Lect. n. 1

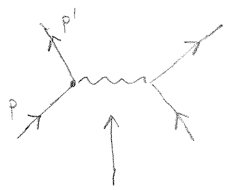
"Effective Theory" $\longrightarrow$ concentrate only on the relevant d.o.f in a given energy / scale domain. Opposite to "Fundamental Th." (Does it exist a F.T.? Not obvious...)

Q.E.F.T. $\mapsto$ E.T. concept within Q.F.T.

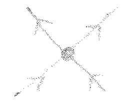
- particularly useful given the U.V. problems of QFT
- new point of view on the renormalization problem

Examples:

1)



$\Rightarrow$ Fermi Theory



$$\frac{1}{q^2-M_W^2} = -\frac{1}{M_W^2}\left[1+\frac{q^2}{M_W^2}+O(q^4)\right] \Rightarrow \mathcal{L}_{eff} = \frac{g^2}{8M_W^2}(\bar\psi\gamma^\mu\psi)^2 + \frac{g^2}{8M_W^4}(\bar\psi\gamma^\mu\psi)\partial^2(\bar\psi\gamma^\mu\psi)+\dots$$

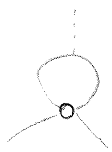
$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Does it make sense beyond the classical limit (tree-level)?

$\Downarrow$

possible new U.V. divergences

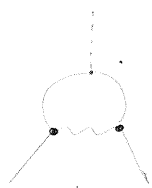
$\boxed{\text{Do the heavy particles decouple when they are inside loops?}}$



$\downarrow$

$G_F \Lambda^2 \bar\psi\gamma^\mu\psi H$

power-like divergence



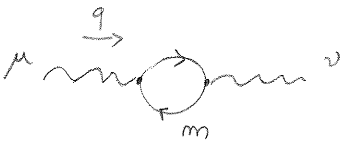
$\downarrow$

$g^2 \log\left(\frac{\Lambda^2}{M_W^2}\right) \times \bar\psi\gamma^\mu\psi H$

logarithmic (renormaliz.) divergence

2) The QED vacuum polarization case:

$\boxed{\text{We want to understand what happens when } m \to \infty \text{ beyond the tree-level}}$



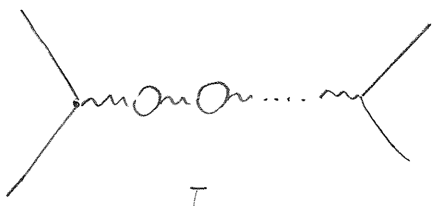
$$\hookrightarrow \quad \Pi^{\mu\nu} = e^2 \int_{\text{"Reg"}} \frac{d^4 \ell}{(2\pi)^4}\, \mathrm{tr}\left[\gamma^\mu \frac{1}{\not\ell + \not q - m} \gamma^\nu \frac{1}{\not\ell - m} \right]$$

$$= i\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \Pi(q^2)$$

Various regularization possible (Pauli-Villard / dim. reg.)

$$\Pi(q^2) = -\frac{e^2}{2\pi^2} \int_0^1 dx\, x(1-x) \left[\ln\left(\frac{m^2 - q^2 x(1-x)}{\Lambda^2} \right) + C \right]$$

$$\left(\text{dim. reg.} \quad C = \frac{2\mu^{d-4}}{4-d} + \gamma_E - \ln(4\pi) \qquad \Lambda^2 = \mu^2 \right)$$



$\hookrightarrow$ effective propagator:

$$\frac{-i g^{\mu\nu}}{q^2} e_0^2 + \frac{-i g^{\mu\nu}}{q^2} e_0^2 \frac{(-i)^2}{\cancel{q^2}} \cancel{q^2}\, \Pi(q^2) + \dots$$

$$= -i \frac{g^{\mu\nu}}{q^2} \underbrace{\frac{e_0^2}{[1 + \Pi(q^2)]}}_{\hookrightarrow \text{ effective charge}} \equiv -i \frac{g^{\mu\nu}}{q^2} (4\pi\alpha)$$

