CHAPTER 2 The Standard Model Effective Theory

2.1 Motivation

In the previous chapter we have learned how it is possible to build an effective quantum field theory, by expanding a given theory around a certain limit. We have done so by assigning a power-counting to the various fields in the theory and thereby classifying them by importance at low energy scales. We have then split up the field modes of the theory into the categories *hard* and *soft* and constructed the effective theory by removing the hard modes and absorbing their effects on the soft modes into local operators. These local operators, built out of soft modes exclusively, came with coupling constants that encoded the underlying hard physics that we have removed from the theory. The result was that matrix elements of the effective theory factorized into the soft matrix elements and the Wilson coefficients, which encoded the hard physics.

The consequence of this factorization is that the effective theory knows nothing about the ultraviolet dynamics of the full theory it was made from. The only remnants of the full theory, the Wilson coefficients, carry no deeper information about the UV picture - they only determine the interaction strength of the effective operators. For example, in the Fermi theory, the Wilson coefficients did not uniquely give away the presence of the W boson, it was merely a coupling constant that could be measured in beta decay. A W boson with double the coupling strength g and double the mass m_W would have given the same effective coupling.

What sounds like an apparent shortcoming can be turned into a benefit: Using effective theories, we can derive predictions for low-energy processes as functions of the Wilson coefficients without ever having to know about the UV theory: The UV theory is a detail that we need not care about and might very well not even know. We can be fairly certain that this applies to the SM itself: We know from its inability to answer a number of questions, that the SM can not be a complete theory. However, at scales probed by current experiments, it does provide an accurate description of the phenomena observed. So while it cannot be a full theory, it most certainly can be viewed as an EFT.

This is the underlying traing of thought behind the Standard Model Effective Field Theory (SMEFT): It is a bottom-up EFT, where the fields of the SM take the roles of the soft modes. The symmetries of the theory are exactly those of the SM. Whatever UV theory we can imagine will result in a certain set of Wilson coefficients in the SMEFT, but at low energy we do not need to know about these.

The power-counting is given by $\lambda^2 \sim q^2/\Lambda^2$, where Λ is the scale at which the hard modes have been removed.

You can already see a caveat: We do not actually know Λ, since we do not know what the hard modes were in the UV theory. Therefore, when computing processes in the SMEFT, one has to be careful about the validity of the underlying expansion. We have good reasons to believe that new physics (NP) should be situated at scales at least $\mathcal{O}(\text{TeV})$. If there are new particles at 1 TeV, then the local OPE cannot be trusted for processes at high momentum transfer, like the ones happening at the Large Hadron Collider, which has a maximum center-of-mass energy of 14 TeV [1].

In the following we will briefly review the SM itself and then construct the SMEFT from its building blocks.

2.2 Field content and gauge symmetries

Let us begin by specifying the field content of the SM. Let us begin with the gauge group of the SM. It is given by

$$\mathcal{G}_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y \,, \tag{2.1}$$

where $\text{SU}(3)_C$ gives rise to the strong interactions and $\text{SU}(2)_L \times U(1)_Y$ governs the electroweak interactions. After electroweak symmetry breaking (EWSB), the electroweak symmetry is broken into its diagonal subgroub, $\text{SU}(2)_L \times \text{U}(1)_Y \to U(1)_{\text{em}}$, which is the symmetry group of quantum electrodynamics (QED).

The next step is to list the field content along with its transformation properties under this group. Let us begin with the fermions. There are five types of multiplets, transforming under the gauge group as follows:

$$Q_L^i \sim (3,2)_{1/6},\ u_R^i \sim (3,1)_{2/3},\ d_R^i \sim (3,1)_{-1/3},\ L_L^i \sim (1,2)_{-1/2},\ e_R^i \sim (1,1)_{-1}. \tag{2.2}$$

They correspond to the left-handed quarks, the right-handed up- and down-type quarks, the left-handed leptons and the right-handed charged leptons. There are no right-handed neutrinos in the SM. The first number in the bracket denotes the transformation under $\text{SU}(3)_C$, the second number the transformation under $\text{SU}(2)_L$ and the subscript denotes the hypercharge. For example, the fields Q_L^i transform as triplets under the strong group, are weak-isospin doublets and have a hypercharge of 1/6. The index $i = 1, 2, 3$ is the generation index. For example, the doublet $Q_L^3 = (t_L, b_L)$ contains the left-handed top and bottom quarks.

