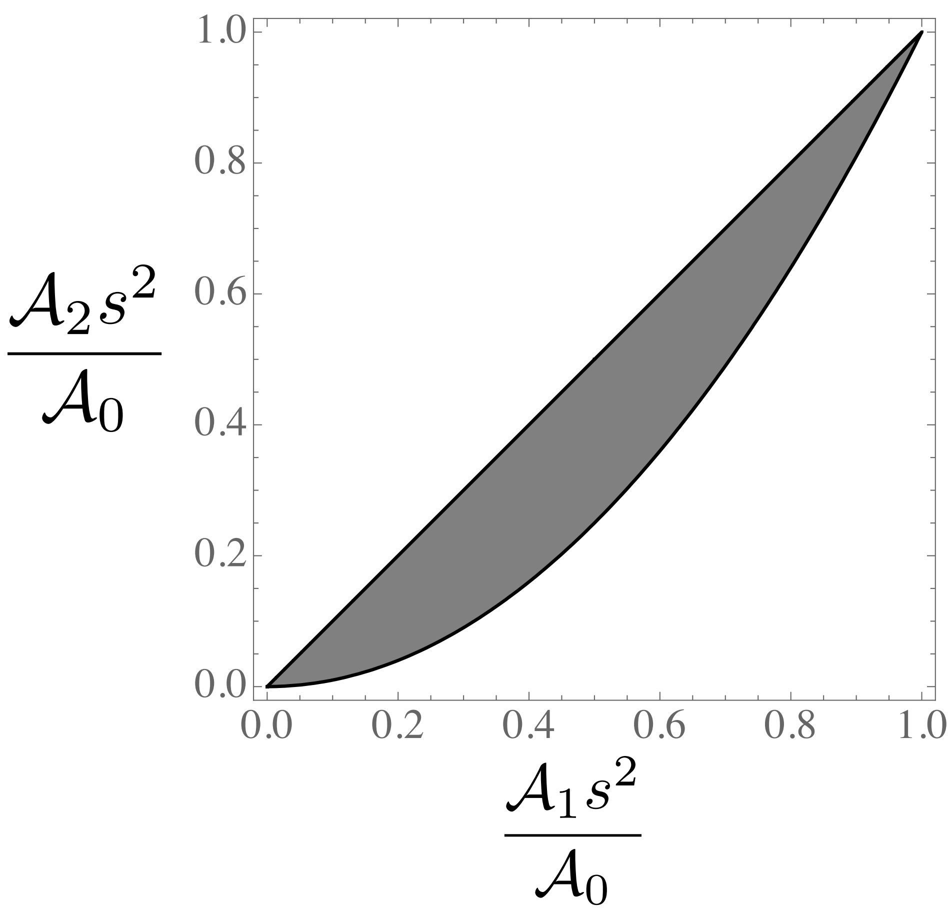
$$H^0_N \succ 0$$
$$H^1_N \succ 0$$
$$H^0_{N-1} - \hat{s}^2 H^1_N \succ 0$$
$$H^1_{N-1} - \hat{s}^2 H^2_N \succ 0$$

e.g. $H^0_4 \equiv \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_1 & \mathcal{A}_2 & \mathcal{A}_3 \\ \mathcal{A}_2 & \mathcal{A}_3 & \mathcal{A}_4 \end{pmatrix}$

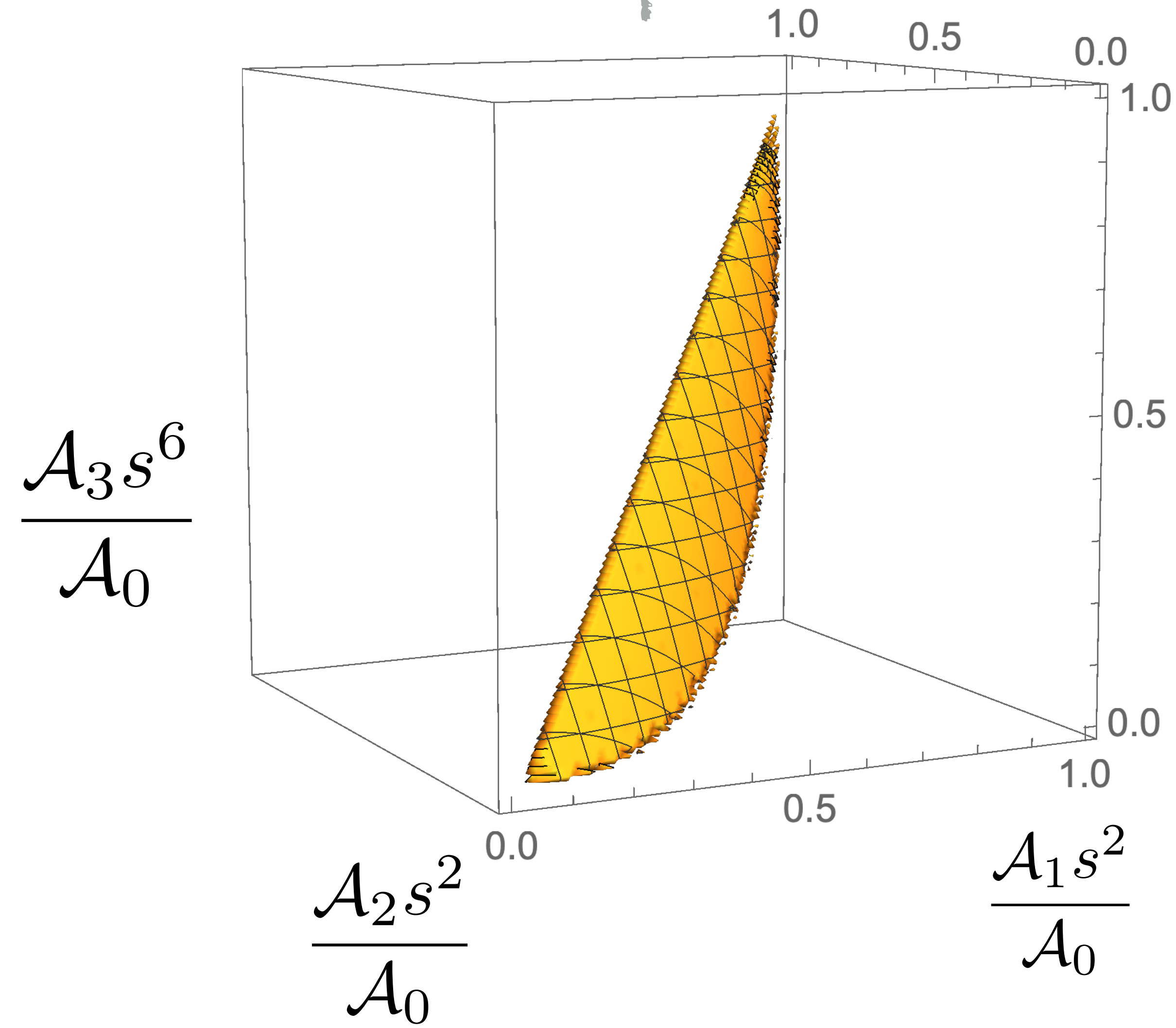
s: any energy within EFT

...up to 3 arcs...

$\mathcal{A}_0 > s^2 \mathcal{A}_1 \qquad \mathcal{A}_1 > s^2 \mathcal{A}_2 \qquad \mathcal{A}_1^2 < \mathcal{A}_2 \mathcal{A}_0$



...up to 4 arcs...



Any moment **two-sided** bounded in terms of $\mathcal{A}_0$ and $\hat{s}$

1. UV → IR

What are arcs in the IR EFT?

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

IR arcs

EFT amplitude (forward)

$\beta_4 = \frac{7c_2^2}{160\pi^2}$

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + \; c_6 s^6 + \cdots$$

▶ **Arcs** $\mathcal{A}_n \equiv \int_{\cap_{\hat{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^{2n+3}}$

Smaller window in which theory looks tree-level

Coupling $c_2 s^2$

Assume higher loops small. e.g. $c_4 c_6 s^8$

$$\mathcal{A}_0 = c_2 + \cdots$$

$$\mathcal{A}_1 = c_4 + \cdots$$

$$\mathcal{A}_2 = c_6 + \cdots$$

Weak Coupling: $\mathcal{A}_n = c_{2n+2}$, all $\mathcal{L}$ couplings captured by arcs

Strong Coupling (e.g ChiPT): high arcs dominated by c_2 loop effects!

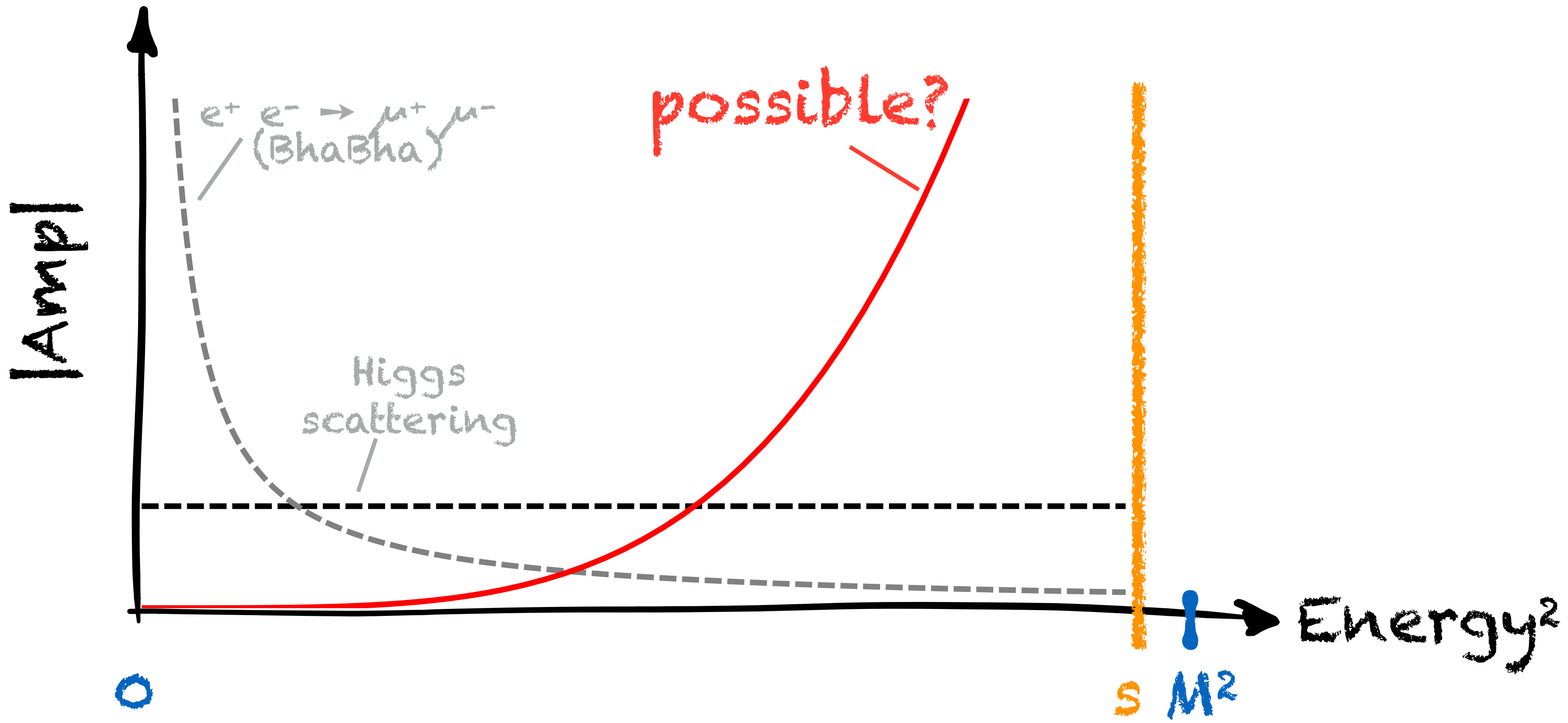
▶ Information unaccessible

2. Applications

(at weak coupling)

1. How soft can EFTs be?
(weak coupling)

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + c_6 s^6 + c_8 s^8 + \cdots$$



Naively:

- all coefficients comparable (units of M²)
- indeed, c's mixed by running

1. How soft can EFTs be?
(weak coupling)

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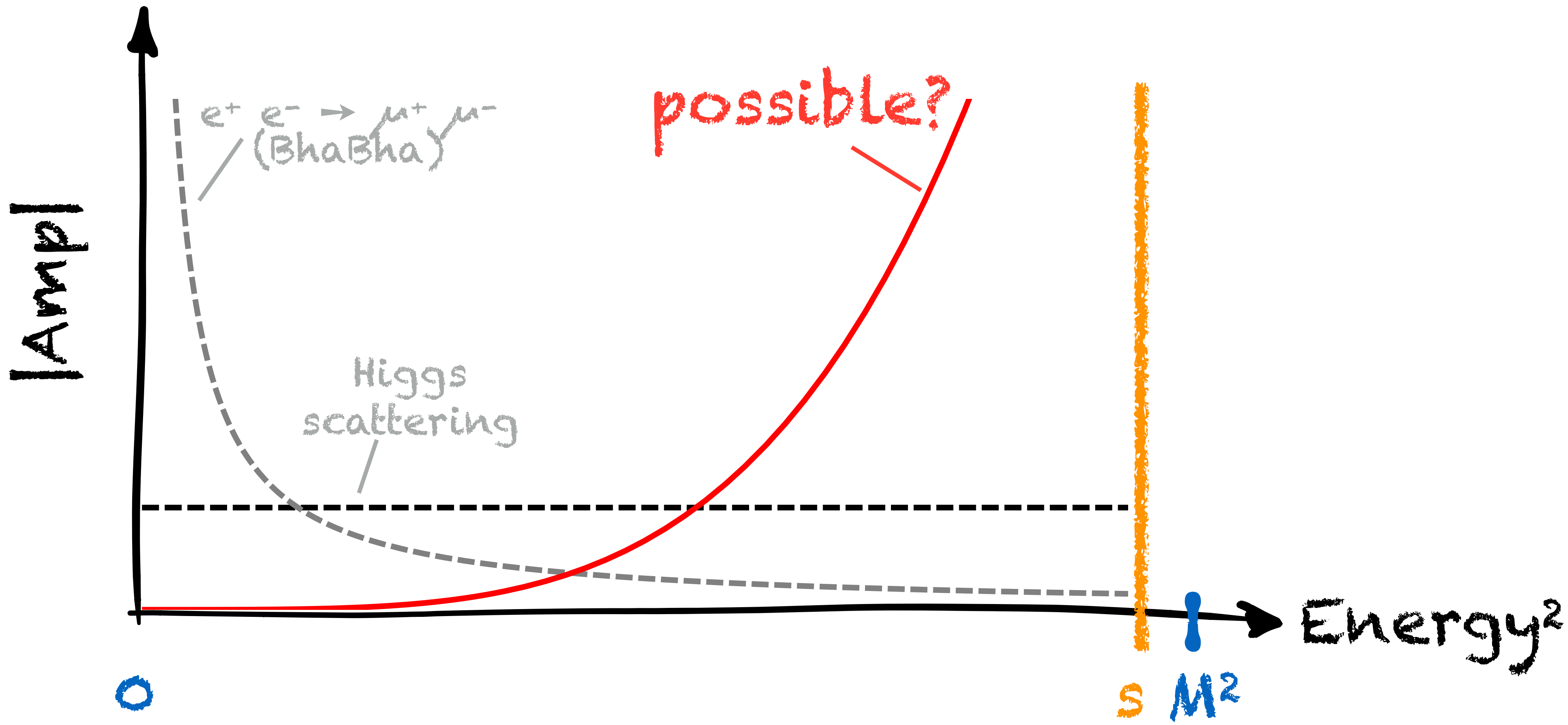


