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Applications of General Relativity

Sheet 7

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Exercise 1 [A spinning rod]

A thin rod with mass M and length l spins at a frequency ω around one axis.

(a) Let the rod be in the $x-y$ plane, rotating about the $z-$axis (which goes through the center of mass of the rod). Let μ denote the mass per unit length. Calculate the mass quadrupole tensor of the rotating rod.

(b) Calculate the emitted power of gravitational radiation.

(c) Assuming that the rod has a cross-sectional area A, and that the centripetal and electrostatic forces are balanced,

$$e\nabla\phi = rm\omega^2, \tag{1}$$

(where ϕ denotes the electric potential, and where m and e are the mass and the charge of the electron, respectively) the charges in the rod will deplete and accumulate on the endpoints, effectively making the rod an electric quadrupole. Calculate the quadrupole moment Q_E, neglecting the diffuse positive charge in the internal regions of the rod.

(d) Because the charges are accelerating, power is radiated. Express this luminosity $P_E \sim \omega^6 Q_E^2$ in terms of the parameters of the system.

(e) If the rod has a density of 10g/cm^3, and rotates at 1KHz, will electromagnetic or gravitational radiation be the dominant energy-loss channel? I.e. evaluate L_{EM}/L_{GW}.