HIGGS PRODUCTION AT HADRON COLLIDERS

As in e^+e^- collisions, at hadron colliders the Higgs production mechanisms all make use of the fact that the Higgs boson couples predominantly to heavy particles.

The main production channels are thus 1) gluon-gluon fusion 2) vector-boson fusion 3) associated production with a vector boson 4) associated production with a $t\bar{t}$ pair

We discuss the four production channel in turn.

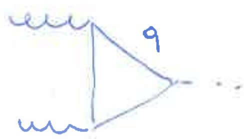
GLUON FUSION

Gluon fusion is the dominant production channel for Higgs boson production in the SM, due to the large probability to extract gluons from the incoming protons.

It is a process which already starts at one loop level in the Born approximation.

The LO cross section can be written as

$$\sigma_{LO} = \frac{\alpha_S^2}{\pi} \frac{m_H^2}{256 v^2} |A|^2 \delta(s - m_H^2)$$



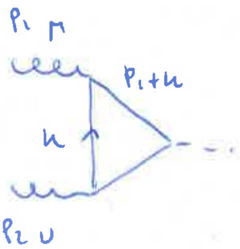
where

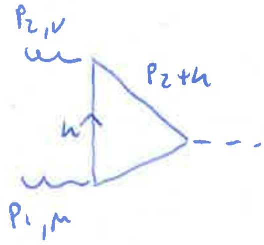
$$A = A_{1/2} = \sum_q \tau_q \left(1 + (1-\tau_q) f(\tau_q)\right) \qquad \tau_q = \frac{4 m_q^2}{m_H^2}$$

and

$$f(\tau_q) = \begin{cases} \arcsin^2 \sqrt{1/\tau_q} & \tau_q \geq 1 \\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau_q}}{1-\sqrt{1-\tau_q}} - i\pi\right]^2 & \tau_q < 1 \end{cases}$$

We now discuss in detail the LO computation - the Feynman diagrams are depicted in the Figure





The amplitude can be written as ← gluon polarisations

$$\mathcal{M}_{gg\to h}(P_1,P_2) = \varepsilon_1^{\mu}\,\varepsilon_2^{\nu}\,T_{\mu\nu}(P_1,P_2)$$

$$T_{\mu\nu}(P_1,P_2) = a_1 g_{\mu\nu} + a_2 P_{1\mu}P_{2\nu} + a_3 P_{2\mu}P_{2\nu} + a_4 P_{1\mu}P_{2\nu} + a_5 P_{1\nu}P_{2\mu}$$

But the gluon polarisations are transverse $(P_1\cdot\varepsilon_1 = P_2\cdot\varepsilon_2 = 0)$

$\Rightarrow$ a_2, a_3, a_4 give vanishing contribution

Moreover gauge invariance provides us with the additional conditions

$$P_1^{\mu}T_{\mu\nu} = P_2^{\nu}T_{\mu\nu} = 0$$

$\Rightarrow$ $T_{\mu\nu} \sim g_{\mu\nu}\,P_1P_2 - P_1^{\nu}P_2^{\mu}$ this condition holds to all orders

$$T_1^{\mu\nu}(P_1,P_2) = (-i)^3 g_s \frac{m_Q}{v}\,\mathrm{Tr}[T^aT^b]\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\Big[\gamma^{\mu}\,\frac{i(\not k+\not P_1+m_Q)}{(k+P_1)^2-m_Q^2+i\varepsilon}$$
$$\times\,\frac{i(\not k-\not P_2+m_Q)}{(k-P_2)^2-m_Q^2+i\varepsilon}\,\gamma^{\nu}\,\frac{i(\not k+m_Q)}{k^2-m_Q^2+i\varepsilon}\Big]$$

first diagram

contribution from the first diagram

$$\mathrm{Tr}\left[\gamma^{\mu}(\not k+\not P_1+m_Q)(\not k-\not P_2+m_Q)\gamma^{\nu}(\not k+m_Q)\right]$$

$$= 4m_Q\left[g^{\mu\nu}\left(m_Q^2-k^2-m_h^2/2\right)+4k^{\mu}k^{\nu}+P_1^{\nu}P_2^{\mu}\right]$$

The trace vanishes in the $m_Q\to 0$ limit : helicity violation in the Higgs coupling!