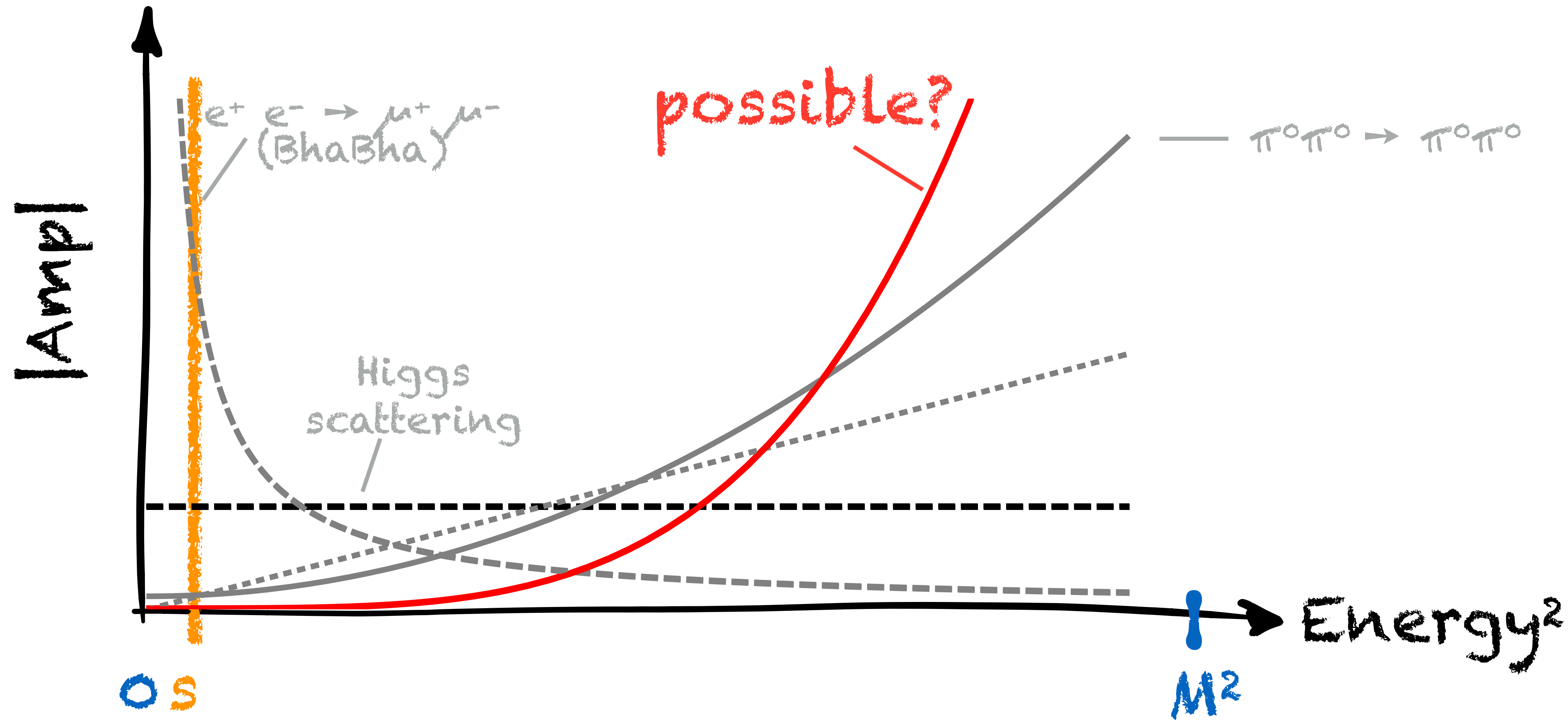
Naively: ...but... Symmetries:

- all coefficients comparable (units of M²)
- indeed, c's mixed by running

make hyerarchies natural

1. How soft can EFTs be?
(weak coupling)

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + c_6 s^6 + c_8 s^8 + \cdots$$



Symmetries:

make hyerarchies natural

Goldstone $\pi \to \pi + \alpha$

$$\mathcal{L} = c_2(\partial\pi)^4 \;\blacktriangleright\; A \sim c_2 s^2 + \cdots$$

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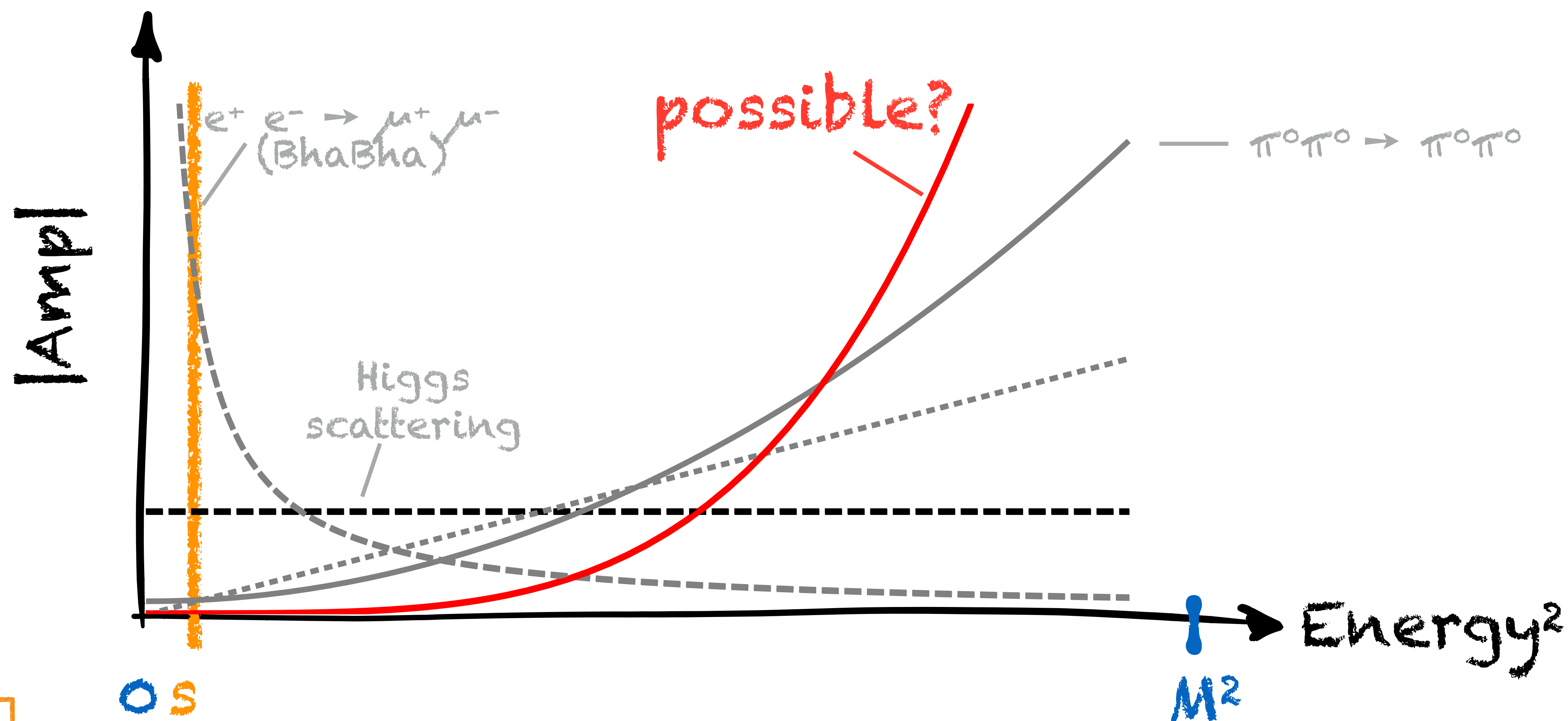

Galileons $\pi \to \pi + \alpha + \beta_\mu x^\mu$

Nicolois,Rattazzi,Trincherini'08

$\mathcal{L} = c_4(\partial\partial\pi)^4$ ▶ $A \sim c_4 s^4 + \cdots$

Super-Soft $\pi \to \pi + \alpha + \beta x \cdots + \gamma x^n$

▶ $A \sim c_N s^{2N}$

Symmetries:

make hyerarchies natural

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1. How soft can EFTs be? (weak coupling)

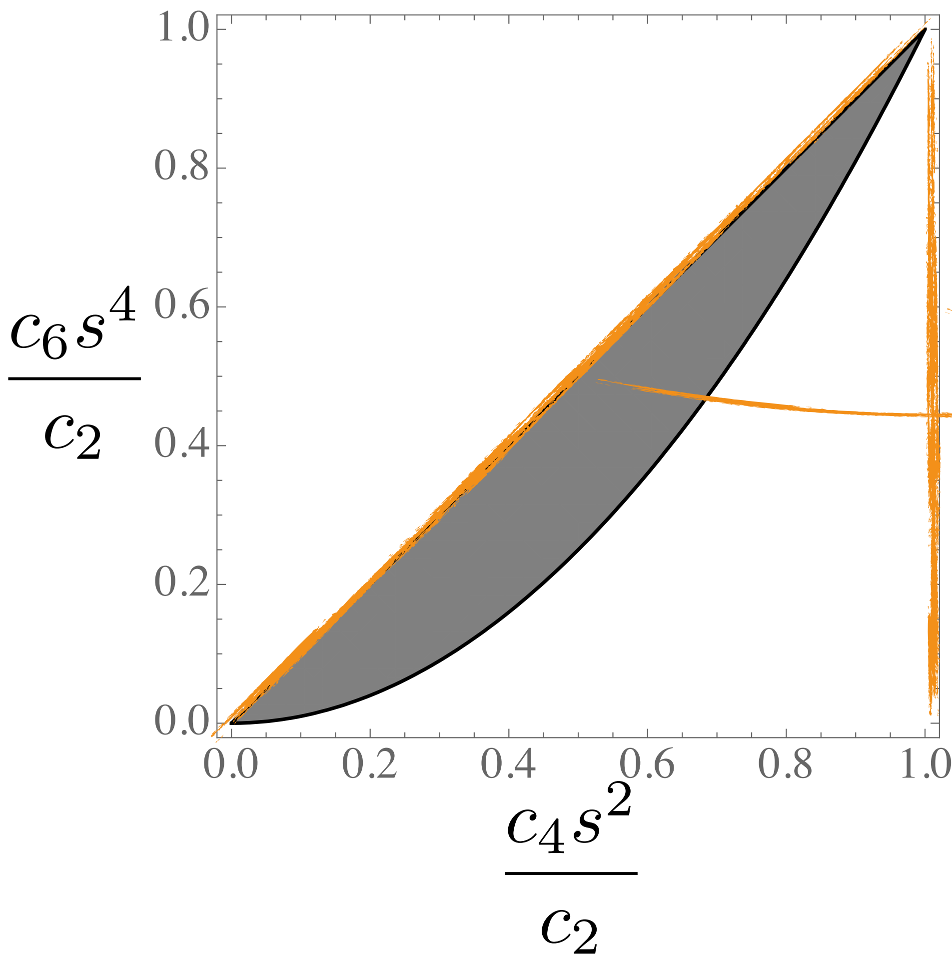
Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

see also Englert, Giudice, Greljo, McCullough'19, Bellazzini, Serra, Sgarlata, FR'19

Arcs in tree-level approximation:

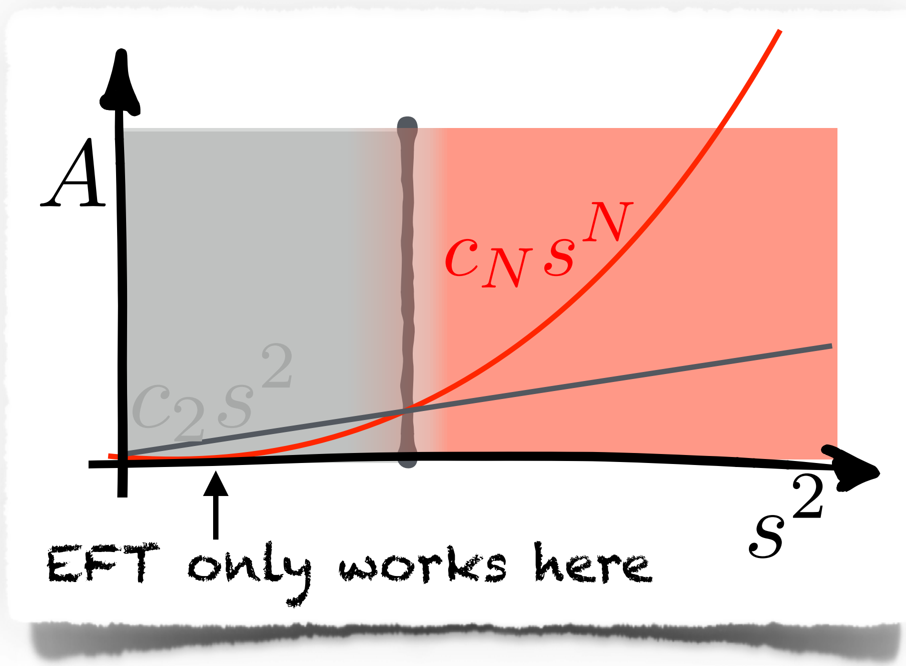
$$A(s) = \cancel{c_0} + c_2 s^2 + c_4 s^4 + c_6 s^6 + \cdots$$

▶ $$\mathcal{A}_n \equiv \int_{\cap_s} \frac{ds}{\pi i} \frac{A(s,t)}{s^{2n+3}} = c_{2n+2}$$



$$c_2 s^2 > c_4 s^4 > c_6 s^6 > \cdots$$

Froissart



▶ info on theory cutoff

Supersoft theories have low cutoff...

