① CHIRAL PERTURBATION THEORY

In the previous lecture we have been discussing effective field theories (EFT)

We have seen that the traditional discrimination against EFT, due to the fact that they are non-renormalizable has been overcome in the last decades.

This is indeed motivated by the observation that all known quantum field theories are probably just effective field theories, in the sense that they are low energy approximations of some underlying, more fundamental theories.

Although all quantum field theories must be renormalizable, they differ in their sensitivity to the UV. We distinguish two possibilities

- Asymptotically free theories

  These are theories without a limit in energy above which they cannot really applied. QCD is the classical example.

- Ultraviolet unstable theories

  These theories contain information about their limited validity. This information, is sometimes, like in QED, of little practical use, since the breakdown of the theory appears well beyond the scale at which the theory is embedded in a more fundamental theory.

Effective field theories are obtained from a more fundamental theory by integrating out the heavy degrees of freedom. We distinguish two kinds of EFT

- DECOUPLING EFT: For energies well below a certain scale Λ all degrees of freedom above Λ are integrated out and NO light particle are generated (Example, Fermi theory, Euler-Heisenberg)

- NON DECOUPLING EFT: The transition to EFT occurs via SSB: light degrees of freedom are generated, pseudogoldstone bosons

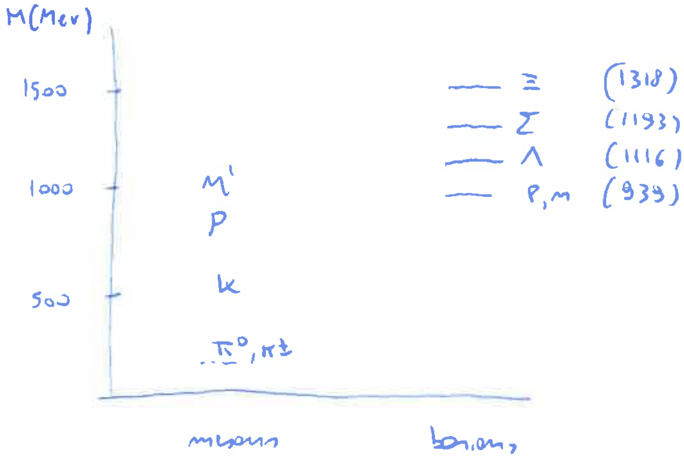
  $\Rightarrow$ Standard Model at $E \ll 1$ GeV

(2) QCD is described by the lagrangian $\mathcal{L} = \bar{\psi} i \not{D} \psi - \frac{1}{4} G_{\mu\nu a} G^{\mu\nu}_{a}$ (massless limit)

However, because the strong interactions are indeed strong, at low energy we see only bound states: mesons and baryons

mesons : $q\bar{q}$ bound states

baryons : qqq bound states



$\Rightarrow$ CHIRAL PT provides a field theoretical descriptions of baryons, mesons and their interactions at relatively low energies

Note that bound states of quarks are qualitatively different than the atomic bound states, which are driven by the electromagnetic force. An atom has typically a rather low binding energy, compared to the mass of the constituents. In a QCD bound state, the binding energy is much larger than the sum of the masses of the constituents.

In order to write the effective lagrangian we have to

1) Enlarge the chiral symmetry group to accomodate 8 mesons (SU(2) → SU(3)

2) we have to understand how to introduce explicit chiral symmetry breaking

3) We must include baryons

We start from the QCD lagrangian with $N_f = 3$

$$\mathcal{L} = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \qquad \psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}$$

Such massless QCD lagrangian has the $SU(3)_L \otimes SU(3)_R$ chiral invariance

$$\psi_L \to e^{i \alpha_L^a \frac{\lambda^a}{2}} \psi_L \qquad \psi_R \to e^{i \alpha_R^a \frac{\lambda^a}{2}}$$

3 Here λ_a are the Gell-Mann SU(3) matrices (the generators of SU(3) in the fundamental representation)

We can now follow what we did in the non-linear σ model and write

$$U = \exp\left\{ i \lambda_a \varphi^a / v \right\} \qquad \underline{\pi} \cdot \underline{\tau} \to \lambda_a \varphi^a$$

SU(2) SU(3)

$$\frac{1}{\sqrt{2}} \lambda_a \varphi^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}_0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

As in the non linear σ model, we require U to transform as $U \to U_L U U_R^+$ under $SU(3)_L \otimes SU(3)_R$ and the ϕ_i transform non linearly under general chiral rotations, but linearly under the diagonal (unbroken) $SU(3)_V$ subgroup.