The denominators can be treated by using the Feynman parametrization

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} dy \, \frac{1}{\left(Ax + By + C(1-x-y)\right)^3}$$

$$A = (k+p_1)^2 - m_Q^2 \qquad B = (k-p_2)^2 - m_Q^2 \qquad C = k^2 - m_Q^2$$

and the linear terms can be eliminated with the replacement $k = k' - x p_1 + y p_2$

By using standard formulae for loop integrals we obtain

$$T_1^{\mu\nu} = i \, \frac{m_Q^2}{v} \, \frac{g_s^2}{8\pi^2} \, \delta^{ab} \left(g^{\mu\nu} m_H^2/2 - p_1^\nu p_2^\mu \right) \int_0^1 dx \int_0^{1-x} dy \, \frac{1-4xy}{m_Q^2 - xy\, m_H^2}$$

The second diagram can be obtained with the exchange $p_1 \leftrightarrow p_2$ $\mu \leftrightarrow \nu$ and leads to the same result, so we obtain

$$T^{\mu\nu} = \frac{i}{v} \, \frac{\alpha_s}{2\pi} \, \delta^{ab} \left(g^{\mu\nu} m_H^2/2 - p_1^\nu p_2^\mu \right) A_{1/2}(\tau_Q)$$

$$A_{1/2}(\tau_Q) = 2\int_0^1 dx \int_0^{1-x} dy \, \frac{1-4xy}{1 - xy\, m_H^2/m_Q^2} \xrightarrow[\tau_Q \to \infty]{} 2\int_0^1 dx \int_0^{1-x} dy \, (1-4xy) = \frac{2}{3}$$

COMMENTS

- The expression for $T_1^{\mu\nu}$ contains two powers of m_Q, one from the Yukawa coupling m_Q/v and one from the helicity violation
- Despite the loop integral is superficially divergent the final result is finite because it is proportional to $g^{\mu\nu} p_1 p_2 - p_1^\nu p_2^\mu$

The limit $\tau_q \to 0$ corresponds to light quarks, and the cross section vanishes as $\tau_q \log \tau_q$.

The dominant contribution is given by heavy quarks, and in particular by the top quark.

In the standard model the bottom quark contributes about 10% of the cross section.

The limit $\tau_q \to \infty$ is called heavy top limit: in this limit we have

$$A_{1/2} \underset{\tau_q \to \infty}{\approx} \tau_q \left(1 - (1-\tau_q)\left(\frac{1}{\tau_q} + \frac{1}{3\tau_q^2} \right)\right) \to \frac{2}{3}$$

and we obtain

$$\sigma_{LO} \to \frac{\alpha_s^2}{\pi} \frac{m_H^2}{576 v^2} \delta(s - m_H^2)$$

The limit in which the heavy-quark mass becomes much larger than the Higgs mass corresponds to INTEGRATE OUT the heavy-quark field in the theory, and to shrink the quark loop to a point

This means that, effectively, the gluons become directly coupled to the Higgs through a new interaction, which is driven by an effective Lagrangian which is obtained by integrating out the heavy quark.

The explicit form of the effective Lagrangian can be obtained by observing that the Higgs boson couples to the trace of the energy-momentum tensor.

In an exactly scale invariant theory the dilatation transformation $x \to x e^{-\varepsilon}$ is a symmetry and the corresponding current is conserved $\partial_\mu D^\mu = 0 = \theta_\mu^{\ \mu}$, $\theta_\mu^{\ \mu}$ being the trace of the energy-momentum tensor. In practice the divergence of the dilatation current does not vanish for two reasons: first, there is an explicit mass term (we assume only the top quark to be massive).

Second, the renormalization procedure forces us to break scale invariance.