... so low that supersoftness unobservable!

(dimension>8 operators cannot dominate)

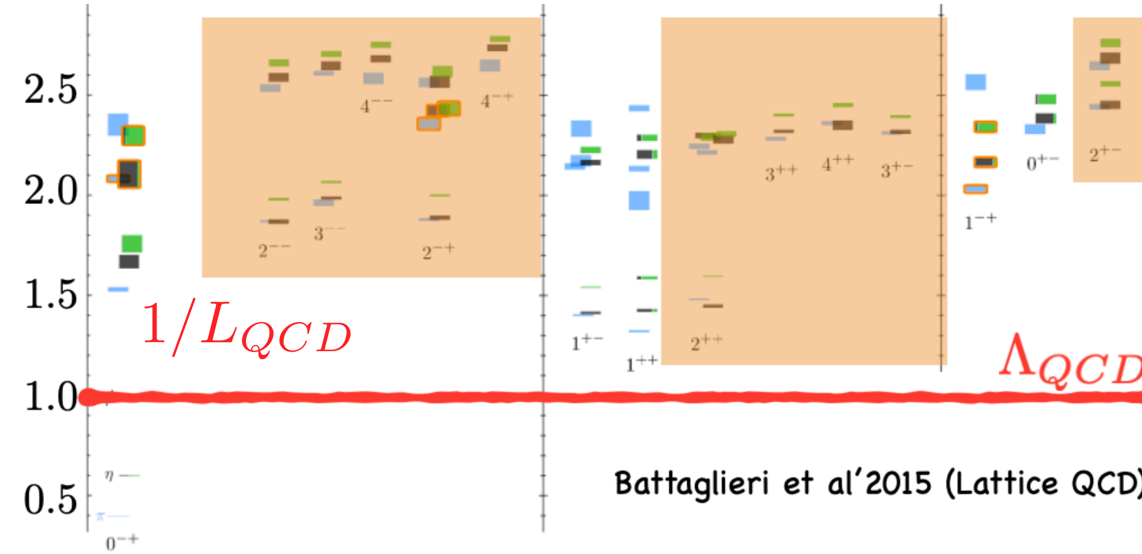
2. Massive Higher Spin

Bellazzini,Serra,Sgarlata,FR'19

$$\Phi^{\mu_1\cdots\mu_J}$$

Higher Spin resonances exist in QCD, Nuclei/atoms, Strings,...
(J>2)

$$m_{HS} \gtrsim \frac{1}{L_{HS}}$$



Can there be lighter HS states?

Interactions grow with $E \lesssim \frac{1}{L_{HS}}$

$$A(\Phi\Phi \to \Phi\Phi) \propto \frac{s^2}{m_{HS}^4} + \cdots + \frac{s^{2J}}{m_{HS}^{4J}}$$

$$m_{HS} \gtrsim \frac{1}{L_{HS}} \longleftarrow \propto \quad c_2 > \bar{s}^{2J-2} c_{2J}$$

Higher Spin always heavier than their size^{-1}

3. Non forward & Gravity

Bellazzini,Riembau,FR'to appear
Tolley,Wang,Zhou'20; Caron-Huot,Mazac,
Rastelli,Simons-Duffin',vanDuong `20-'21

Finite-t

NON-Forward = no bounds?

$$A(s,t) = \cancel{c_0} + c_2 s^2 + c_4 s^4 + \cdots + c_{2,1} s^2 t + c_{2,2} s^2 t^2 + \cdots$$

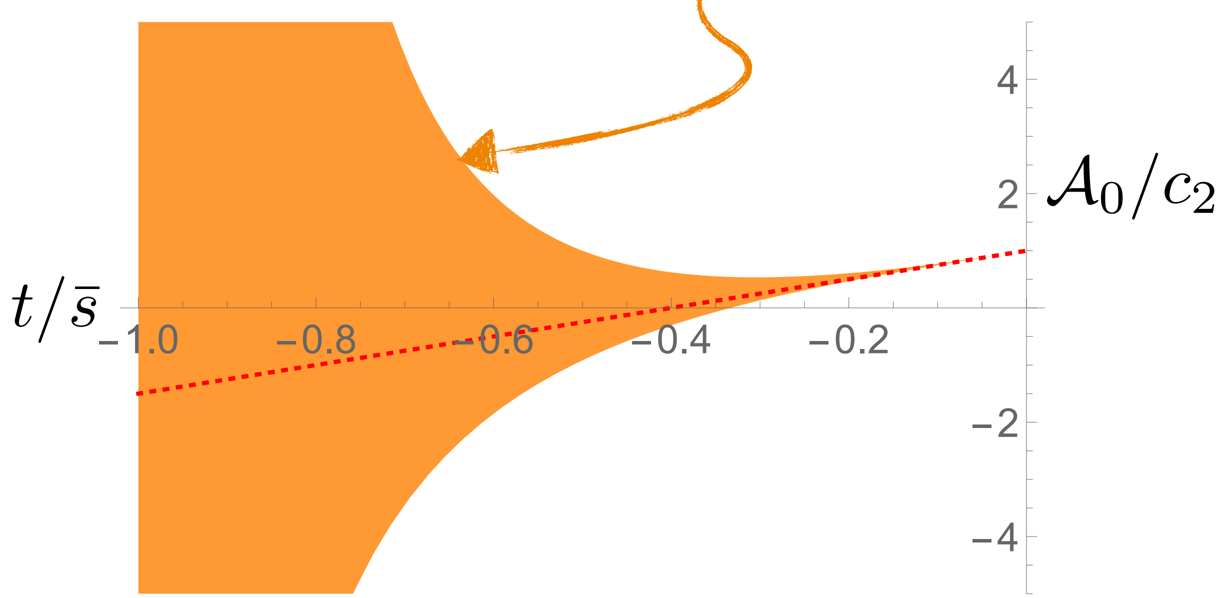
Galileon Nicolois,Rattazzi,Trincherini'08
(appears in massive/modified gravity)

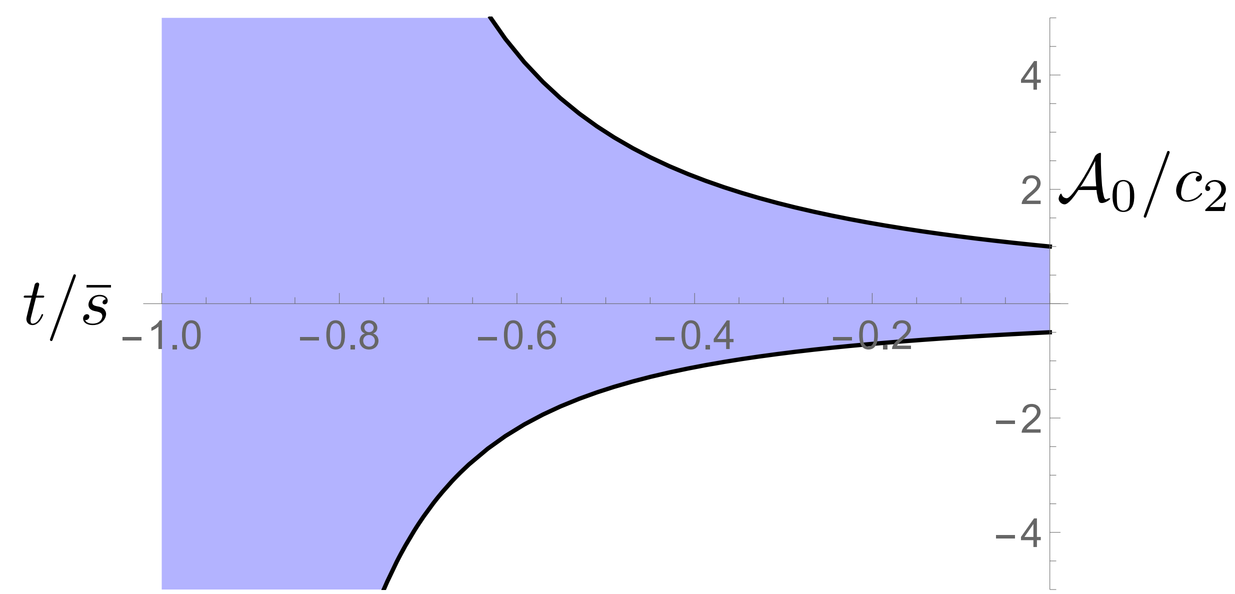
If A(s,t) analytic*: arcs at t≠0 !

$$\mathcal{A}_0(t) \equiv \int_{\cap_{\bar{s}_t}} \frac{ds}{\pi i} \frac{A(s,t)}{(s+\frac{t}{2})^3} = \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \sum_{\ell} \mathrm{Im} f_\ell(s) \frac{P_\ell(1+\frac{2t}{s})}{(s+\frac{t}{2})^3}$$

>0; $-1/2 < \cdot < 1$

$$c_2 + c_{2,1} t + \cdots$$





*Proved for lightest particle in theory, not in general

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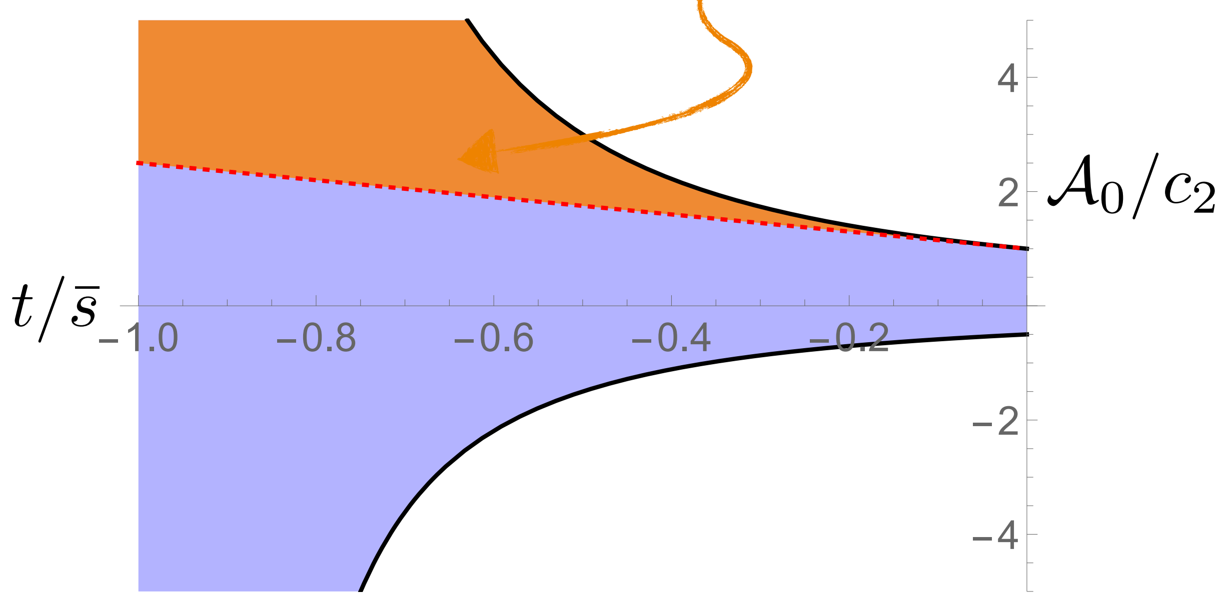
$\mathrm{Im} f_\ell(s)$: >0

$P_\ell(1+\frac{2t}{s})$: $-1/2 <$ · < 1

$$c_2 + c_{2,1} t + \cdots$$



$$-\frac{3}{2} c_2 < c_{2,1} s$$

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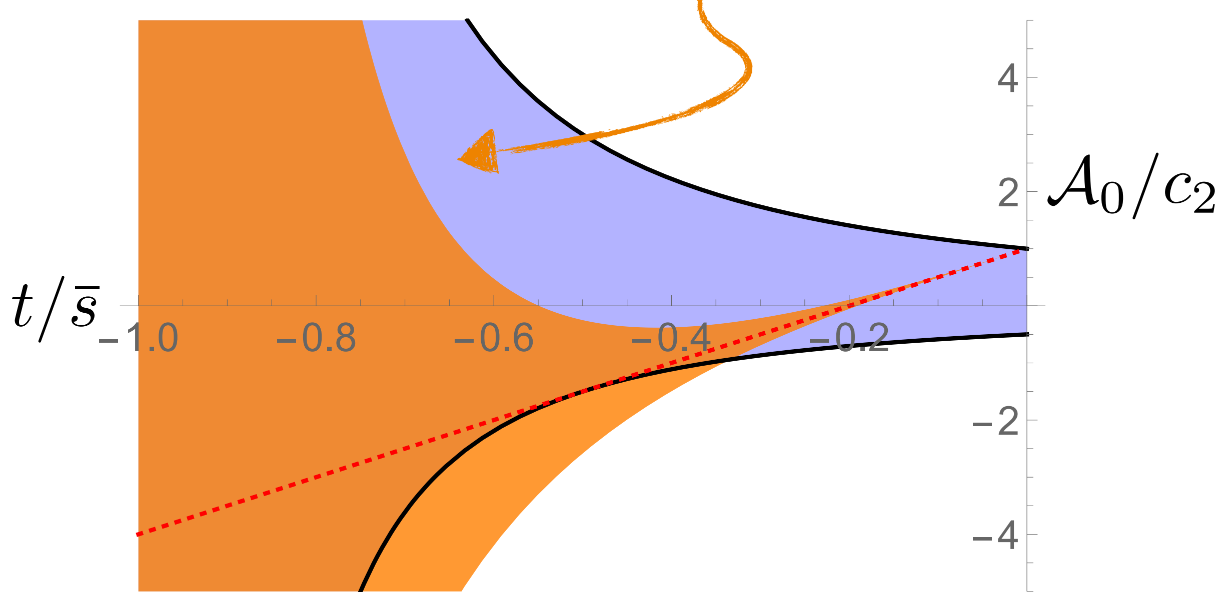
$\mathrm{Im} f_\ell(s)$: >0; $P_\ell(1+\frac{2t}{s})$: $-1/2 <$ ● < 1