$\downarrow$

Renormalization at $q^2 = 0$ $\Rightarrow$ $e_0^2 = 4\pi\alpha(0)\,[1 + \Pi(0)]$

Renormalization at $q^2 = -Q^2$ $\Rightarrow$ $e_0^2 = 4\pi\alpha(Q^2)\,[1 + \Pi(-Q^2)]$

$$\alpha(Q^2) = \frac{\alpha(0)}{[1 + \underbrace{\Pi(Q^2) - \Pi(0)}_{\Delta\Pi(-Q^2)}]}$$

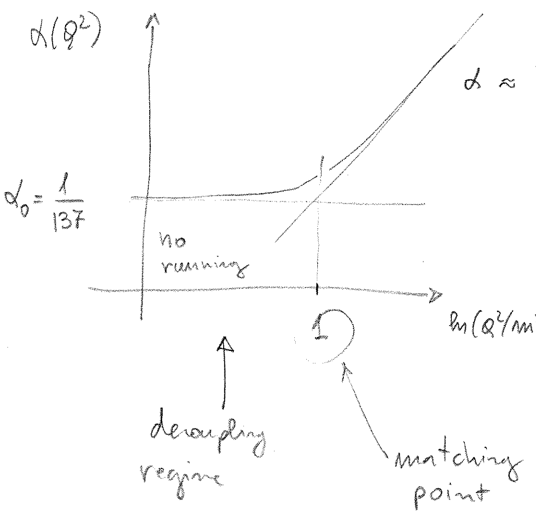
$\Delta\Pi(-Q^2) \longrightarrow$ indep. of regularization prescriptions

$$\Delta\Pi(-Q^2) = -\frac{2\alpha}{\pi}\int_0^1 dx\; x(1-x)\,\ln\left(\frac{m^2+Q^2x(1-x)}{m^2}\right)$$

$$\Delta\Pi(-Q^2) \xrightarrow{Q^2 \ll m^2} -\frac{2\alpha}{\pi}\int_0^1 dx\,[x(1-x)]^2\,\frac{Q^2}{m^2} = -\frac{\alpha}{15\pi}\,\frac{Q^2}{m^2}$$

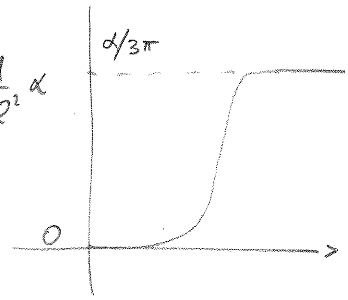
$$\Delta\Pi(-Q^2) \xrightarrow{Q^2 \gg m^2} -\frac{2\alpha}{\pi}\int_0^1 dx\; x(1-x)\,\ln\left(\frac{Q^2}{m^2}\right) = -\frac{\alpha}{3\pi}\left[\ln\left(\frac{Q^2}{m^2}\right)-\frac{5}{3}\right]$$

$$+\,x(1-x)\,\ln(x(1-x))$$



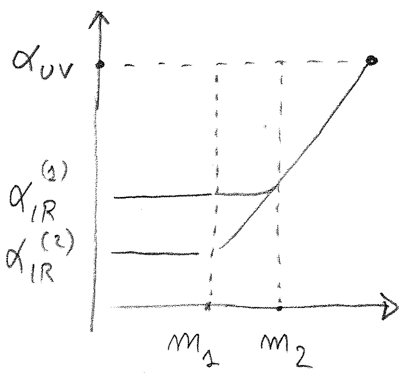
$$\alpha \approx \frac{\alpha_0}{1-\frac{\alpha_0}{3\pi}\ln\left(Q^2/m^2\right)+\dots}$$

$$\beta_\alpha = Q^2\frac{d}{dQ^2}\alpha$$



The heavy state "decouple" → no influence on physics at $Q^2 \ll m$

BUT for redefinition of the renormalizable coupling constant at low energies



we can observe "physical" effects of the heavy statutes only when $Q^2 \to m^2$

$\Rightarrow$ <u>NO NEED TO KNOW</u>

<u>ULTRAVIOLET PHYSICS</u>

⊛ <u>General th.</u> [Appelquist – Carrazone P.R.D. 11 (1974) 2856]

⇒ Given any renormalizable QFT with different mass scales, the role of the heavy fields in the low-momentum behaviour of Green's functions without external heavy fields is <u>ONLY</u> the coupling-constant and field-strength renormalization [DECOUPLING of the HEAVY fields].