We can thus write

$$\partial_\mu D^\mu = \theta_\mu{}^\mu = (1+\gamma_m)\, m_t^0\, \bar{t}^0 t^0 + \frac{\beta(\alpha_s)}{2\alpha_s} G^a_{\mu\nu} G^{\mu\nu}_a$$

The first term corresponds to the explicit breaking (γ_m is the mass anomalous dimension) while the second term is the so called TRACE ANOMALY (its form can be understood by observing that under the scale transformation we have $\mu^2 \to e^{2\epsilon}\mu^2$ and $\delta\alpha_s = \frac{\delta\mu^2}{\mu^2}\beta(\alpha_s) = 2\epsilon\,\beta(\alpha_s)$ such that $\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\alpha_s}\delta\alpha_s \sim \frac{\beta(\alpha_s)}{\alpha_s} G^a_{\mu\nu}G^{\mu\nu}_a$)

We now observe that the matrix element $\langle 0|\theta_\mu{}^\mu|gg\rangle$ vanishes at zero momentum transfer

$$\lim_{q^2\to 0} \langle 0|\theta_\mu{}^\mu|gg\rangle = 0$$ IWAZAKI, PRD15 (1977) 1172

Since when the Higgs has vanishing momentum it acts as a constant field we have

$$\lim_{p_H\to 0} \langle h|\theta_\mu{}^\mu h|gg\rangle = 0$$

we can exploit the previous expression for the $\theta_\mu{}^\mu$ to obtain

$$\mathcal{L}_{eff} = \frac{1}{2}\,\frac{\beta^t(\alpha_s)/\alpha_s}{1+\gamma_m}\, G^a_{\mu\nu}G^{\mu\nu}_a\, \frac{h}{v}$$

where the top contribution to the β function is obtained by taking the limit $p_H\to 0$ (which is indeed what we need to justify the effective field theory approach). (*) ↗ ALTERNATIVE DERIVATION USING LOW-ENERGY THEOREM

The approach we have used is very powerful, because it tells us that, to all orders in α_s, the coefficient in the effective Lagrangian is determined by the top contribution to the QCD β function and by the mass anomalous dimension.

We have

$$\beta_0 = \frac{11C_A - 2n_F}{12\pi} \qquad \beta_1 = \frac{17C_A^2 - 5C_A n_F - 3C_F n_F}{24\pi^2} \qquad \gamma_m = \frac{3}{2}C_F\frac{\alpha_s}{\pi} + O(\alpha_s^2)$$

$$\beta(\alpha_s) = \frac{d\alpha_s}{d\ln\mu^2} = -\beta_0\alpha_s^2 - \beta_1\alpha_s^3 + O(\alpha_s^4)$$

(*)

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- Low energy theorems

We consider a Higgs boson with vanishing four momentum $p_H \to 0$. In this case we have

$$[P_\mu, H] = i\partial_\mu H = 0$$

and H acts as a constant field. This implies that its effect is equivalent to redefining all the mass parameters of the theory.

$$m_i \to m_i\left(1+\frac{H}{v}\right)$$

It follows that the amplitude for $A \to B$ plus a Higgs boson obeys

$$\lim_{p_H\to 0} \mathcal{M}(A\to B H) = \frac{1}{v}\sum_{i=f,V} m_i \frac{\partial}{\partial m_i}\mathcal{M}(A\to B)$$

We can use such low energy theorem to compute the effective coupling of the Higgs boson to gluons. We need to evaluate the contribution of the top quark to the on-shell gluon self energy.

The result can be written with an effective Lagrangian

$$\mathcal{L}_{gg} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \left(1 + \Pi^t_{gg}(0)\right)$$

$$\Pi^t_{gg}(0) = \frac{\alpha_s}{6\pi}\left(\frac{4\pi\mu^2}{m_t^2}\right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}$$

By using the low energy theorem we obtain

$$\frac{1}{v}\, m_t \frac{\partial}{\partial m_t}\left[-\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \Pi^t_{gg}(0)\right]$$

$$= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \frac{\Gamma(1+\epsilon)}{\epsilon}\frac{1}{v}\, 2m_t^2 \frac{\partial}{\partial m_t^2}\left(e^{\epsilon \log\frac{4\pi\mu^2}{m_t^2}}\right)\frac{\alpha_s}{6\pi}$$

$$= -\frac{1}{4} G^{\mu\nu}_{a} G^{a}_{\mu\nu} \frac{1}{v} 2 \frac{\Gamma(1+\varepsilon)}{\varepsilon} \frac{\alpha_s}{6\pi} e^{\varepsilon \log \frac{4\pi\mu^2}{m_t^2}} (-\varepsilon)$$

$$\rightarrow \boxed{\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} \frac{H}{v} \frac{\alpha_s}{3\pi}}$$