$$c_2 + c_{2,1} t + \cdots$$



$$-\frac{3}{2} c_2 < c_{2,1} s \lesssim 5 c_2$$

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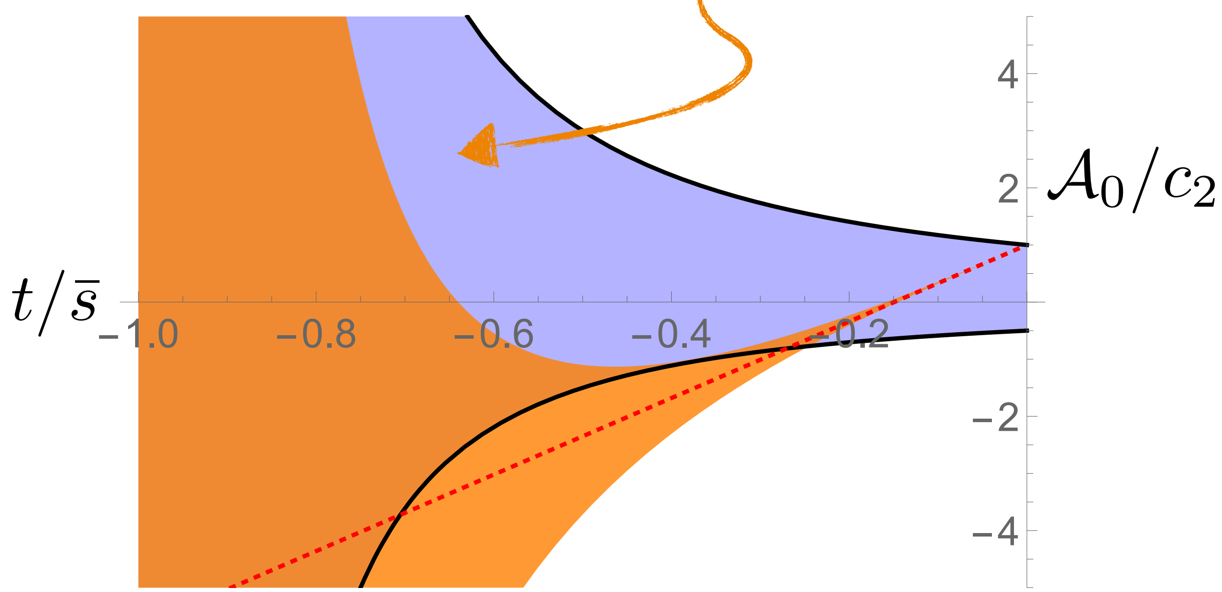


$$-\frac{3}{2}c_2 < c_{2,1}\,s < 5.17 c_2$$

Galileons have small cutoff!

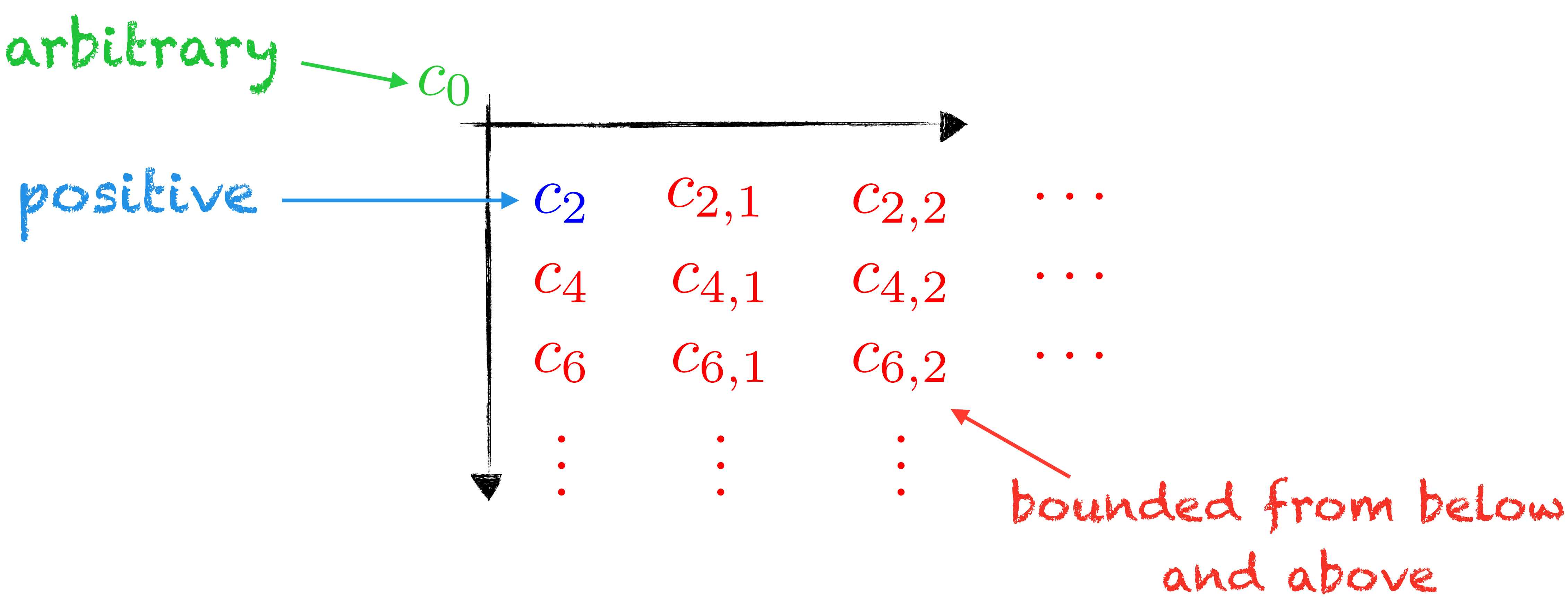
*Proved for lightest particle in theory, not in general

EFTs

Tree-level, beyond forward:

$$A(s,t) = \sum_{p,q} c_{p,q} s^p t^q = \; c_0 \; + \; c_2 s^2 \; + \; c_{2,1} s^2 t \; + \cdots$$

Of ∞ many coefficients, only 2 can lead the amplitude:



3. Massive Gravity

Massive Gravity EFT $\xleftrightarrow{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}$ spin-2, $m_g \neq 0$ → 2 +3 d.o.f.

$$S = \int d^4x \sqrt{g} \frac{M_{Pl}^2}{2}\, [R]$$

$$A(hh \to hh) \sim \left(\frac{m_g^2 s^2}{\Lambda_3^6} + \frac{s^2 t}{\Lambda_3^6}\right)$$

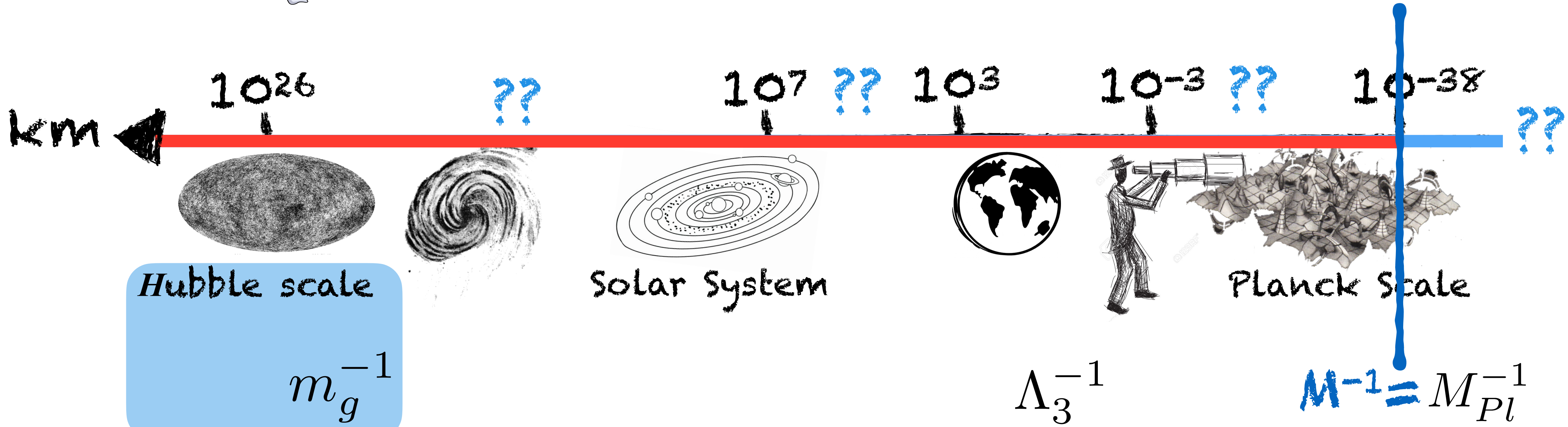
c_2 $\quad$ $c_{2,1}$

$\Lambda_3 \equiv \sqrt[3]{M_{Pl} m_g^2}$

Tuning of coefficients

Fierz,Pauli'1930s, Arkani-Hamed,Georgi,Schwartz'02, deRham,Gabadatze,Tolley'10

$c_{2,1} s_{max} \lesssim 5 c_2$ ▸ $s_{max} \lesssim 5 m_g^2$



- Might Solve the c.c. problem
- Exp bound $m_g^{-1} \gtrsim 0.1 H$

▸ Massive Gravity not compatible with unitarity bounds

4. Strong Coupling

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

IR arcs

EFT amplitude (forward)

$\beta_4 = \frac{7c_2^2}{160\pi^2}$

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + c_6 s^6 + \cdots$$

Coupling $c_2 s^2$

▸ **Arcs** $\mathcal{A}_n \equiv \int_{\cap_{\hat{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^{2n+3}}$

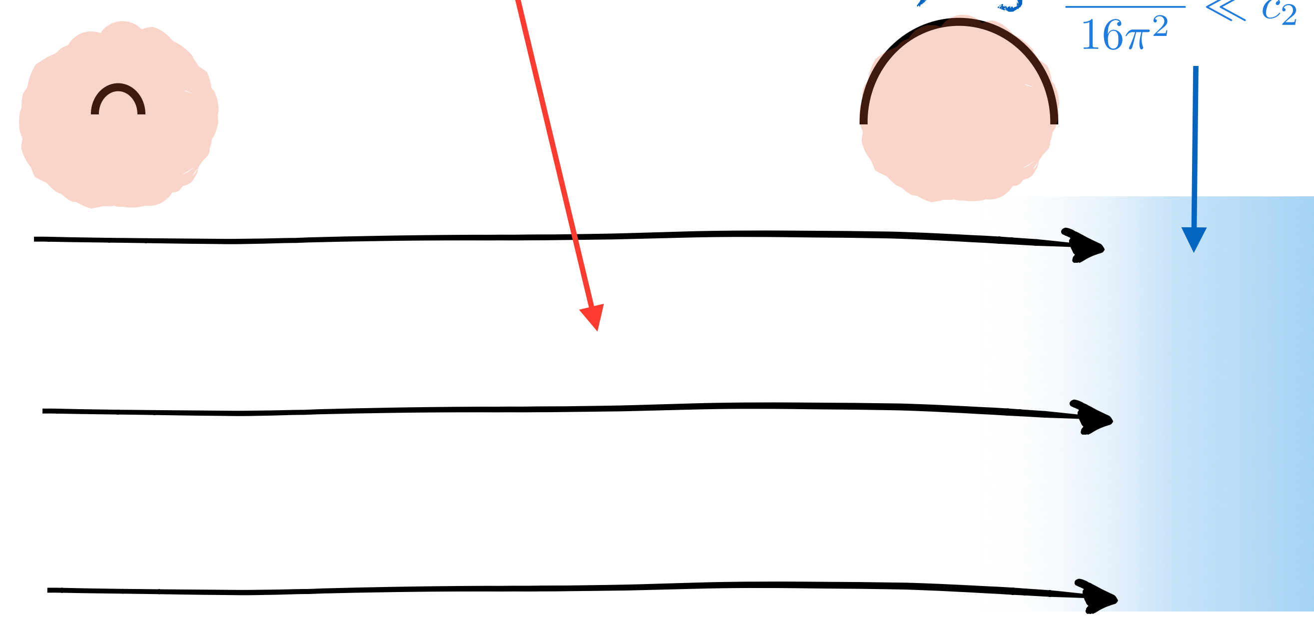
Smaller window in which theory looks tree-level

Assume higher loops small, e.g. $\frac{c_4 c_6 s^8}{16\pi^2} \ll c_2$

$$\mathcal{A}_0 = c_2 + \cdots$$

$$\mathcal{A}_1 = c_4 + \cdots$$

$$\mathcal{A}_2 = c_6 + \cdots$$



Weak Coupling: $\mathcal{A}_n = c_{2n+2}$, all $\mathcal{L}$ couplings captured by arcs

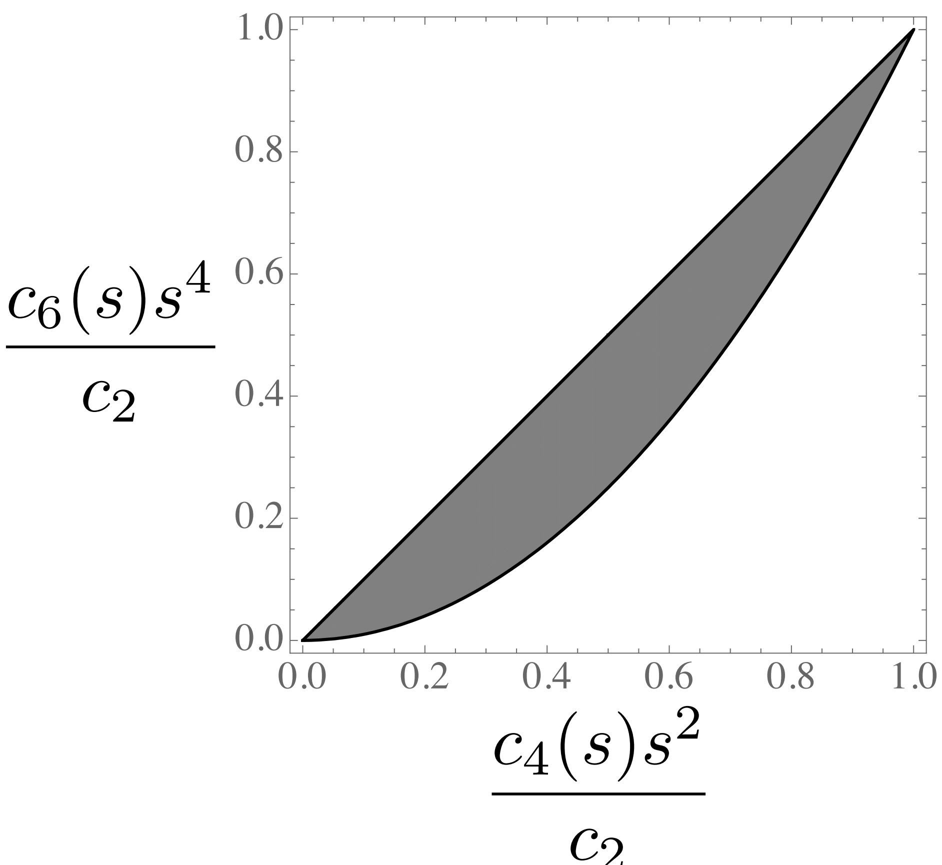
Strong Coupling (e.g ChiPT): high arcs dominated by c_2 loop effects!