The SM also contains a scalar doublet, the Higgs doublet. It transforms as:

$$\Phi \sim (1,2)_{1/2} \,. \tag{2.3}$$

After it acquires a vacuum expectation value, it generates the mass terms for the fermions and the weak bosons W and Z. The SMEFT however is a theory valid

[1] This however does not mean that the momentum scale of the scattering processes are $q^2 = (14\ \text{TeV})^2$. Since the LHC is colliding protons, the scattering of indivdual partons receives only a fraction of the collision energy and a large part is carried away by the proton remnants not participating in the hard scattering.

between the weak scale $v = 246$ GeV and the NP-scale Λ. In this region, we can treat electroweak symmetry as unbroken and the weak bosons as massless.

We are still missing the four vector bosons in our list: The gluons, the W and Z bosons and the photon. However, as they are gauge fields, they do not enter the Lagrangian as indivdual fields but instead automatically arise from derivatives on the fields: Since the fields transform differently at each point in space time, the derivatives on the fields contain an extra term that compares the difference in the gauge phase rather than the difference of the field value itself. One then introduces a vector field, defined to compensate this term and subtract it from the derivative:

$$\partial_\mu \to D_\mu = \partial_\mu - ig\tau^a A^a_\mu \,. \tag{2.4}$$

In short, the gauge fields appear whenever the gauge phase is compared along a path, either between two points - where it enters as a term in the derivative - or along a closed path - where it gives rise to the field strength tensors:

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]\,. \tag{2.5}$$

Now we have gathered all our building blocks: The fermions in (2.2), the Higgs doublet Φ, the covariant derivative D_μ and the field strength tensors.

2.3 The operator basis

2.3.1 Operator reduction

With the building blocks and the symmetries in place, we can proceed to build the operator basis. In principle, this sounds like a simple enough task to perform in a "brute-force" approach by simply piecing together all products of the building blocks in accordance with the symmetries of the theory.

While this is certinaly an appropriate first step, care must be taken about the completeness of the operator basis: Not all operators written down this way will be independent of each other. There are three ways by which operators can be connected with each other:

1. **Integration-by-parts relations:** Terms that can be written as total derivatives cannot contribute to the action, since they correspond to a boundary term at infinity. So by imposing $D_\mu \mathcal{O}^\mu = 0$ and using the product rule, one can find relations among operators containing derivatives. An example:

$$\begin{aligned} 0 = D_\mu(\phi^2 D^\mu \phi^2) &= (D_\mu\phi^2)(D^\mu\phi^2) + \phi^2 D^2\phi^2 \\ \Rightarrow \qquad (D_\mu\phi^2)(D^\mu\phi^2) &= -\phi^2 D^2 \phi^2 \,. \end{aligned} \tag{2.6}$$

   Both of these operators could be written in a Lagrangian but they are identical up to a minus sign.

2. **Equations of motion:** Another relation is given by the equations of motion for the fields. These will allow us to replace terms of the form of the kinetic terms. For a real scalar, the equation of motion give:

$$\partial^2\varphi = -m^2\varphi + (\text{interactions})\,, \tag{2.7}$$

so each instance of $\partial^2\varphi$ can be replaced with the right-hand side of this equation. Similar replacements can be derived for fermion operators involving $\not{\partial}\psi$.

3. **Fierz relations:** For fermion fields, relations of the following type hold:

$$(\bar{\psi}_L\gamma_\mu\psi_L)(\bar{\chi}_L\gamma^\mu\chi_L) = (\bar{\psi}_L\gamma_\mu\chi_L)(\bar{\chi}_L\gamma^\mu\psi_L)\,. \tag{2.8}$$

Therefore, many four-fermion operators can be related to each other. A list of Fierz identities can be found in Ref. [1].

Using these three techniques, one can reduce the number of operators drastically. While the idea of continuing the SM Lagrangian to higher mass-dimension was originally proposed in Ref. [2], the reduction to a minimal set of operators was performed in Ref. [3], lowering the number of operators from 80 to 59, much to the delight of the authors, as can be read off from their conclusions:

It is really amazing that no author of almost 600 papers that quoted Ref. [2] over 24 years has ever decided to rederive the operator basis from the outset to check its correctness. As the current work shows, the exercise has been straightforward enough for an M. Sc. thesis. It has required no extra experience with respect to what was standard already in the 1980's.

2.3.2 Classification

Let us now discuss the classes of operators the theory can have. We will follow the naming scheme of Ref. [3], where ψ^n denotes the power of fermions in the operator, X^n the powers of field strength tensors, φ^n the number of Higgs doublets and D^n the number of covariant derivatives. First, note that by Lorentz invariance, fermions always come in pairs. These pairs are frequently referred to as fermion currents.