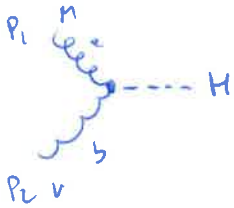
$\Rightarrow$ the top quark contribution comes from the n_F term

$$\beta_0^t = -\frac{1}{6\pi} \qquad \beta_1^t = -\frac{5C_A+3C_F}{24\pi^2}$$

$$\beta^t(\alpha_s) = \frac{\alpha_s^2}{6\pi}\left(1 + \frac{5C_A+3C_F}{4\pi}\alpha_s + \dots\right) = \frac{\alpha_s^2}{6\pi}\left(1 + \frac{19}{4\pi}\alpha_s + \dots\right)$$

$$\frac{1}{2}\,\frac{\beta^t(\alpha_s)/\alpha_s}{1+\delta_m} = \frac{\alpha_s}{12\pi}\left(1 + \frac{11}{4}\frac{\alpha_s}{\pi} + \dots\right)$$

It turns out that the large m_{top} approximation is very good $\Rightarrow$ we can use this approximation for our calculations. The effective ggH vertex is



$$i\,\frac{\alpha_s}{3\pi v}\,\delta^{ab}\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)$$

The corresponding scattering amplitude is

$$\mathcal{M} = i\,\frac{\alpha_s}{3\pi v}\,\delta^{ab}\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)\varepsilon_{1\mu}\varepsilon_{2\nu}$$

$$|\mathcal{M}|^2 = \frac{\alpha_s^2}{9\pi^2v^2}(N_c^2-1)\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)\varepsilon_{1\mu}\varepsilon_{\nu 2}\left(g^{\rho\sigma}p_1p_2 - p_1^\sigma p_2^\rho\right)\varepsilon^*_{1\rho}\varepsilon^*_{2\sigma}$$

$$= \text{summing over the polarizations} = \frac{\alpha_s^2}{9\pi^2v^2}(N_c^2-1)\,\frac{1}{2}m_H^4$$

$$\sigma = \frac{1}{2s}\overline{|\mathcal{M}_0|^2}\,\cancel{(2\pi)^4}\,\cancel{\delta^4(p_1+p_2-p_H)}\,\cancel{\frac{d^4p_H}{(2\pi)^4}}\,2\pi\,\delta(s-m_H^2)$$

$$= \frac{1}{2s}\frac{|\mathcal{M}_0|^2}{(N_c^2-1)^2}\,2\pi\,\delta(s-m_H^2) = \frac{\alpha_s^2}{\pi}\,\frac{m_H^2}{576v^2}\,\delta(s-m_H^2)$$

and the result coincides with what obtained in the heavy-top limit

We now want to compute QCD radiative corrections to this process. These corrections were first evaluated exactly by Djouadi, Graudenz, Spira and Zerwas already in 1995. We will consider the calculation in the effective field theory approach.

One has to consider <u>REAL CORRECTIONS</u>

and <u>VIRTUAL CORRECTIONS</u>

These corrections are affected by different kinds of singularities: <u>UV singularities</u> are dealt with through the renormalization procedure, and are reabsorbed into the redefinition of the QCD coupling α_s.

<u>IR singularities</u> affect both virtual and real corrections. They are due to the emission of <u>SOFT</u> and/or <u>COLLINEAR</u> particles. Separately, real and virtual contributions are IR divergent, and it is only in the sum that the singularities cancel out. More precisely, it is soft and <u>final state</u> collinear singularities that cancel. Initial state collinear singularities do not cancel and they must be reabsorbed into the PDFs. Dealing with IR and UV divergences requires a REGULARIZATION: we use dimensional regularization, by working in $D=4-2\varepsilon$ dimensions, and in particular, in the CDR scheme, in which there are 2 independent polarizations for massless quarks and $2(1-\varepsilon)$ for gluons. The subtraction of both UV and collinear poles is done in the $\overline{MS}$ scheme, which works as follows. The poles appear as singularities in $\frac{1}{\varepsilon}$ that, in practice, come in the form

$$\Gamma(1+\varepsilon) \simeq 1-\gamma_E\varepsilon$$

$$\frac{1}{\varepsilon}(4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon) \simeq \frac{1}{\varepsilon} - \gamma_E + \log(4\pi)$$