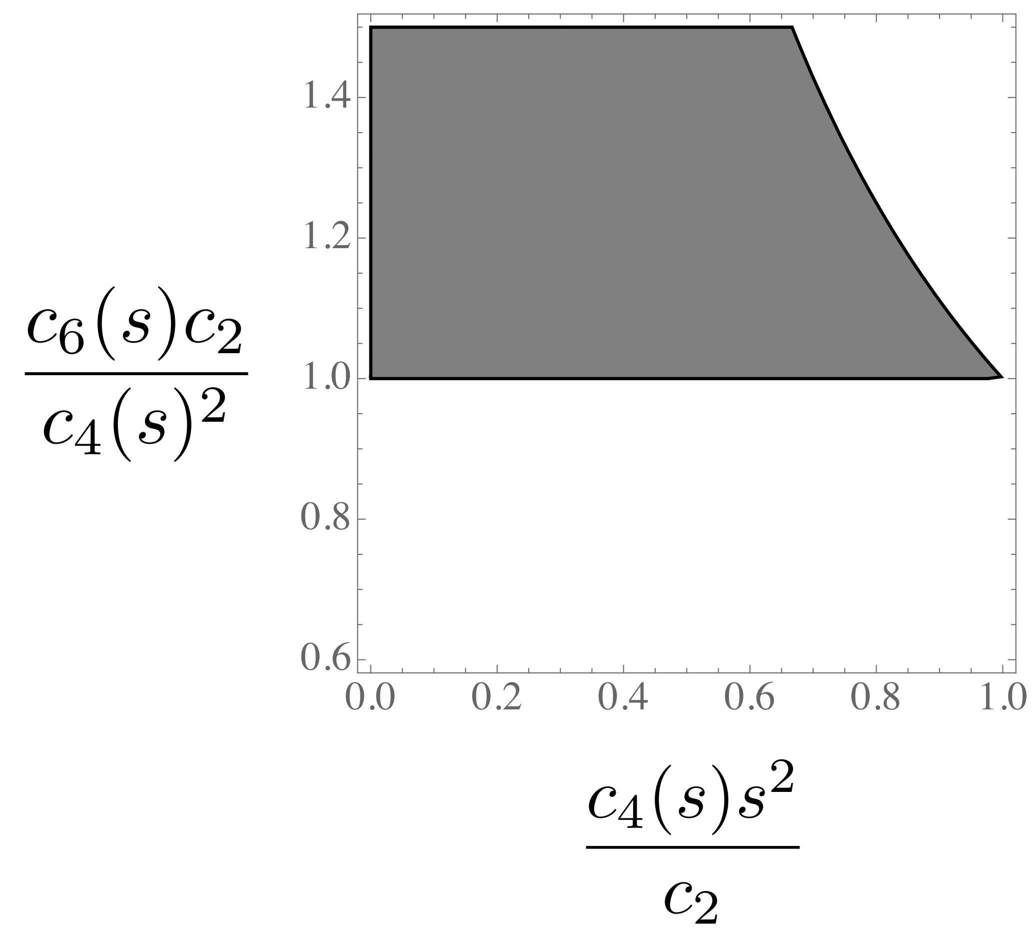
▸ Information unaccessible

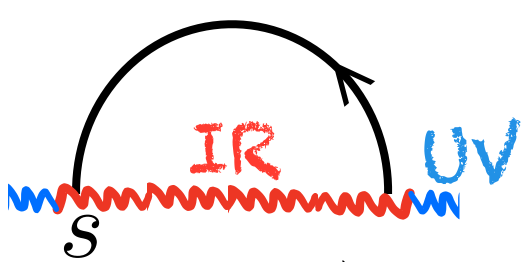
Running

Do bounds apply for running coefficients $c_n(s)$?

$$A(s) = c_2 s^2 + s^4 \underbrace{[c_4 + \beta_4 \log(-is)]}_{c_4(s)} - i\pi s^5 \beta_5/2 + s^6 \underbrace{[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)]}_{c_6(s)} + \cdots$$







Arcs: suitable to access running coefficients

$$\mathcal{A}_0 = c_2 + \cdots$$

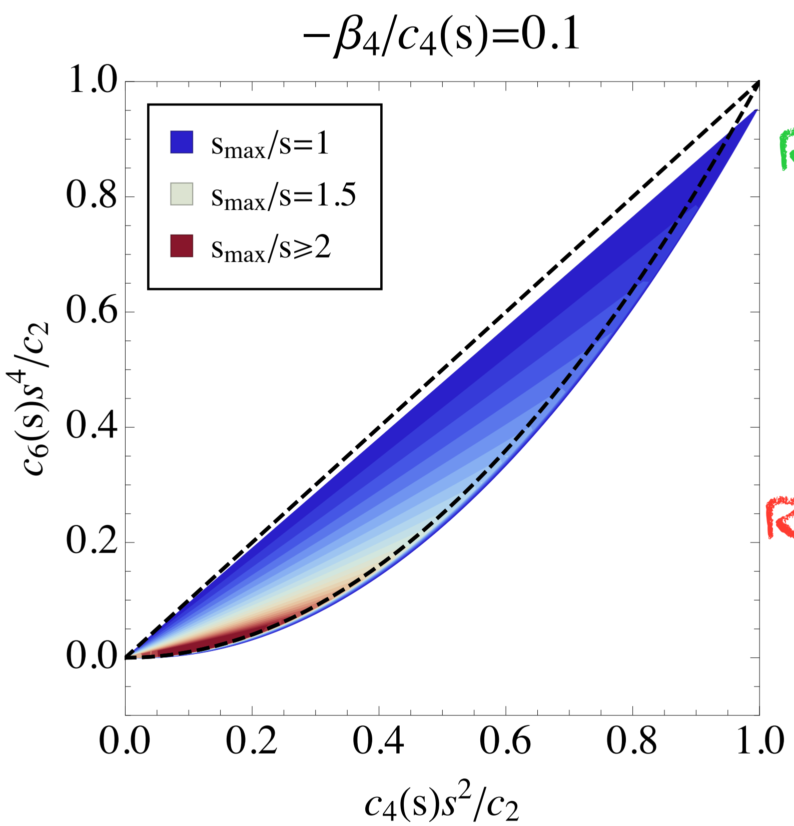
$$\mathcal{A}_1 = c_4(s) + \cdots$$

$$\mathcal{A}_2 = -\frac{\beta_4}{2s^2} + c_6(s) + \cdots$$

Running

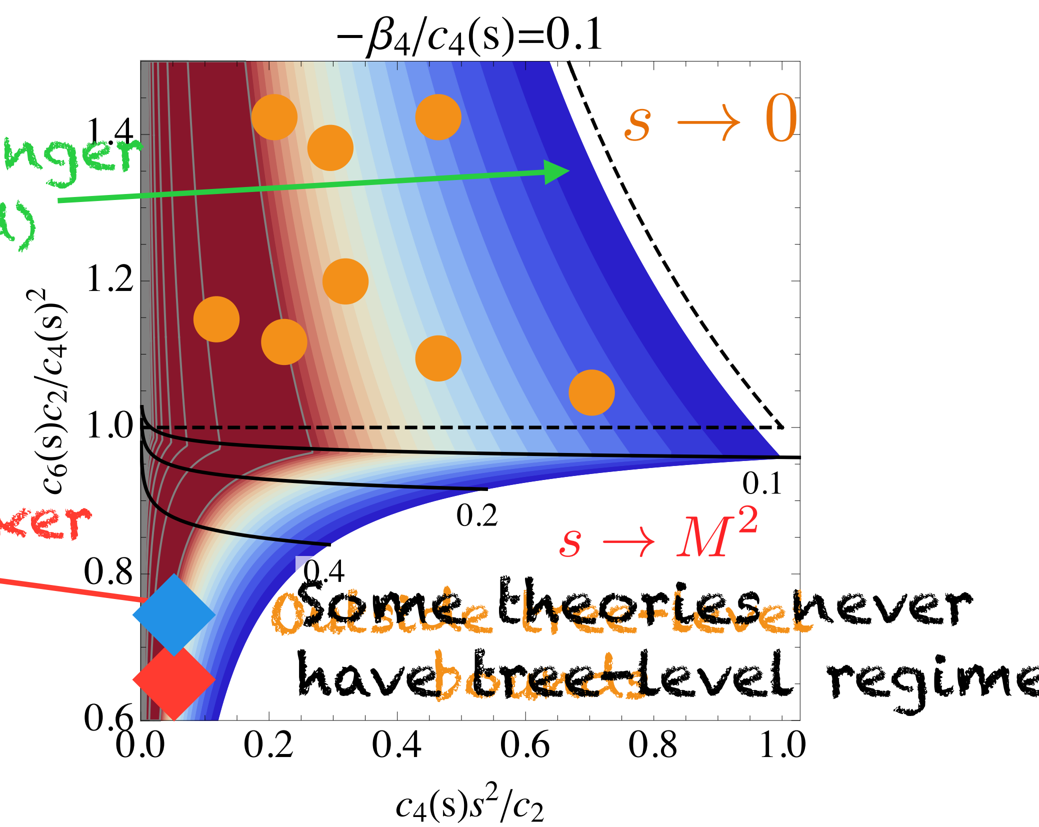
EFT Wilson coefficients run: Do bounds apply for $c_n(s)$?

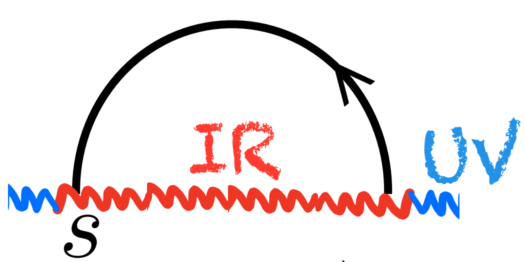
$$A(s) = c_2 s^2 + s^4 \underbrace{\left[c_4 + \beta_4 \log(-is)\right]}_{c_4(s)} - i\pi s^5 \beta_5/2 + s^6 \underbrace{\left[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)\right]}_{c_6(s)} + \cdots$$



Real bounds little stronger
(supersoftness still dead)

Real bounds much weaker
e.g. $c_6(s) < 0$ ok





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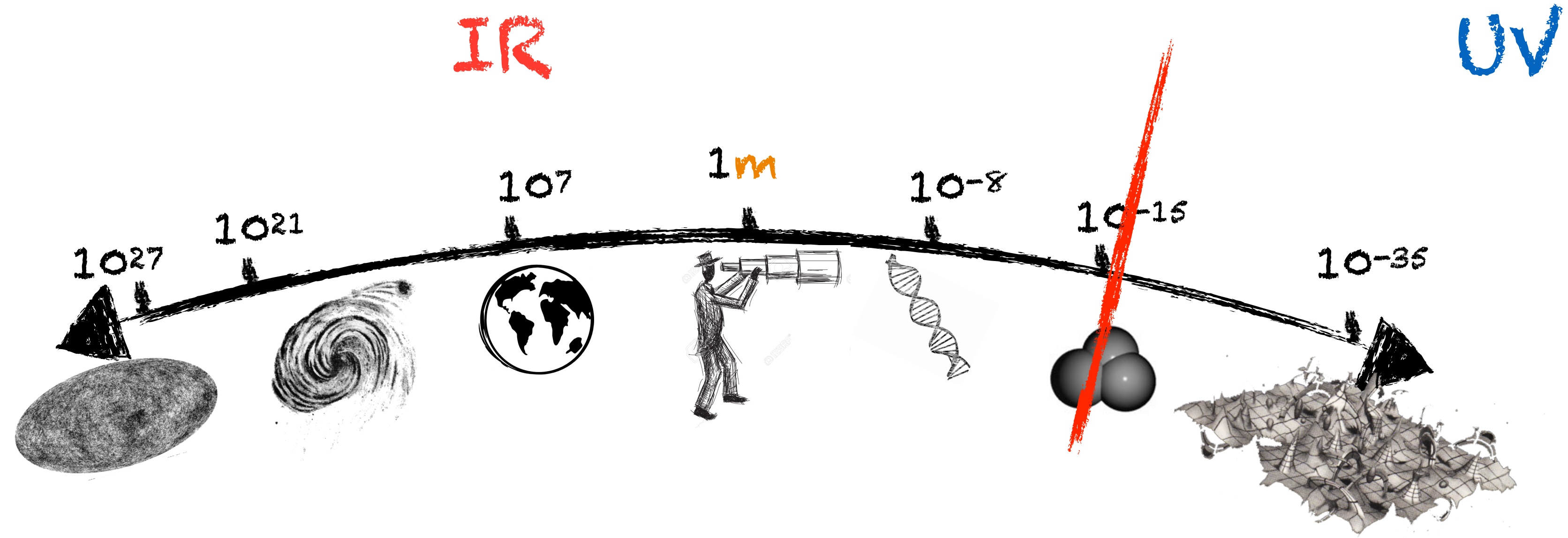
powers of t / powers of s

c_2	$c_{2,1}$	$c_{2,2}(s)$	$\cdots$
$c_4(s)$	$c_{4,1}(s)$	$c_{4,2}(s)$	$\cdots$
$c_6(s)$	$c_{6,1}(s)$	$c_{6,2}(s)$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	

