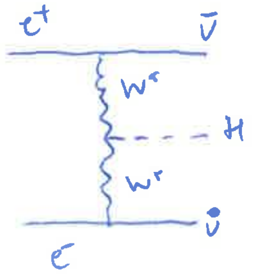
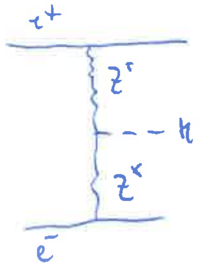
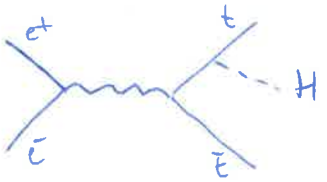
# HIGGS PRODUCTION IN $e^+e^-$ COLLISIONS

The main production mechanisms for Higgs particles at $e^+e^-$ colliders are the Higgsstrahlung process and the WW fusion process.


Less important channels are the ZZ fusion process and the associated production with a $t\bar{t}$ pair




## – $e^+e^- \to ZH$

The production cross section for this process is

$$\frac{d\sigma}{d\cos\vartheta} = \frac{G_F^2 m_Z^4}{96\pi s^2} \lambda^{1/2}(s, m_Z^2, m_H^2) \frac{\left(1 + (1 - 4\sin^2\vartheta_W)^2\right)}{(s - m_Z^2)^2} \left(\frac{3}{4}\lambda \sin^2\vartheta + 6 m_Z^2 s\right)$$

where $\vartheta$ is the polar scattering angle in the $e^+e^-$ cm frame. The function $\lambda(a,b,c)$ is the Källen function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

Since ZH is a two-body final state the energies of the Z and Higgs boson are completely determined as a function of their masses and $\sqrt{s}$

$$E_Z = \frac{s - m_H^2 + m_Z^2}{2\sqrt{s}} \qquad E_H = \frac{s + m_H^2 - m_Z^2}{2\sqrt{s}} \qquad (*)$$

while the momentum of the Z and Higgs bosons is $|\underline{p}|^2 = \frac{1}{4s}\lambda(s, m_Z^2, m_H^2)$.

We note that the cross section is proportional to the phase-space factor $\lambda^{1/2}(s, m_Z^2, m_H^2)$ which is typical for the production of a scalar Higgs boson.

$$\sigma(e^+e^- \to ZH) \sim \lambda^{1/2} \sim \sqrt{s-(m_Z+m_H)^2}$$

This behavior would be different for a CP odd Higgs, for which the cross section would behave as

$$\sigma(e^+e^- \to ZA) \sim \lambda^{3/2}$$

The total cross section for ZH production is obtained by integrating over $\cos\theta$ and we get

$$\sigma = \frac{G_F^2 m_Z^4}{96\pi s^2} \lambda^{1/2}(s, m_Z^2, m_H^2) \frac{\left(1+(1-4s\sin^2\theta_W)^2\right)}{(s-m_Z^2)^2} \left(\lambda + 12 s m_Z^2\right)$$

The cross section has a maximum for $\sqrt{s} \sim 240$ GeV and then falls off as $\sqrt{s}$ increases.

Let us now go back to the $\cos\theta$ distribution. In the large $s$ limit it is the term in $\sin^2\theta$ that dominates

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} \xrightarrow[s\to\infty]{} \frac{3}{4}\sin^2\theta$$ → spin-0 distribution!

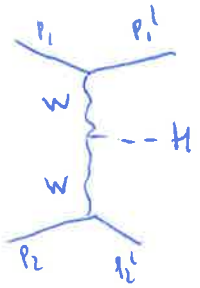
This is in agreement with what we expect from the equivalence theorem, which states that the amplitude in the high-energy limit can be computed by replacing the Z boson with the corresponding Goldstone boson. This corresponds to a dominance of the contributions from longitudinally polarised Z bosons.

By using $\varepsilon_L^\mu = \left(\frac{|\vec{p}|}{m_Z}, 0, \frac{E}{m_Z}\sin\theta, \frac{E}{m_Z}\cos\theta\right)$ we obtain

$$\frac{\sigma_L}{\sigma_{TOT}} = 1 - \frac{8 s m_Z^2}{12 m_Z^2 s + \lambda} \xrightarrow[s\to\infty]{} 1$$