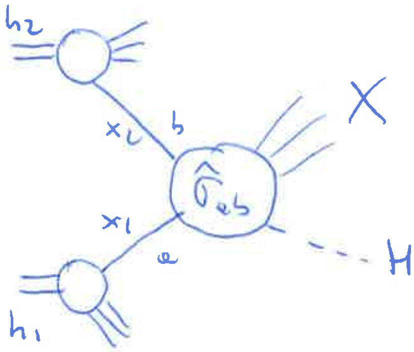
$\Rightarrow$ WE SIMPLY SUBTRACT THESE ADDITIONAL CONSTANTS TOGETHER WITH THE $1/\varepsilon$ POLES

Before starting to discuss the calculation let us recall what the factorization theorem tell us for this process

$$\sigma(s, m_H^2) = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \int_0^1 dz\, \delta\left(z - \frac{\tau}{x_1 x_2}\right)$$

$$\cdot\, \hat{\sigma}_{ab}\left(z;\, \alpha_s(\mu_R^2),\, m_H^2/\mu_R^2,\, m_H^2/\mu_F^2\right)$$

$$\tau = \frac{m_H^2}{S}$$



The partonic CM energy squared s is related to the hadronic one by the relation

$$s = x_1 x_2 S$$

The variable z give us a measure of the "inelasticity" of the process and it is defined as $m_H^2 = z s$.

Since at Born level there is no additional radiation, we have $z=1$

The partonic cross section can be computed as a perturbative expansion in α_s

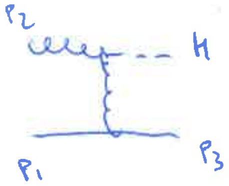
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right) \hat{\sigma}_{ab}^{(1)} + \dots$$

Where the zero-order contribution, that we have already evaluated, is

$$\hat{\sigma}_{ab}^{(0)} = \frac{\alpha_s^2}{\pi} \frac{1}{576 v^2} \delta(1-z)\, \delta_{ag}\, \delta_{bg}$$

NLO corrections

We start the calculation from the qg channel, and there is only one Feynman diagram



$$\overline{|M_{qg\to qH}|^2} = -\frac{\alpha_s^3}{18\pi v^2}\,\frac{1}{N(1-\varepsilon)}\,\frac{s^2+u^2-\varepsilon(s+u)^2}{t}$$

$\uparrow$ collinear singularity

$$s=(p_1+p_2)^2 \qquad t=(p_1-p_3)^2 \qquad u=(p_2-p_3)^2$$

To compute the cross section we have to integrate the matrix element on the two particle phase space, which can be expressed as

$$\sigma_{qg} = \frac{1}{2s}\int \overline{|M_{qg\to qH}|^2}\, d\phi_2 \qquad d\phi_2 = \frac{1}{8\pi}\left(\frac{4\pi}{s}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\, z^{\varepsilon}(1-z)^{1-2\varepsilon}\, y^{-\varepsilon}(1-y)^{-\varepsilon}\,dy$$

$$y=\frac{1+\cos\vartheta_3}{2} \qquad z=\frac{m_H^2}{s}$$

By doing the phase space integral we obtain

$$\sigma_{qg} = (4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon)\left(\frac{\mu^2}{m_H^2}\right)^{\varepsilon}\frac{\alpha_s}{2\pi}\,\sigma_{LO}(\varepsilon)\left[\cancel{-\frac{1}{\varepsilon}P_{gq}(z)} + P_{gq}(z)\log\frac{(1-z)^2}{z} + \frac{4}{3}z - 2\frac{(1-z)^2}{z}\right]$$