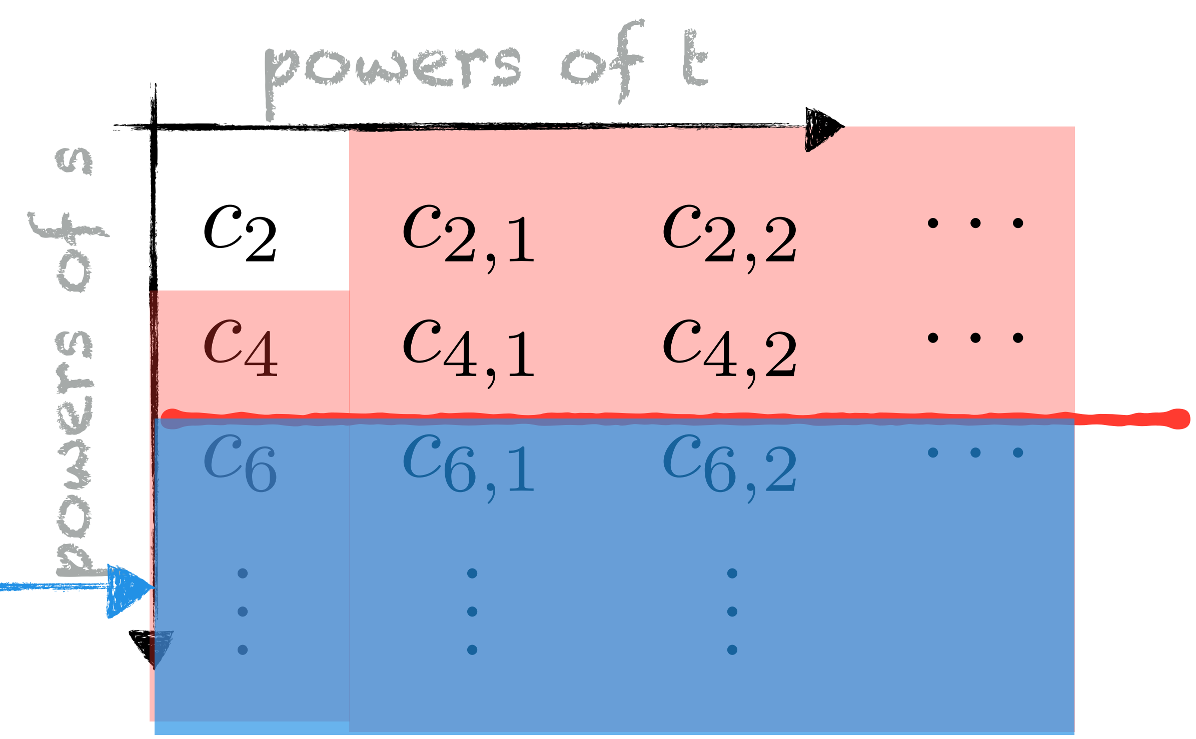
Summary



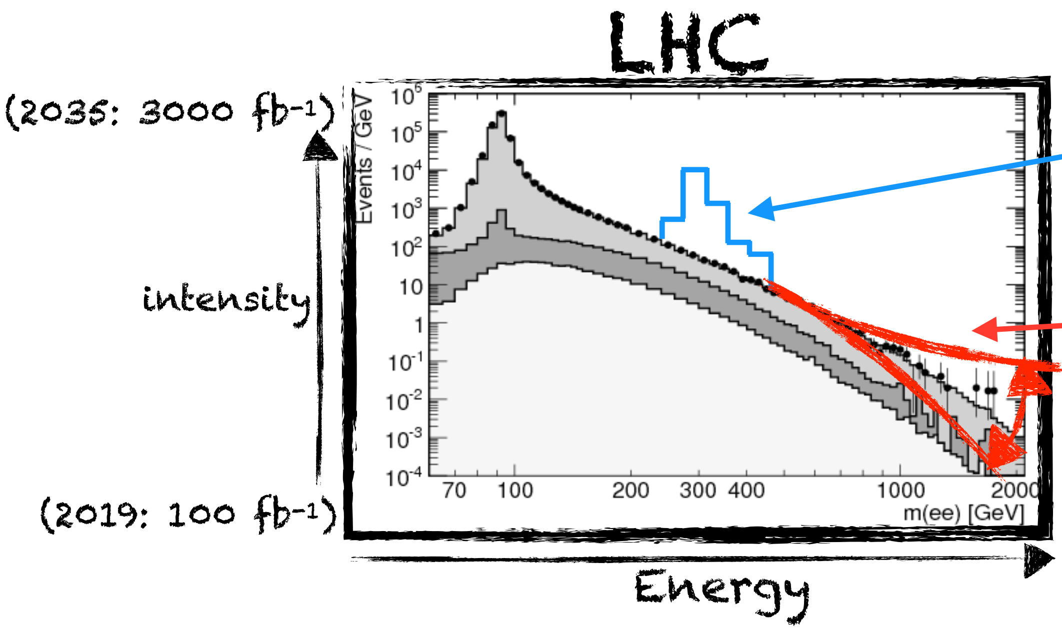
Constrained EFTs ← moments ← Causality, Unitarity, Lorentz invariance, Locality

- Only 3(2) coefficients can dominate
- ~~Supersoftness~~
- $m_{HS} > 1/L_{HS}$
- massive gravity X

IR running important →



SM Precision tests



New Physics searches (so far)

New Physics searches (future) : EFTs

Operator Dimension

Standard Model - 4

Experimental bounds

5

6

Experiments target leading effects
(generically dim-6)

7

Overlap?

8

Froissart Bound

BSM

9

10

11

2-sided bounds
(can't be the leading effect)

12

violate B,L

...

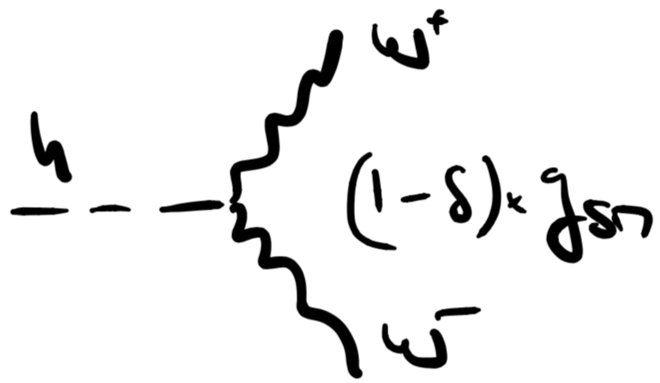
Theory Positivity Bounds

SM Precision tests

Low,Rattazzi,Vichi'12
Falkowski,Rychkow,Urbano'12
Remmen,Rodd'20
Zhang,Zhou'20

UV Assumptions

Stronger UV convergence → Bounds/sum rules on dimension-6



$\delta<0$ → Isospin-2 light resonance

Standard Model - 4

Experimental bounds

5

6 Experiments target leading effects

7

8

Froissart Bound

BSM

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...

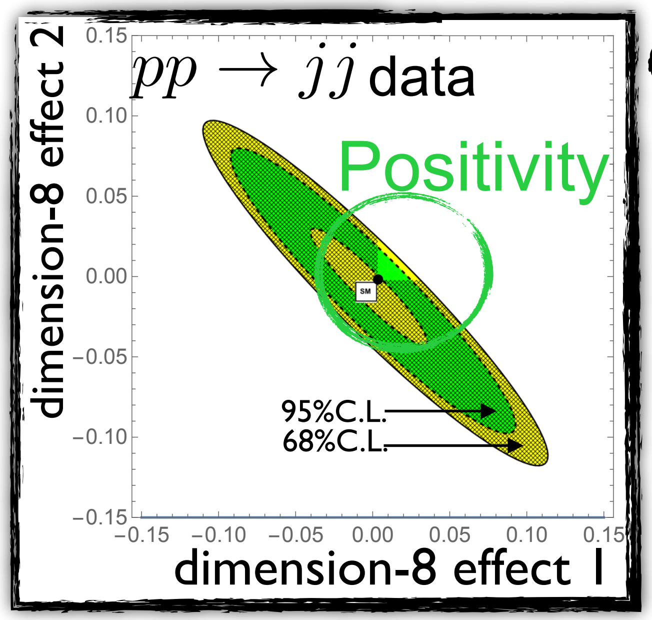
violate B,L

2-sided bounds
(cant be the leading effect)

Theory Positivity Bounds

SM Precision tests

IR Features Non-linear IR symmetries: suppress dim-6
→experiments target dim-8

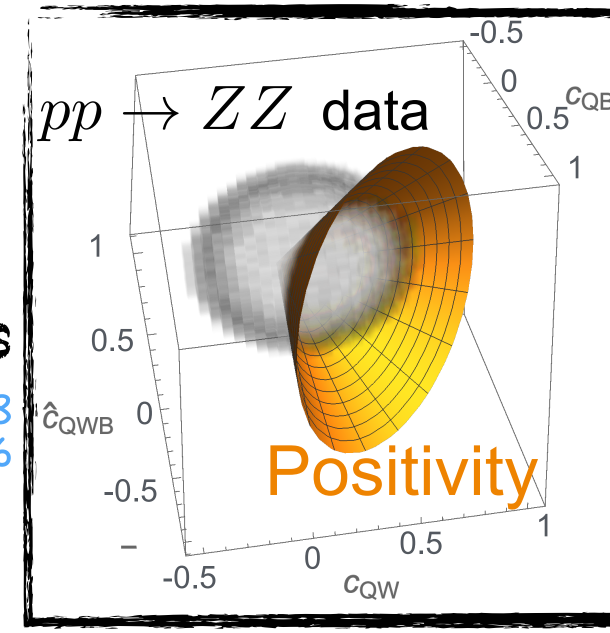


Quarks as PseudoGoldstini:
Bellazzini,FR,Sgarlata,Serra'17

$$\mathcal{L}_{int} \sim (\bar{\psi}\gamma^\mu \partial^\nu \psi)^2$$

Pseudo-Goldstone Higgs (flat coset):
Bellazzini,FR'18
Liu,Pomarol,Rattazzi,FR'16

$$\mathcal{L}_{int} \sim D_\mu H^\dagger D_\nu H W^{\mu\tau} W^\nu_\tau$$



Standard Model - 4 | Experimental bounds
Bellazzini,FR'18

5
6
7
8 — Experiments target leading effects
9
BSM
10
11
12
...

Froissart Bound

violate B,L

2-sided bounds
(cant be the leading effect)

Theory Positivity Bounds

SM Precision tests

IR Features processes without dim-6

→experiments target dim-8

- $e^+e^-/\bar{q}q \to ZZ$ no dimension-6! Bellazzini,FR'18 Gu,Wang,Zhang'20
- Helicity selection rules:

Azatov,Contino,Machado,FR'16

A_4	$h(SM)$	$h(\mathcal{O}^6)$	$h(\mathcal{O}^8)$
$VVVV$	0	4,2	4,0
$VV\phi\phi$	0	2	2
$VV\psi\psi$	0	2	2
$V\psi\psi\phi$	0	2	2

← total helicity

← dim-6 dont interfere with SM, dim-8 do

- Positivity guide experiment

Standard Model - 4

Experimental bounds

5

6

7

8

Experiments target leading effects

Froissart Bound

BSM

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...

violate B,L

2-sided bounds

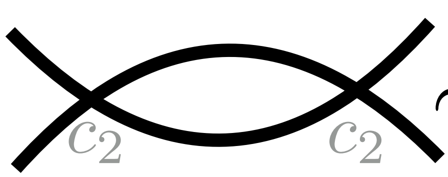
(cant be the leading effect)

Theory Positivity Bounds

IR Effects alter Bounds

Change relation Wilson coeff. ⟷ arcs (on which bounds apply)

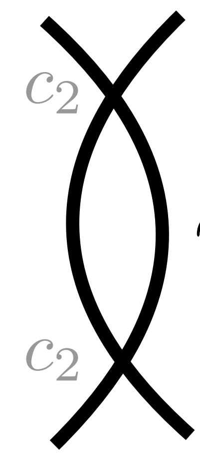
Running



$$\sim \frac{c_2^2 s^4}{16\pi^2} \log \frac{s}{\mu}$$

$$\delta \mathcal{A}_n \sim \frac{c_2^2}{16\pi^2 s^{2n-2}}$$

Collinear Divergences



$$\sim \frac{c_2^2 s^2 t^2}{16\pi^2} \log \frac{s}{t}$$

$$\delta \partial_t^k \mathcal{A}_n \sim \left(\frac{c_2^2}{16\pi^2} \log \frac{s}{m^2} \right)^{k-2}$$

k powers of t

n powers of s

c_2	$c_{2,1}$	$c_{2,2}$	$\cdots$
c_4	$c_{4,1}$	$c_{4,2}$	$\cdots$
c_6	$c_{6,1}$	$c_{6,2}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	

Polygons vs Polynomials

Arkani-Hamed,Huang[2],2020

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Positivity from geometry

Different "functional" approach

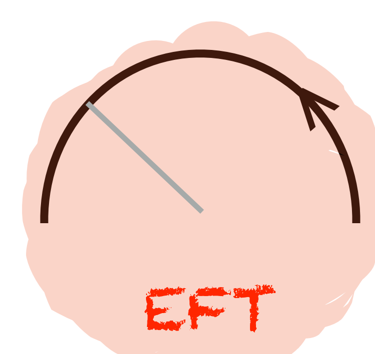
Forward Bounds for infinite arcs same

$$1 - x/2 - x^2/8 + \cdots = q(x) = \sqrt{1-x}$$

- Focus on "Optimal" bounds for finite many arcs, (both forward and at finite-t)
- Two-sided bounds

Residues

Arcs



- Suitable for EFT cutoff estimate
- Ideal for running

3. Finite-t supersoftness and Galileons

Beyond forward:

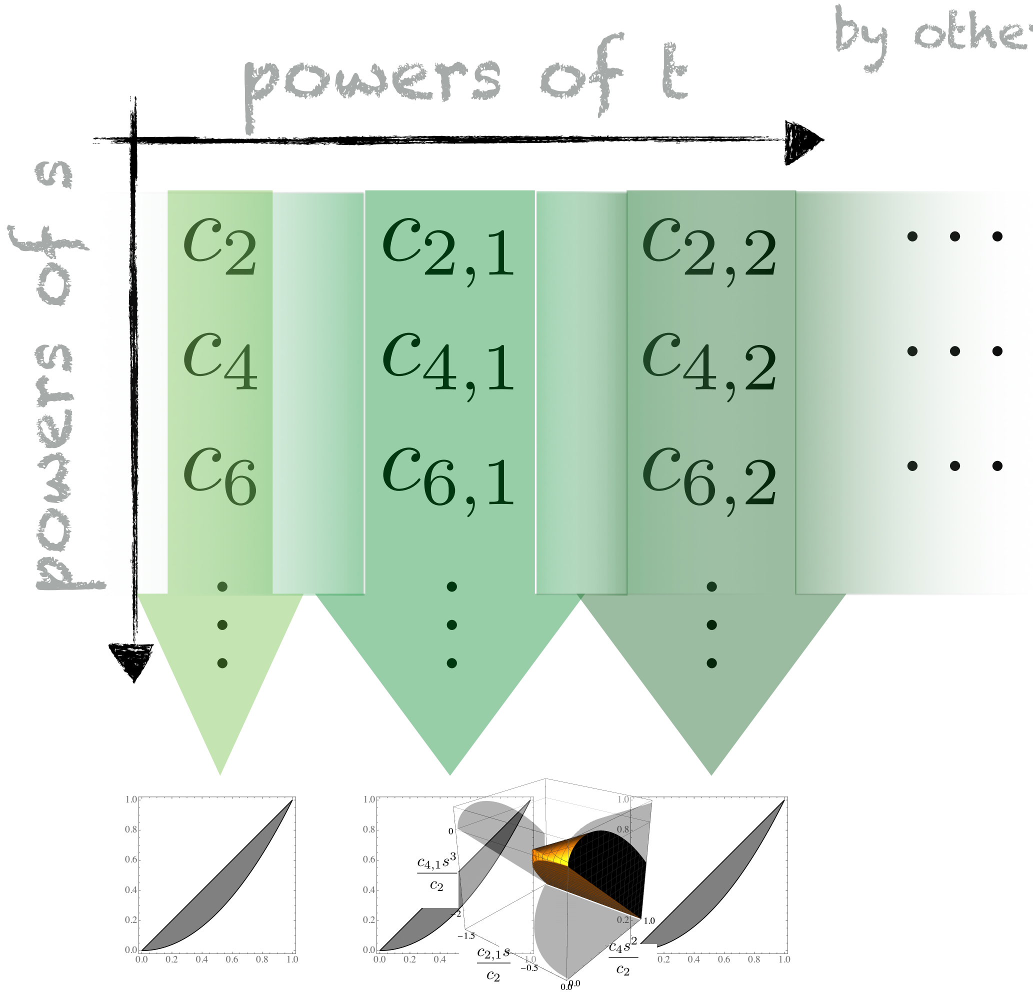
Galileon Nicolois,Rattazzi,Trincherini'08

$$A_{2\to2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

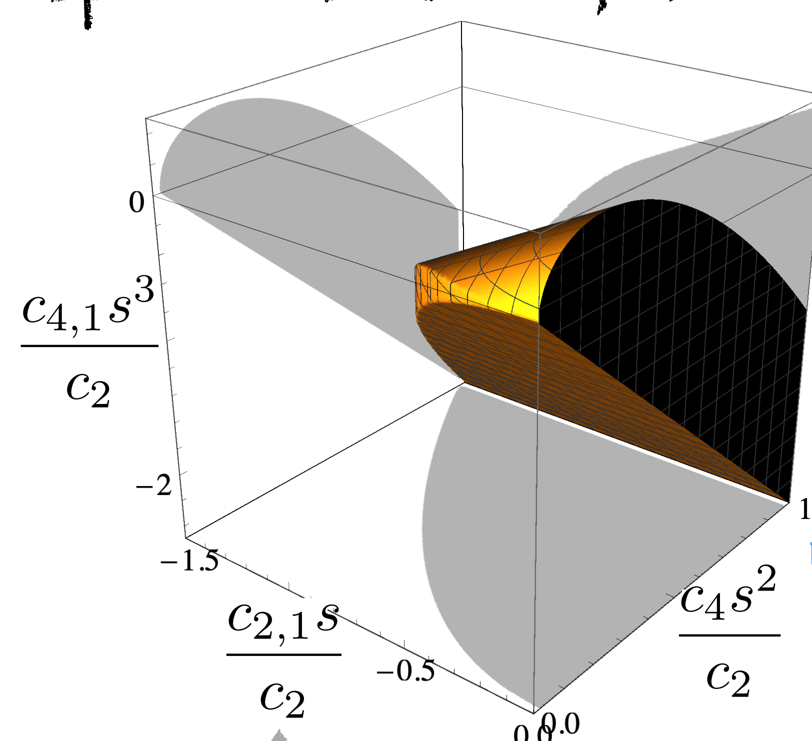
At tree level:

- $c_{p,q} = \partial_t^q A_n(s,t) = \partial_t^q \frac{2}{\pi}\int_s^\infty ds' \frac{\mathrm{Im}A(s',t)}{(\hat{s}' + \frac{t}{2})^{2n+3}}$ $\partial_t^q \mathrm{Im}A|_{t=0} > 0$ Martin'65

Negative, but limited by other moments



Optimal bounds for single t-derivative:



Bellazzini,Elias-Miro, Rattazzi,Riembau,FR'20

Can be slightly negative $c_{2,1} > -\frac{3}{2}\sqrt{c_4 c_2}$

also deRham,Melville,Tolley,Zhou'17 : $> -3c_2/2s$

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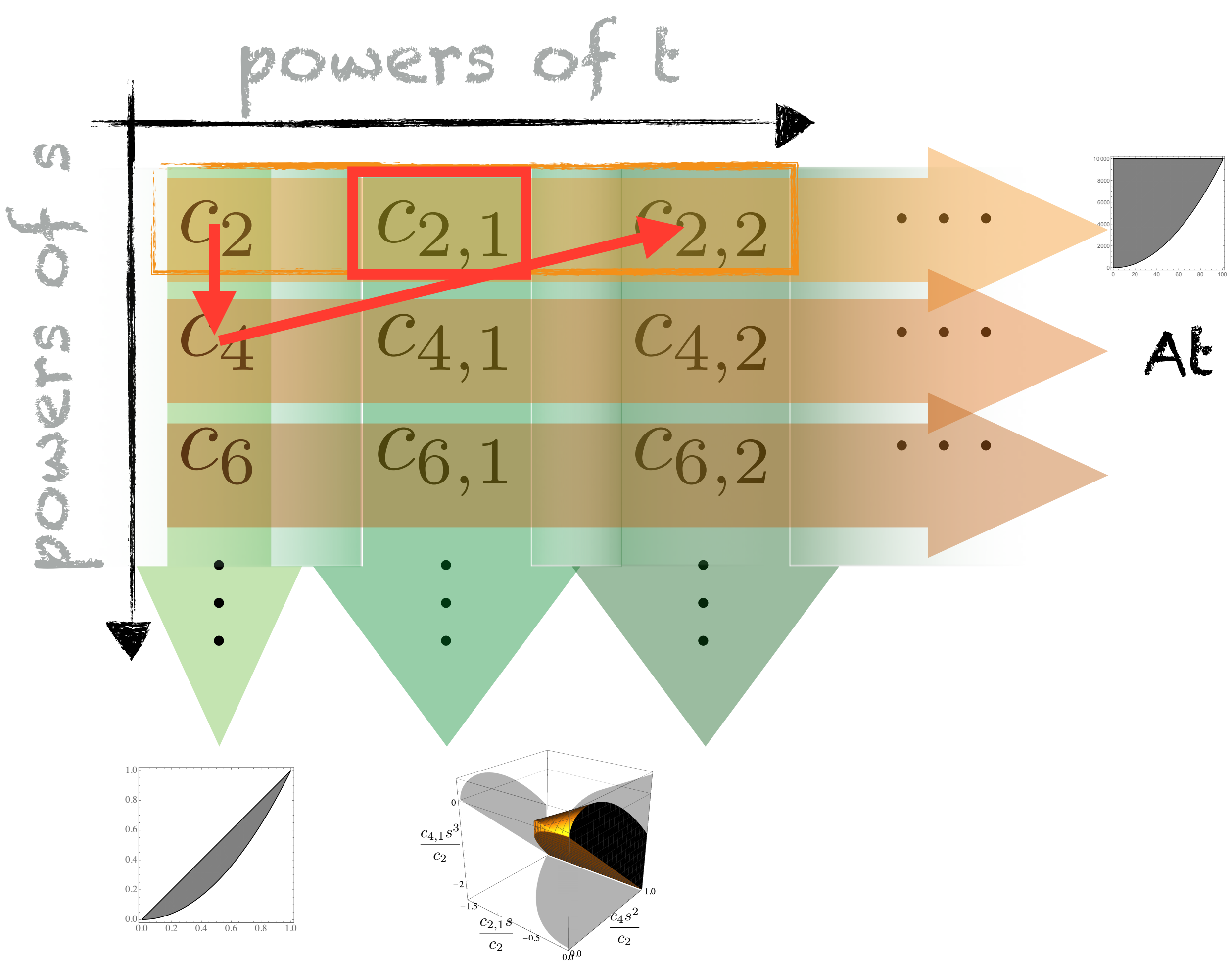
$$\partial_t^q \mathrm{Im}A|_{t=0} = \sum_{\ell=0}^{\infty} \frac{(\ell+q)!}{(\ell-q)!q!}\mathrm{Im} f_\ell(s)$$

Arkani-Hamed,Huang²,2020

$$\sim \int_0^\infty d\mu(l)\, l^{2q}$$

Moments in l

Bellazzini et al, to appear



At tree-level* x-ing symmetry implies

$$A(s,t) = \cdots + g_4(s^2+t^2+u^2)^2 + \cdots$$

$$c_{2,2} = 2c_4$$

$$-\frac{3}{2}c_2 < c_{2,1}s < 8c_2$$

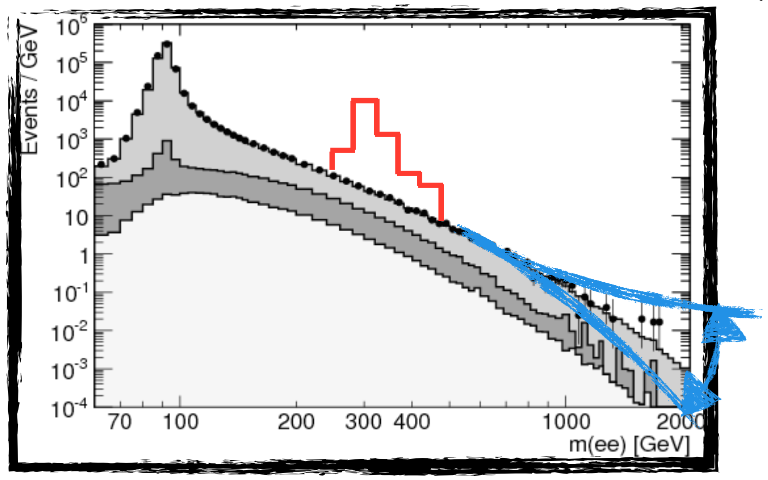
Tolley,Wang,Zhou'20
Caron-Huot,vanDuong'20

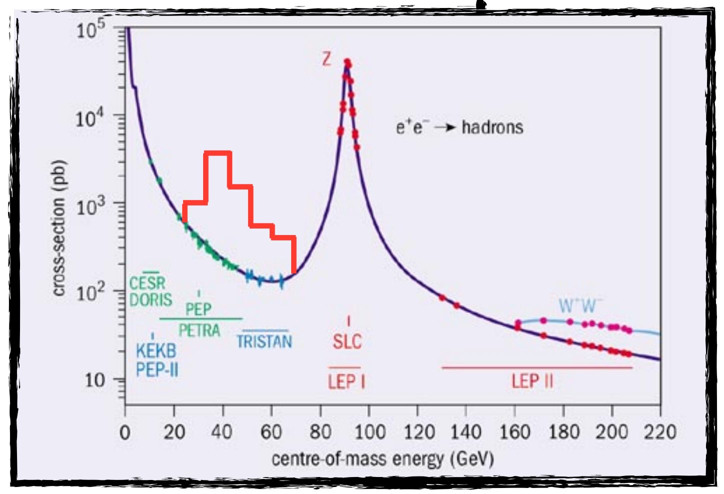
Galileons tightly bounded!

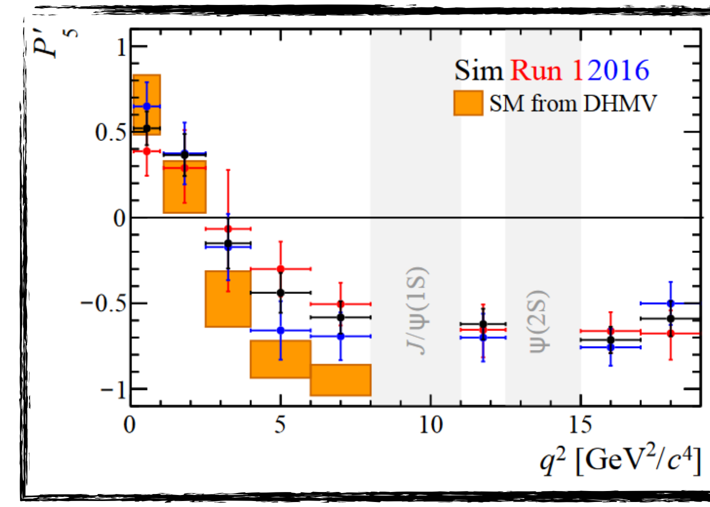
* See loop effects later

Precision Measurements

At the edge of experimental capabilities:



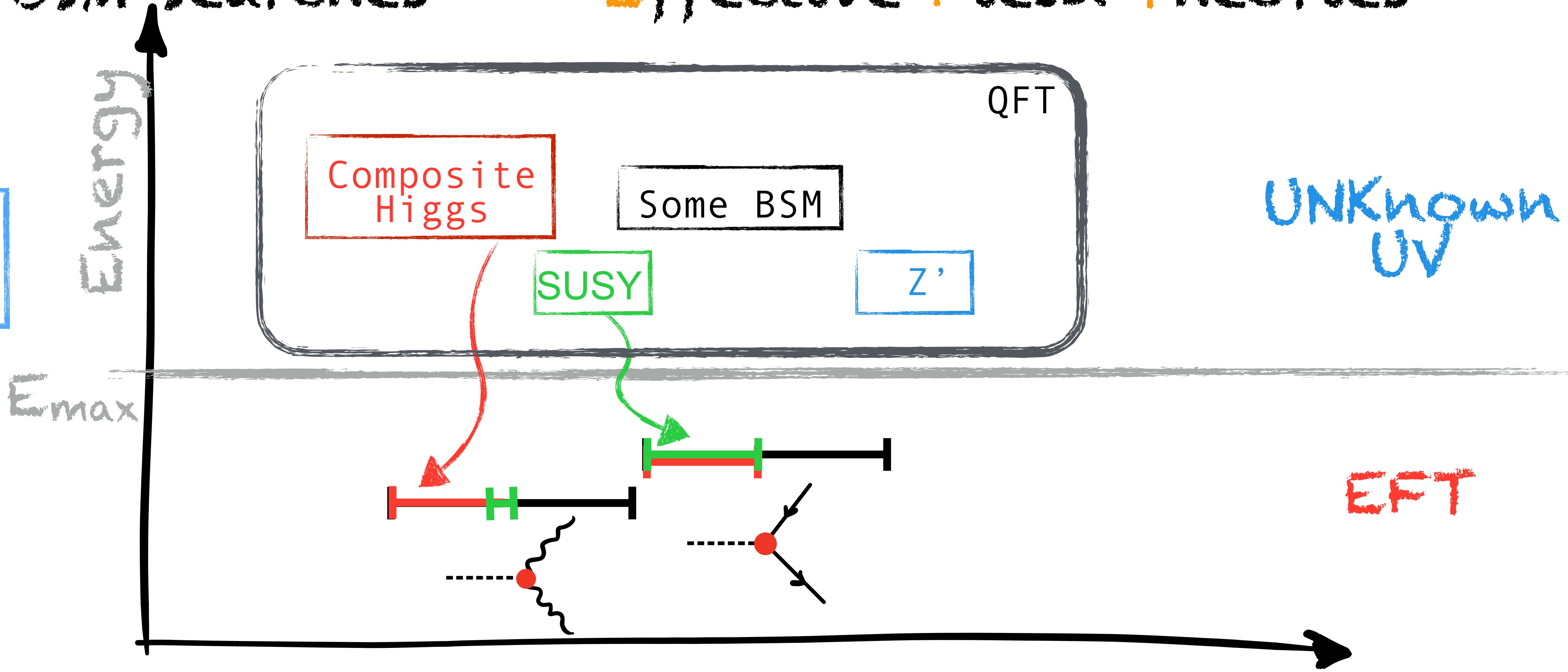




BSM Searches ↔ Effective Field Theories

What can be learned?