## - The WW fusion process

The WW fusion process has an increasing cross section with $\sqrt{s}$ and thus complements the ZH associated production. The matrix element squared for this process is




$$\overline{|M|^2} = \frac{g_{WWH}^2 g^4}{32} \frac{C_+ (p_1p_2)(p_1'p_2') + C_- (p_1p_2')(p_2p_1')}{(q_1^2 - m_W^2)^2 (q_2^2 - m_W^2)^2}$$

with $g_{WWH} = g m_W$ $\qquad C_\pm = (g_{V_1}^2 + g_{A_1}^2)(g_{V_2}^2 + g_{A_2}^2) \pm 4 g_{V_1} g_{V_2} g_{A_1} g_{A_2}$

with $g_V = g_A = \sqrt{2}$ for WW

The total cross section is obtained by integrating over the three-particle phase space

$$\sigma = \frac{G_F^3 m_W^4}{64\sqrt{2}\pi^3} \int_{k_H}^{1} dx \int_x^1 \frac{dy}{(1+(y-x)/k_V)^2} \left[ (g_V^2 + g_A^2)^2 f(x,y) + 4 g_V^2 g_A^2 g(x,y) \right]$$

with $$f(x,y) = \left( \frac{2x}{y^3} - \frac{1+2x}{y^2} + \frac{2+x}{2y} - \frac{1}{2} \right) \left( \frac{z}{z+1} - \log(1+z) \right) + \frac{x}{y^3} \frac{z^2(1-y)}{1+z}$$

$$g(x,y) = \left( -\frac{x}{y^2} + \frac{2+x}{2y} - \frac{1}{2} \right) \left( \frac{z}{1+z} - \log(1+z) \right)$$

with $k_H = m_H^2/s$ , $k_V = m_V^2/s$ and $z = y(x-k_H)/(k_V x)$.

It is interesting to observe that we can obtain an explicit and simple expression for the cross section at large $s$ ($s \gg m_W^2$). In this limit the incoming fermions are not very much deflected upon the emission of the vector boson. Thus we can compute the cross section by using the small angle approximation, or equivalently, by assuming that the transverse momentum of the outgoing fermions are small.

By writing $\quad p_1 = (E, \underline{0}, E) \qquad p_2 = (E, \underline{0}, -E)$

$$p_1' = \left( x_1 E + \frac{p_{T_1}^2}{2x_1 E}, \underline{p_{T_1}}, x_1 E \right) \qquad p_2' = \left( x_2 E + \frac{p_{T_2}^2}{2x_2 E}, \underline{p_{T_2}}, -x_2 E \right)$$

small $p_T$ expansion

We can approximate the matrix element as

$$\overline{|\mathcal{M}^2|} \approx \frac{g^2_{VHH}\, g^4}{32} \; \frac{(C_+ + C_-)\, x_1^3 x_2^3\, s^2/4}{(p_{T1}^2 + x_1 m_V^2)^2 (p_{T2}^2 + x_1 m_V^2)^2}$$

The phase space can also be approximated as

$$dPS_3 \sim \frac{d^3p_1'}{E_1'} \frac{d^3p_2'}{E_2'} \frac{d^3k}{E_k}\, \delta^4(p_1+p_2-p_1'-p_2'-k)$$

$$\sim \frac{1}{s} \frac{dx_1}{x_1} \frac{dx_2}{x_2}\, \delta((1-x_1)(1-x_2)-\tau)\, d^2\underline{p}_1'\, d^2\underline{p}_2'$$

We thus obtain

$$\sigma \approx \frac{G_F^3 m_W^4}{128\sqrt{2}\,\pi^3} (C_+ + C_-) \left[ (1+\tau) \log\frac{1}{\tau} - 2(1-\tau) \right]$$

This formula exhibits the characteristic $\log \frac{s}{m_H^2}$ behavior of the cross section.

It is important to observe that this approximation corresponds to considering only longitudinally polarized W's, which provide the dominant contribution at high energy.

In the high energy limit transversely polarized bosons cannot be emitted at small angles (helicity conservation!)


One would expect that the contribution from longitudinally polarized bosons is vanishing, because at high energy $\varepsilon_L^\mu \approx \frac{p^\mu}{m_V}$, and the fermions are massless. However the expression for the longitudinal polarization vector is

$$\varepsilon_L^\mu = \left( \frac{|\underline{p}|}{m_V}, 0, 0, \frac{E}{m} \right)$$

in the high energy limit we can write

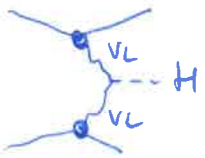
$$\varepsilon_L^\mu \approx \left( \frac{E}{m_V}\left(1 - \frac{m_V^2}{2E^2}\right), 0, 0, \frac{|\underline{p}|}{m_V}\left(1 + \frac{m_V^2}{2|\underline{p}|^2}\right) \right)$$