$\uparrow$ collinear pole

The collinear pole, proportional to the $q\to g$ DGLAP splitting function $P_{gq}(z)=C_F\frac{1+(1-z)^2}{z}$

has to be cancelled with the FACTORIZATION COUNTERTERM

In this way the collinear singularity is reabsorbed in the quark PDF

The function $\sigma_{LO}(\varepsilon)$ is the lowest order cross section computed in $D=4-2\varepsilon$

and has the form $\sigma_{LO} = \frac{\alpha_s^2}{\pi}\,\frac{z}{576 v^2}\,\frac{1}{1-\varepsilon}\,\delta(1-z)$

We note that after the subtraction of the collinear singularity, there is a logarithmic term $\log\frac{(1-z)^2}{z}$ which remains, which is still controlled by P_{gq}

This term corresponds to the left over of the collinear singularity, which has been integrated from m_H to $q^2_{T\,MAX}\sim\frac{(1-z)^2}{z}m_H^2$ which is the maximum transverse momentum allowed by kinematics.

We now go on with the calculation in the gg channel.

The amplitude squared is

$$|\mathcal{M}_{gg\to gH}|^2 = \frac{\alpha_s^3}{v^2}\left(\frac{32}{3\pi}\right)\left\{\frac{m_H^8+s^4+t^4+u^4}{stu}(1-2\varepsilon)+\frac{\varepsilon}{2}\frac{(m_H^4+s^2+t^2+u^2)^2}{stu}\right\}$$

which has to be multiplied by a factor $\frac{1}{4}\cdot\frac{1}{(N^2-1)^2}\frac{1}{(1-\varepsilon)^2}$ to average over colour and spin.

By doing the integral over the two particle phase space we get

$$\sigma_{gg}^{real} = \frac{1}{576\pi^2}\frac{\alpha_s^3}{v^2}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\varepsilon}\Gamma(1+\varepsilon)\, z^{+\varepsilon}\left(1-\frac{\pi^2\varepsilon^2}{3}\right)(1-z)^{-1-2\varepsilon}\times$$

$$\times\left[-\frac{3}{\varepsilon}\left(1+z^4+(1-z)^4\right)-\frac{11}{2}(1-z)^4-6\left(1-z+z^2\right)^2-6\varepsilon\right]$$

OK, checked from the expression of the matrix element

As in the qg channel, the integration produces a pole in $\frac{1}{\varepsilon}$, however, in this channel we also get a factor $(1-z)^{-1-2\varepsilon}$. This term cannot be expanded at $\varepsilon\to 0$ because it would lead to singularities as $z\to 1$. The limit $z\to 1$ indeed probes the SOFT REGION, in which the radiation recoiling against the Higgs boson is forced to be soft ($p_1+p_2=p_3+p_H \Rightarrow (p_1+p_2-p_3)^2=m_H^2 \Rightarrow s-2p_3(p_1+p_2)=m_H^2$

$\Rightarrow 1-z=\frac{2p_3(p_1+p_2)}{s} \Rightarrow 1-z\to 0$ forces p_3 to be soft)

The term $(1-z)^{-1-2\varepsilon}$ has to be treated as a DISTRIBUTION: let's see how it acts onto a test function $f(z)$.

$$\int_0^1 (1-z)^{-1-2\varepsilon} f(z) = \int_0^1 (1-z)^{-1-2\varepsilon}\left[f(z)-f(1)+f(1)\right] = f(1)\left(-\frac{1}{2\varepsilon}\right)+\int_0^1\frac{f(z)-f(1)}{1-z}$$

$$-2\varepsilon\int_0^1\frac{f(z)-f(1)}{1-z}\log(1-z) \quad +O(\varepsilon^2)$$

$$\Rightarrow (1-z)^{-1-2\varepsilon} = -\frac{1}{2\varepsilon}\delta(1-z)+\left(\frac{1}{1-z}\right)_+ -2\varepsilon\left(\frac{\log(1-z)}{1-z}\right)_+ +O(\varepsilon^2)$$