What can be measured?



Are there generic predictions common to all BSM?

Precision Measurements

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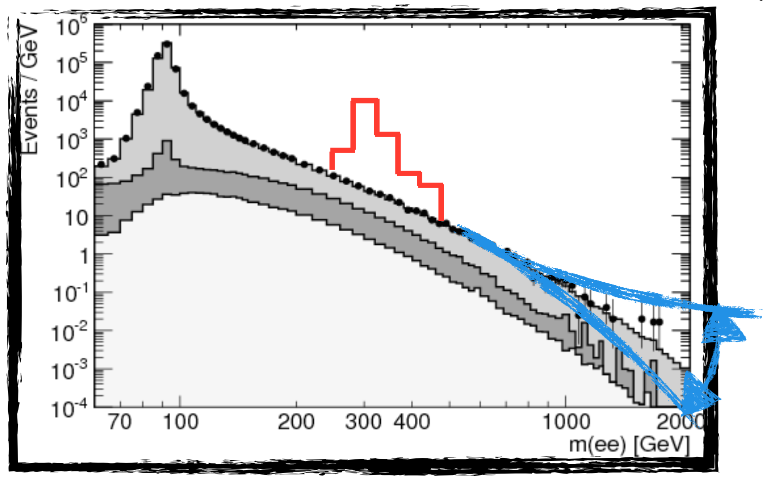

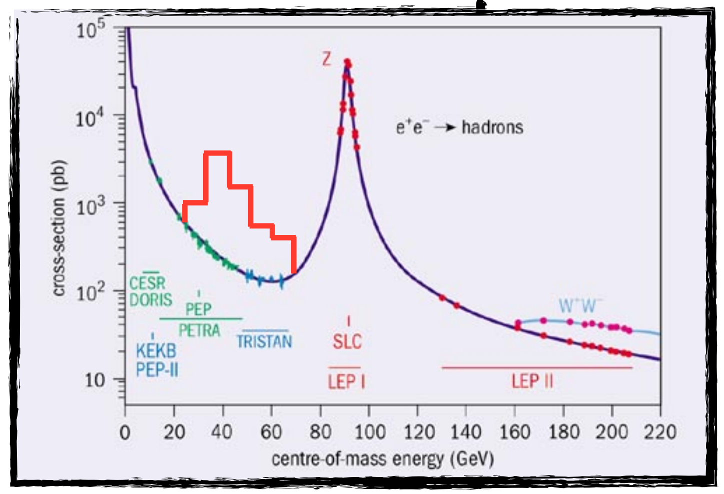

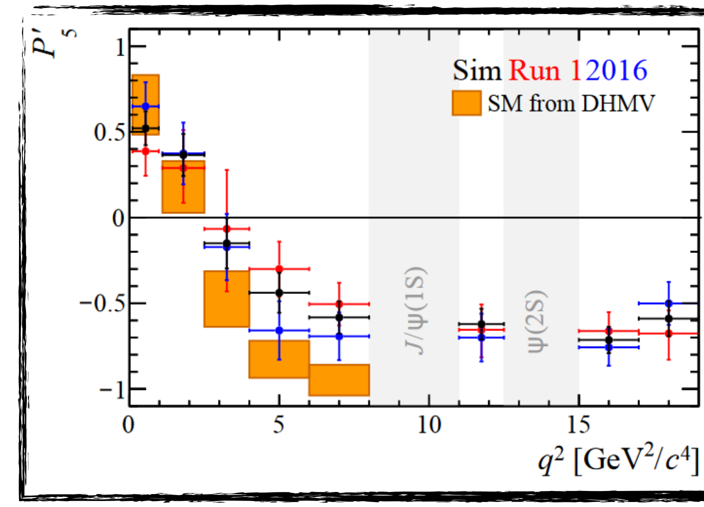

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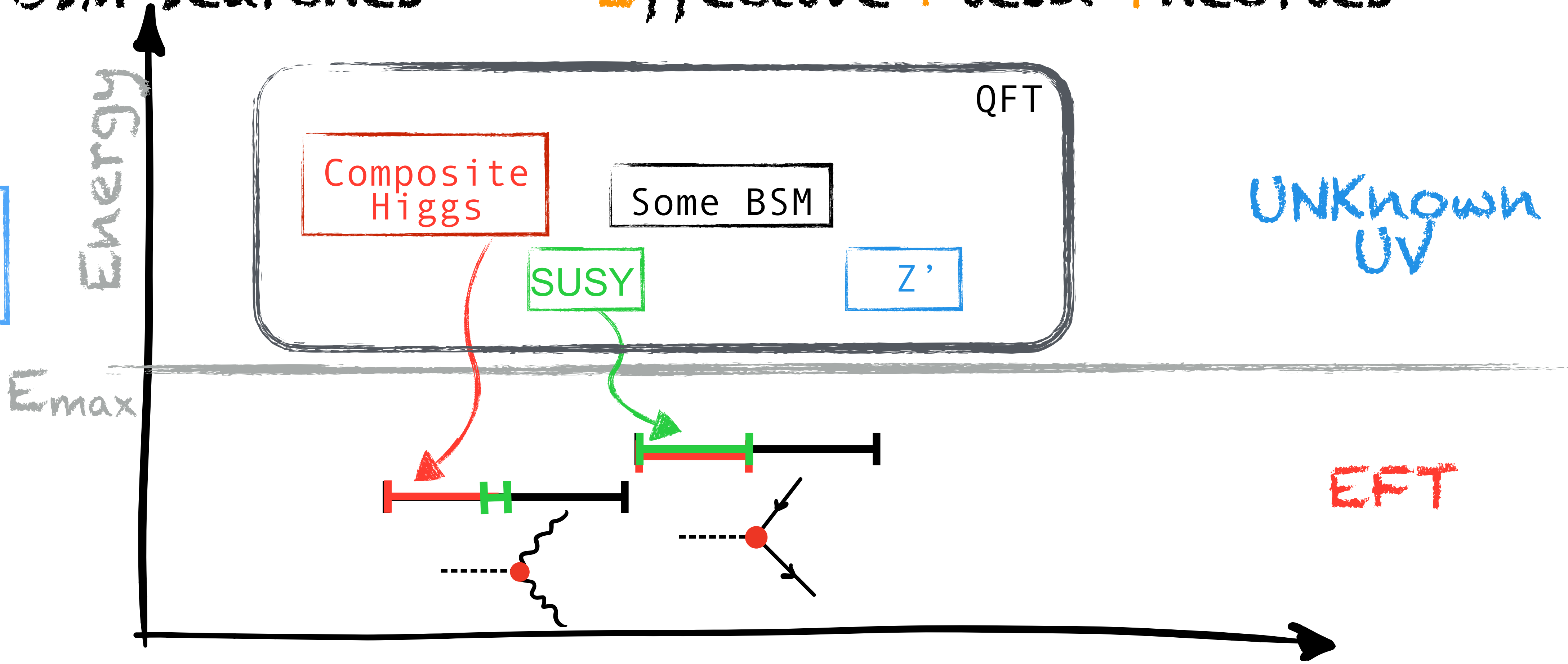

What can be measured?

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Arkani-Hamed,Huang[2],2020
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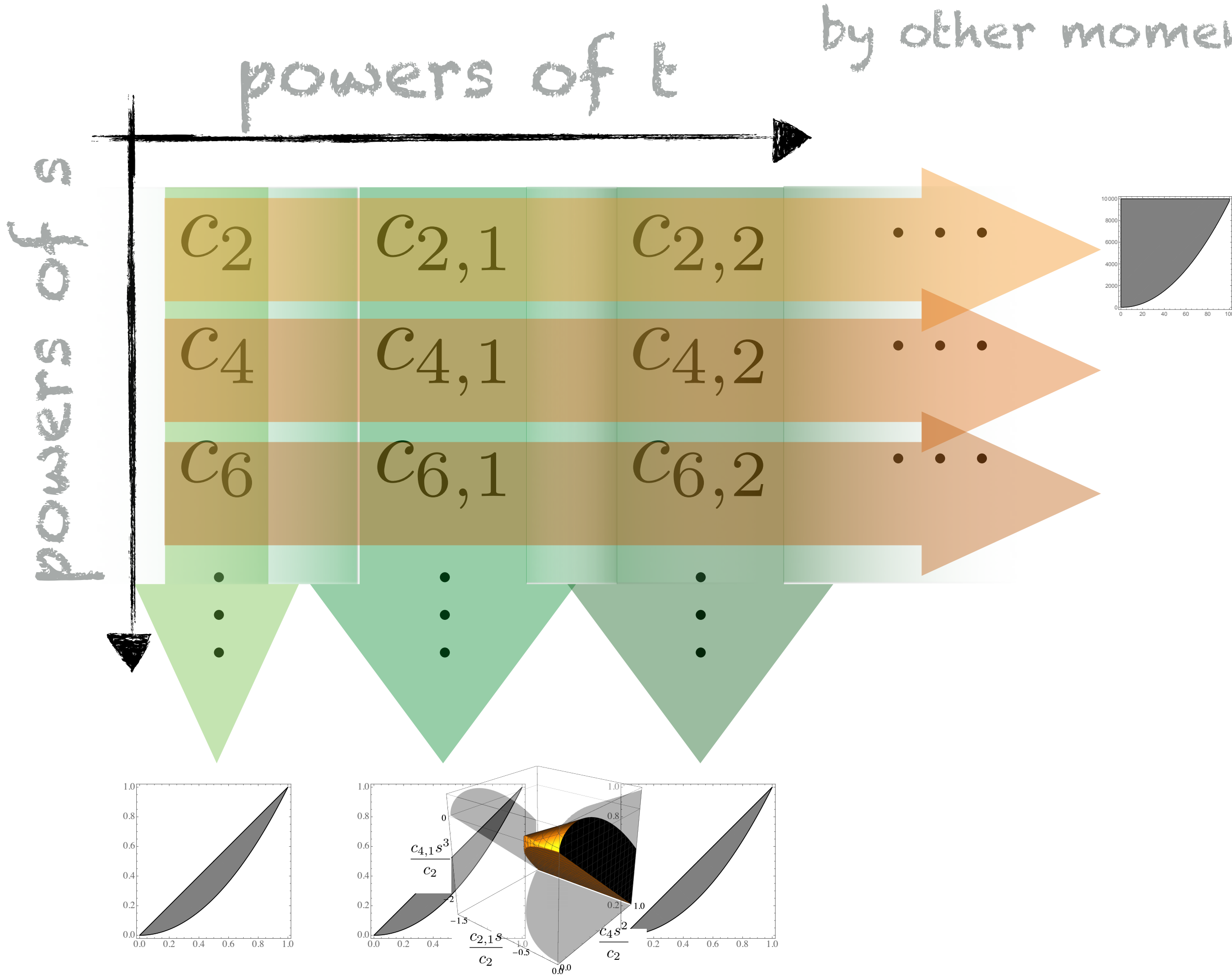

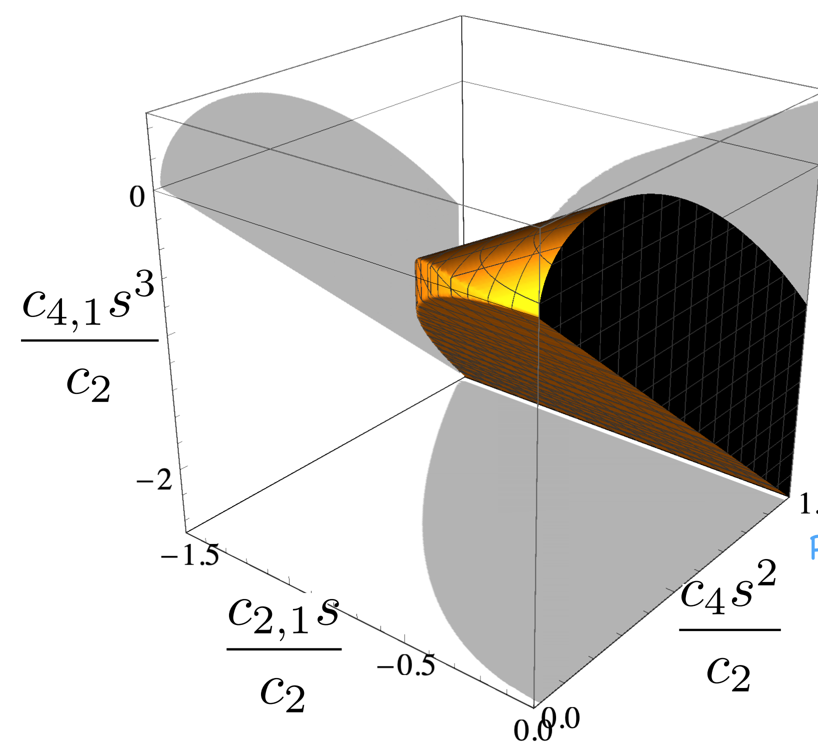

Bellazzini,Elias-Miro,
Rattazzi,Riembau,FR'20