Four-fermion, ψ^4:
Since a pair of fermion counts as three powers of mass, the most obvious class of operators at dimension six is the one of the four-fermion operators. We can further subdivide them by the chirality of the four fermions. The five categories we have are:

$$\bar{L}L\,\bar{L}L,\ \bar{R}R\,\bar{R}R,\ \bar{L}L\,\bar{R}R,\ \bar{L}R\bar{L}R,\ \mathrm{B-violating}\,. \tag{2.9}$$

The first three categories are products of same-chirality fermion currents, meaning vector currents $J^\mu \sim \bar{\psi}_L\gamma^\mu\psi'_L$. The mixed-chirality currents in the fourth category can either by scalar currents $J_S \sim \bar{\psi}_L\psi'_R$ or tensor currents $J_T^{\mu\nu} \sim \bar{\psi}_L\sigma^{\mu\nu}\psi'_R$. The baryon-number violating operators are products scalar currents as well. The full list of four-fermion operators is laid out in Tab. 2.1. It is important to note that the operators in this list have four indices specifying the flavor of the fermions, defined in eq. (2.2). These indices are only shown in the definition as p, r, s, t. Each operator thus corresponds to twelve operators when flavor is distinguished.

The four-fermion operators offer a rich amount of phenomenology: They are relevant to flavor physics processes like hadron decays and meson-antimeson mixing, which are studied to understand the flavor structure of the SM and its possible

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Table 2.1: Four-fermion operators in the SMEFT, taken directly from Ref. [3].

extensions.

Fermion-boson, $\psi^2\varphi^3$, $\psi^2 X\varphi$, $\psi^2\varphi^2 D$:
There are a number of operators composed of one fermion current and non-fermion structures. For the phenomenological implications, it is important to note that we can replace each Higgs doublet by its vacuum expectation value (vev) once we go to the broken phase of the SM. The first type, referred to as $\psi^2\varphi^3$, is composed of a Yukawa-like current dressed with two additional Higgs doublets, so they are of the form

$$\sim (\bar{\psi}_L\,\varphi\,\psi_R)\,(\varphi^\dagger\varphi)\,. \tag{2.10}$$

After EWSB and replacing two Higgs doublets by the vev, these operators produce power-corrections to the Yukawa couplings of fermions to the Higgs, and to the fermion masses once we also replace the third Higgs doublet by the vev.

The operator type $\psi^2 X\varphi$ consists of a tensor current with the two Lorentz indices saturated by a field-strength tensor. An additional Higgs doublet is required by gauge invariance. The operators are of the form

$$(\bar{\psi}_L\,\varphi\,\sigma_{\mu\nu}\psi_R)F^{\mu\nu}\,. \tag{2.11}$$

Once the Higgs is replaced by its vev, this operator generates the dipole-type interactions, entering observables like the anomalous magnetic moment of the the muon $(g-2)_\mu$, and flavor-changing neutral current transitions like $b \to s\gamma$ and $\mu \to e\gamma$, the latter being forbidden in the SM.

The last type of operators $\psi^2\varphi^2 D$, is simlar to the $\psi^2\varphi^3$ but with a vector current for the fermions. Only two Higgs doublets can be inserted here and the Lorentz index of the current is contracted with a covariant derivative:

$$\sim \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)(\bar{\psi}_L\gamma^\mu\psi_L)\,. \tag{2.12}$$

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Table 2.2: Dimension-six operators in the SMEFT other than the four-fermion ones, taken directly from Ref. [3].

An important phenomenological implication of this type of operator is the alteration of the couplings of fermions to gauge bosons, as we will see below.

Bosonic, X^3, φ^6, $\varphi^4 D^2$, $X^2\varphi^2$:
Lastly, we have operators without fermions. The first type X^3 involves contractions of field strength tensors:

$$\sim f^{abc} G^a_{\mu\nu} G^{b\,\nu\rho} G^{c\,\mu}_{\rho} \,. \tag{2.13}$$

The modify three- and higher-point interactions of gauge bosons amongst themselves.

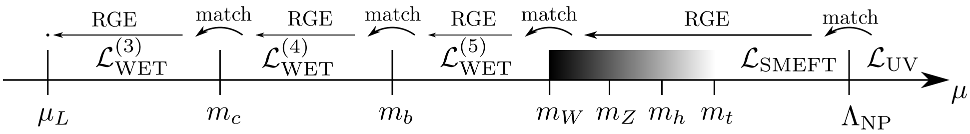
The φ^6 and $\varphi^4 D^2$ operators are self-interactions of the Higgs doublets, either as six-point interactions or quartic interactions with additional derivatives. They impact the self-interactions of the Higgs boson.