⇒ The effective theory obtained neglecting $1/m$ corrections is always a <u>RENORMALIZABLE</u> QFT with the light fields only

<u>Caveat:</u>

A) The decoupling does not occur if the $m \to \infty$ limit breaks the symm. of the "fundamental th." (e.g. $m \to \infty$ only for one comp of a multiplet) or if $m \to \infty$ correspond to $g \to \infty$ (e.g. m_t in SM) ⇒ eff. th. non renormalizable in the classical sense

B) The renormalizability of the "fundamental theory" is not a necessary condition for the th. to apply.

C) The strict $1/m$ limit is also not a limiting point (connection with renormalizability)

↓

modern EQFT (systematic $1/\Lambda_{Heavy}$ expansion)

General formulation [Polchinski - Kaplan]

- Suppose we have a QFT for which we can define a path-integral formulation

- We are interested to study phenomena at energies (scales) $E \ll \Lambda$ and we are able to identify a limited set of "low-frequencies" modes:

$$\phi = \phi_L + \phi_H \qquad \phi_L^{free} \sim e^{i\omega_L t} \quad \omega < \Lambda$$
$$\phi_H^{free} \sim e^{i\omega_H t} \quad \omega_H > \Lambda$$

$$\Downarrow$$

All the information we need is encoded in

$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \; e^{iS(\phi_L, \phi_H) + i\int d^4x \, J_L \cdot \phi_L}$$

(Generic Green's function $\quad \langle 0 | T(\phi_L(x_1) \ldots \phi_L(x_n)) | 0 \rangle \sim \dfrac{\delta^n Z(J_L)}{\delta J_L(x_1) \ldots \delta J(x_n)}$)

NOTES:

⊛ the QFT does not need to be an ordinary renormalizable QFT (e.g. Lattice path integral)

⊛ the separation between ϕ_L & ϕ_H can be complicated in terms of the original fields of the th. (e.g. composite fields)

$\Rightarrow$ ⊛ I assumed we can identify the frequencies of the (low freq.) modes from their free action $\Rightarrow$ weakly coupled regime

$\Rightarrow$ ⊛ Mass-gap (separation of frequencies) is needed

The two steps for the construction of the QEFT are the following

1. Define the Wilsonian action

$$e^{i S_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H \, e^{i S(\phi_L, \phi_H)}$$

"integrate-out" the heavy fields

N.B.: explicit dependence on the "cut-off" Λ

2. Expand $S_\Lambda(\phi_L)$ in series of LOCAL OPERATORS

$$S_\Lambda(\phi_L) = \int d^4x \, \sum_i g_i \, \mathcal{O}_i(\phi_L) = \int d^4x \, \mathcal{L}_{eff}(\phi_L)$$

- All operators allowed by the symmetry of the system
- Usually an infinite sum since $S_\Lambda(\phi_L)$ is not local on scales of $O(1/\Lambda)$

KEY POINT : Only a limited number of operators is sufficient to describe physical processes (at $E < \Lambda$) with arbitrary precision

Easily understood by means of NAIVE DIM. ANALYSIS

$$\mathcal{L}_{eff} = \sum_i g_i \cdot \mathcal{O}_i^{(d_i)} = \sum_i c_i \frac{1}{\Lambda^{d_i - 4}} \mathcal{O}_i^{d_i}$$

canonical dimension (defined by the Kin. term)

adimensional expected to be $O(1)$ (naturalness argument)

The contribution to the <u>action</u> given by a generic operator $O_i^{d_i}$ is

$$\delta S_i \sim c_i \left(\frac{E}{\Lambda}\right)^{d_i-4} \qquad E = \text{energy of the process}$$

d_i	Contribution to S_Λ for $E \to 0$	"classical" renormalization properties	name
<4	grows	Super-renorm. (finite contrib. beyond a certain order in perturb. theory)	RELEVANT
$=4$	constant	renormalizable	MARGINAL
>4	decreases	non-renormaliz.	IRRELEVANT

Usually the number of relevant & marginal operators is very small, the number of irrelevant ops. grows with their "degree of irrelevance"

$\hookrightarrow$ <u>FINITE</u> n° of operators ($\leftrightarrow$ couplings) at <u>FIXED</u> level of precision (which can be arbitrarily small)