

S. Balmelli, R. Bondarescu, D. Fiacconi

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**Exercise 1** [Gravitational waves from a Binary System]

A binary system with masses  $m_1$  and  $m_2$  is in a circular configuration with radius  $R$ . Consider the orbit to be adequately described by Newtonian gravity. We will use this description to compute the leading-order effects to gravitational-wave emission.

(Hint: Don't forget that orbits in a problem of this type are most easily described using the 'reduced system': a body of mass  $\mu = m_1 m_2 / (m_1 + m_2)$  in a circular orbit around a body of mass  $M = m_1 + m_2$ .)

- (a) Compute the gravitational-wave tensor  $h_{ij}^{TT}$  as measured by an observer looking down the angular momentum axis of the system.
- (b) Compute the rate at which energy is carried away from the system by gravitational waves.
- (c) By asserting *global* conservation of energy in the following form,

$$\frac{d}{dt} (E_{\text{kinetic}} + E_{\text{potential}} + E_{\text{GW}}) = 0, \tag{1}$$

derive an equation for  $dr/dt$ , the rate at which the orbital radius shrinks.

- (d) Derive the change of the orbital angular frequency  $\Omega$ , caused by the gravitational wave emission. You should find that the masses appear only in the combination  $\mu^{3/2} M^{2/5}$ , perhaps raised to some power. This combination of masses is known as the 'chirp mass', perhaps raised to some power, since it sets the rate at which the frequency chirps<sup>1</sup>.
- (e) Integrate the  $d\Omega/dt$  you obtained in part (d), to obtain  $\Omega(t)$ , the time evolution of the binary's orbital frequency. Let  $T_{\text{coal}}$  (*coalescence time*), be the time at which the inspiral is over, and the frequency goes to infinity.
- (f) Compare the gravitational wave power emitted by an Earth-Sun system with that of a binary Neutron star system ( $m_1 = m_2 = 1.4M_{\odot}$ ) at the same separation. The electromagnetic radiation given by the sun is about  $\sim 4 \times 10^{26}$  W. Next, calculate the shrinking of the orbit in each case due to the gravitational wave emission.
- (g) Chickens also chirp. Draw a funky chicken.

<sup>1</sup>chirp = zwitschern