The last type of operators $X^2\varphi^2$ is then the product of a gauge-kinetic term and two Higgs doublets, impacting for example masses of the electroweak gauge bosons. All operators involving two or less fermions are given in Tab. 2.2.

2.4 Outlook

2.4.1 From the SMEFT to lower energies

As stated before, the SMEFT is written in the unbroken phase, meaning that the Higgs has not acquired a vev and thereby all fermions and gauge fields remain massless. This is clearly not appropriate for physics at very low energies, say at scales of $\mathcal{O}(\text{GeV})$, where for example the W and Z bosons can be treated as infinitely heavy. Therefore one needs to match the SMEFT onto a generalization of the Fermi theory, which is very often called the *weak effective theory* (WET) or the *low-energy* *effective theory* (LEFT). In this theory, one decouples the electroweak bosons W, Z and h, and all fermions that are heavier than the corresponding scale. In the first step therefore, also the top-quark is integrated out. When the scale is then lowered, one undergoes a series of matching steps in which also the bottom-quark and eventually the charm-quark are decoupled. In between the matching steps, the Wilson coefficients are transformed usng their renormalization group running. The procedure is sketched here:

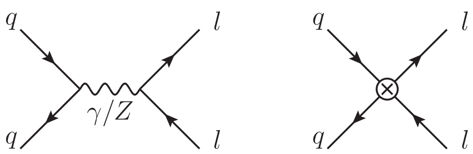


Here the index on the WET Lagrangian denotes the number of active quark flavors. It affects the possible quark flavors out of which the fermionic operators are composed and the renormalization group running of the couplings. A detailed list of operators along with their renormalization group equations are found in Ref. [4].

2.4.2 SMEFT and SM input parameters

When deriving predictions with the SMEFT Lagrangian, care needs to be taken about the consistency of the input parameters. Naively one would compute observables in terms of the SM pieces and then add contributions from the higher-dimensional operators. The SM contributions are then functions of the couplings and masses of the SM Lagrangian, which have been extracted from measurements. However, if we assume the presence of non-zero contributions from the dimension-six operators, the extraction of these parameters will be impacted by them. What we thought we measured as the value of a certain parameter is in reality the value of this parameter plus corrections from SMEFT operators.

Let us clarify this at an example and consider dilepton production at the LHC, generated by the parton-level process $q\bar{q} \to l^+l^-$. There are two types of diagrams: In the SM, this process occurs through Drell-Yan scattering with a virtual photon or Z-boson. Then there are corrections from four-fermion operators:



The SM contribution to the amplitude, given by the first diagram, will be a function of the electroweak gauge coupling α_{ew}, the Z-boson mass m_Z and the electroweak mixing-angle $s_W \equiv \sin\theta_W$. The contribution from four-fermion operators will be a function of their Wilson coefficients, namely $C_{lq}^{(1)}$, $C_{lq}^{(3)}$, C_{eu}, C_{ed}, C_{lu}, C_{qe} and C_{ld}. The input parameters can be extracted from measurements: The Z-boson mass and the electroweak coupling constant have been measured directly. The Weinberg mixing angle θ_W is typically extracted from the relation between the Z-boson mass, the gauge coupling and the Higgs vev:

$$m_Z = \frac{gv}{2\cos\theta_W}\,. \tag{2.14}$$

The Higgs vev can be directly related to the Fermi constant, as we have seen in section 1.4.1. We therefore find:

$$s_W^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha_{\text{ew}}}{\sqrt{2}G_F m_Z^2}} \,. \tag{2.15}$$

The Fermi constant can be very well determined from the decay of the muon, which we discussed in section 1.4.1. Using the results there we can determine the decay rate of the muon as a function of the Fermi constant, $\Gamma_\mu = \Gamma_\mu(G_F)$, and compare it to the experimental result to obtain a number for G_F. However, when computing muon decay in the SMEFT, there are additional contributions from the operators Q_{ll} and $Q_{\varphi l}^{(3)}$. The first operator generates the four-fermion interaction directly whereas the second one alters the coupling of the W boson to leptons. That means, the muom decay rate actually is a function of G_F and the Wilson coefficients of these operators, $\Gamma_\mu = \Gamma_\mu(G_F, C_{ll}, C_{\varphi l}^{(3)})$. The SM value extracted from measurements therefore changes to account for this fact. In the end, this means that the SMEFT cross section of $q\bar{q} \to l^+ l^-$ depends for example on the Wilson coefficients of four-lepton operators C_{ll}, which cannot possibly enter the process directly as a Feynman diagram at tree-level. The same considerations apply also to the parameters α_{ew} and m_Z. These effects have been studied for example in Ref. [5], where the shifts in the input parameters due to SMEFT Wilson coefficients are tabulated.

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