We now must consider the virtual correction. The one-loop contribution is

$$\overline{\mathcal{M}^{(1)}_{UN}\mathcal{M}^{(0)*}} + c.c. = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_h^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\, C_A\left(-\frac{2}{\epsilon^2}+\frac{11}{3}+\pi^2\right)\overline{|\mathcal{M}^{(0)}|^2}$$

where $\mathcal{M}^{(1)}_{UN}$ is the UNRENORMALIZED amplitude

The UV renormalization is accounted for by subtracting the UV pole as

$$\overline{\mathcal{M}^{(1)}\mathcal{M}^{(0)*}} + c.c. = \overline{\mathcal{M}^{(1)}_{UN}\mathcal{M}^{(0)*}} + c.c. - \frac{\alpha_s}{\epsilon}\, 2\,(4\pi)^{\epsilon}\,\Gamma(1+\epsilon)\,\beta_0\,\overline{|\mathcal{M}^{(0)}|^2}$$

2 powers of α_s!

and $\beta_0 = \frac{11}{12\pi}C_A - \frac{n_f}{6\pi}$

we thus get

$$\overline{\mathcal{M}^{(1)}\mathcal{M}^{(0)*}} + c.c. = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_h^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{6}{\epsilon^2}-\frac{4\pi\beta_0}{\epsilon}+11+2\pi^2\right)\overline{|\mathcal{M}^{(0)}|^2}$$

The factorization counterterm to cancel the initial state collinear singularities is

$$\sigma_{CT} = \sigma_0(\epsilon)\,\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_h^2}\right)^{\epsilon}\Gamma(1+\epsilon)\,\frac{2}{\epsilon}\,P_{gg}(z)$$

with $P_{gg}(z) = 2C_A\left[\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)\right]+2\pi\beta_0\,\delta(1-z)$

Summing everything all the singularities cancel out and we obtain

$$\sigma_{gg} = \frac{\alpha_s}{\pi}\,\sigma_0\left[\left(\frac{11}{2}+\pi^2\right)\delta(1-z)+12\left(\frac{\log(1-z)}{1-z}\right)_+ + P_{gg}^{reg}(z)\,\ln\frac{(1-z)^2}{z}-\frac{6\ln z}{1-z}-\frac{11}{2}\frac{(1-z)^3}{z}\right]$$

contains a factor z

where $P_{gg}^{reg} = 6\left(\frac{1-z}{z}-1+z(1-z)\right) = 6\left[\left(\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)\right)-\frac{1}{(1-z)_+}\right]$

is the P_{gg} splitting function after having subtracted the soft singularity.

The final contribution to be considered is the one of the $q\bar{q}$ channel, where only one diagram contributes

This diagram produces no divergences $\overline{|M|^2} = \frac{16}{9} \frac{\alpha_s^3}{\pi v^2} \frac{u^2+t^2-\epsilon(u+t)^2}{s}$

$$\Rightarrow \quad \sigma_{q\bar{q}} = \frac{32}{27} \sigma_0 \frac{\alpha_s}{\pi} \frac{(1-z)^3}{z}$$

$\uparrow$ contains a factor z

The impact of the $q\bar{q}$ channel is very small, since the q density is much suppressed with respect to the gluon density.

The impact of NLO corrections is very large, and increase the LO result by about 100% at LHC energies. The impact of QCD radiative corrections can be presented by using K-factors

$$K_{NLO} = \frac{\sigma_{NLO}(\mu_F,\mu_R)}{\sigma_{LO}(\mu_F=\mu_R=m_H)} \qquad K_{NNLO} = \frac{\sigma_{NNLO}(\mu_F,\mu_R)}{\sigma_{LO}(\mu_F=\mu_R=m_H)}$$

The following plot is obtained by varying $\frac{1}{2} m_H < \mu_F, \mu_R < 2 m_H$ with $\frac{1}{2} < \frac{\mu_F}{\mu_R} < 2$

This choice, which is to some extent arbitrary defines a way to estimate perturbative uncertainties. We see that the LO and NLO bands do not overlap, thus implying that this procedure can only give a lower limit of the true perturbative uncertainty. However the NNLO result overlaps with the NLO one, thus suggesting that perturbation theory is under control.

The recent computation of N^3LO corrections [Anastasiou et al. 2015] shows that indeed higher order corrections are rather small.

[Anastasiou-Melnikov 2002]
[Harlander-Kilgore 2002]
[Van Neerven et al 2003]
[Spira et al. 1995]
[Dawson, 1991]
$m_t \to \infty$