$$= \frac{p^\mu}{m_V} + \left(-\frac{m_V}{2E}, 0, 0, \frac{m_V}{2|p|}\right)$$

$$\simeq \frac{p^\mu}{m_V} - \frac{m_V}{2E}(1,0,0,-1)$$

The first term indeed provides a vanishing contribution when coupled to massless fermions. The second term gives a suppressed contribution at high energy, but the suppression factor is compensated by a factor $\frac{\Delta}{m_V^2}$ coming from the phase space integration. Thus, one can obtain the approximated result, as in the Weizsäcker-Williams approximation, by evaluating the cross section for Higgs boson produced through $V_L V_L$ scattering, and then folding the result with the probability to extract a $V_L$ from each fermion

$$P_{V_L/q}(z) = \frac{\alpha}{\pi}\frac{1-z}{z}\left(g_V^2+g_A^2\right)$$

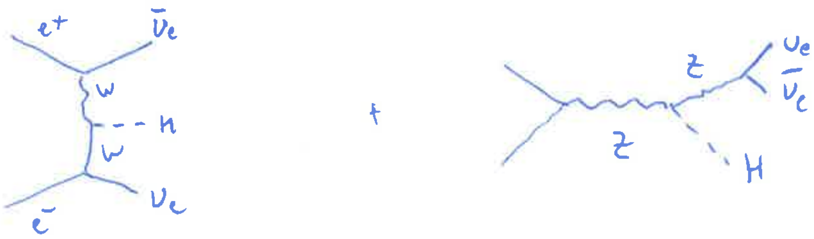



(see Chanowitz, Gaillard NPB 261 (1985) 379)

NOTE THAT SINCE $V_L V_L \to H$ GIVES $\delta(1-x)$ ONE OBTAINS THE PREVIOUS RESULT THROUGH $\frac{1-z}{z} \otimes \frac{1-z}{z}$ !

## Interference between WW fusion and ZH production

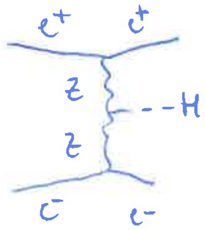
To compute the cross section for $e^+e^- \to H\nu\bar{\nu}$ we need to take into account both WW fusion and ZH production with $Z \to \nu\bar{\nu}$.




Note that experimentally we need to consider $Z \to \nu\nu$ with all the three species of neutrinos. One finds that Higgsstrahlung is dominant at low energies, while WW dominates at high energies. The interference effect is important in the intermediate region.

## - ZZ fusion process

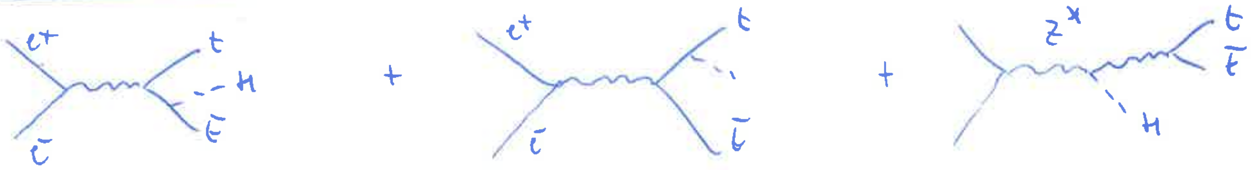
The ZZ fusion process is completely analogous to the WW fusion process discussed before




and the cross section can be computed in an analogous way. Due to the difference of the neutral current couplings with respect to the charged current couplings, the ZZ fusion cross section is an order of magnitude smaller than the WW fusion cross section.

The lower rate is compensated by another clean signature, due to the $e^+e^-$ pair in the final state. As it happens in the WW fusion process, the cross section for $e^+e^- \to He^+e^-$ receives contribution not only from ZZ fusion but also from ZH production, with $Z \to e^+e^-$, and the interference has to be properly taken into account.

## - Associated production with a $t\bar{t}$ pair




In the SM the associated production of the Higgs boson with a pair of heavy fermions proceeds through two kind of diagrams. The Higgs can be radiated off the t and $\bar{t}$ lines, or it can be emitted by the Z boson, that eventually decays in a $t\bar{t}$ pair.
The dominant contribution to the cross section comes from the Higgsstrahlung diagrams, and, in particular, from the $e^+e^- \to \gamma^* \to t\bar{t} \to t\bar{t}H$ process. This implies that this process would allow a clean measurement of the $t\bar{t}H$ coupling.

# Higgs cross sections in $e^+e^-$ collisions