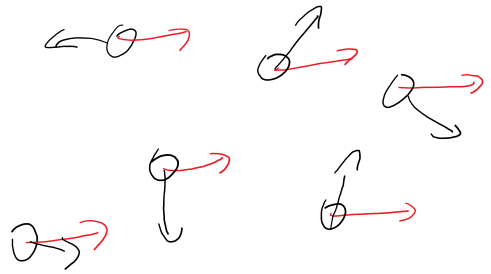
# Impuls-Satz

Ansammlung von Punktmassen

$$\frac{d\vec{P}_{tot}}{dt} = \sum_{i=1}^{N} \frac{d\vec{p}_i}{dt} = \sum_i \vec{F}_{a_i}$$



$$\sum \vec{p}_i \cong 0$$

$$\sum p_i \cong N p_i$$

Schwerpunkt:

$$\vec{r}_s = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\int \vec{r}\, dm}{M}$$

$$\vec{v}_s = \frac{d\vec{r}_s}{dt} = \frac{\sum m_i \frac{d\vec{r}_i}{dt}}{\sum m_i} = \frac{\sum \vec{p}_i}{\sum m_i}$$

$$\vec{P}_{tot} = \sum_{i=1}^{N} \vec{p}_i = M \cdot \vec{v}_s$$

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$$\sum \vec{F}_a = \frac{d\vec{P}_{tot}}{dt}$$ Impuls-Satz

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$$\int_{\text{Stoss}} \vec{F}_{tot}\, dt = \Delta \vec{p}_{tot} = \vec{\bar{F}} \cdot \tau$$

$\tau$: Stosszeit

$$\vec{\bar{F}} = \frac{1}{\tau} \int_0^{\tau} \vec{F}_{tot}\, dt$$

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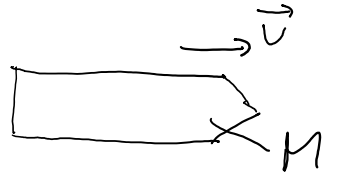
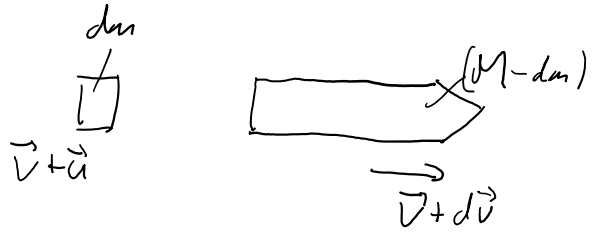
## Rakete

$$\vec{F}_a = \frac{d\vec{p}_{tot}}{dt}$$

$\vec{p}_{tot}(t) = M\vec{v}$ zur Zeit $t$




zur Zeit $t+dt$

$$\vec{p}_{tot}(t+dt) = dm(\vec{v}+\vec{u}) + (M-dm)(\vec{v}+d\vec{v})$$

$$= \vec{u}\,dm + M\,d\vec{v} + M\vec{v}$$

$$\frac{d\vec{p}}{dt} = \vec{u}\frac{dm}{dt} + M\frac{d\vec{v}}{dt} = -\vec{u}\frac{dM}{dt} + M\frac{d\vec{v}}{dt}$$

ohne äussere Kräfte: $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{u}\frac{dM}{dt} = M\frac{d\vec{v}}{dt}$

$$\vec{u}\int_{M_0}^{M} \frac{dM}{M} = \int_0^{\vec{v}} d\vec{v} \quad \rightarrow \quad \vec{u}\ln\left(\frac{M(t)}{M_0}\right) = \vec{v}(t)$$

$$M(t) = M_0 - \Delta M \qquad \Delta M \ll M_0$$

$M(t) = M_0 - \Delta M$ oder $M_0$

$\Rightarrow \ln\left(\frac{M}{M_0}\right) = \ln\left(1 - \frac{\Delta M}{M_0}\right) \overset{\text{Taylor}}{\approx} -\frac{\Delta M}{M_0}$

$-\vec{u}\frac{\Delta M}{M_0} = \vec{v} \quad \rightarrow \quad -\vec{u}\Delta M = M_0\vec{v}$

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$\Delta M = \rho A u \cdot \Delta t$

$\Rightarrow F_R = \rho A u^2 = 1\,kg/m^3 \cdot 4m^2 \cdot (400\,m/s)^2$

$= 6{,}4 \cdot 10^5\,N$

$M \approx 5 \cdot 10^4\,kg$

$\Rightarrow a_R = \frac{F_R}{M} = \frac{6{,}4 \cdot 10^5}{5 \cdot 10^4}\,\frac{N}{kg} \approx 13\,m/s^2$

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