The effective lagrangian is constructed by including all the possible terms that respect the symmetry group

$$\mathcal{L} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \dots$$

where the first term is

$$\mathcal{L}_{(2)} = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U^+ \partial^\mu U \right)$$

Such effective lagrangian has three remarkable properties:

- The symmetry requires interactions with arbitrarily many mesons
- One parameter, v, determines all meson interactions
- Derivative couplings: the interaction vanishes if the momentum becomes soft.

Up to now the effective lagrangian is valid only in the chiral limit $m_q = 0$

⇒ we should account for the quark masses that break the symmetry

In the QCD lagrangian the mass terms appear as

$$\mathcal{L}_m = -\bar{\psi}_R M^\dagger \psi_L - \bar{\psi}_L M \psi_R \qquad \text{with} \qquad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

To make this term invariant we could "promote" the mass matrix M to an external field and ensure that it transforms like $M \to U_L M U_R^\dagger$

⇒ in this way the QCD lagrangian would become invariant!

We can now add a term in $\mathcal{L}_{eff}$ which contains M and is $SU(3)_L \otimes SU(3)_R$ invariant. This term is

$$\frac{v^2}{2} B \, \mathrm{Tr}\left(M^\dagger U + M U^\dagger\right)$$

where the constant B is arbitrary. This term will give a mass to the mesons.

In the case of $SU(2)$ we get

$$\frac{v^2}{2} B \, \mathrm{Tr}\left(M^\dagger\left(1 - \frac{\Sigma^2}{2v^2}\right) + M\left(1 - \frac{\Sigma^2}{2v^2}\right)\right) = \frac{v^2}{2} B \left(m_u + m_d\right)\left(-\frac{\Sigma^2}{v^2}\right)$$

So the three pions have the same mass $m_\pi^2 \sim (m_u + m_d)$

This is indeed a consequence of the properties of the τ matrices $\{\tau^i, \tau^j\} = 2\delta^{ij}$

In the case of $SU(3)$ $\{\lambda^a, \lambda^b\}$ is non trivial!

To understand the meaning of B we consider M as an effective source and take the functional derivative of the full and EFT generating functional.

$$\frac{1}{i} \frac{\delta Z_{QCD}}{\delta m_{ij}} = -\langle 0 | \bar{\psi}_{Li} \psi_{Rj} + \bar{\psi}_{Rj} \psi_{Li} | 0 \rangle$$

$$\frac{1}{i} \frac{\delta Z_{eff}}{\delta m_{ij}} = \frac{v^2}{2} B \langle 0 | U_{ij} + U^\dagger_{ji} | 0 \rangle$$

(5) Setting $\xi_i = 0$ we get ($\xi_i = 0 \Rightarrow U = 1$)

$$v^2 B\, \delta_{ij} = -\langle 0 |\, \bar{\psi}_{Li}\psi_{Rj} + \bar{\psi}_{Rj}\psi_{Li} \,| 0 \rangle$$

$\Rightarrow$ B corresponds to the quark condensate in the $m_q \to 0$ limit

By defining $\chi = 2B\mathcal{M}$ we can write the EFT lagrangian as

$$\mathcal{L}^{(1)} = \frac{v^2}{4} \operatorname{Tr}\left[\partial^\mu U^+ \partial_\mu U + \chi^+ U + \chi U^+\right]$$

Expanding the lagrangian up to the second order in ϕ we get

$$4 m_K^2 - 3 m_\eta^2 - m_\pi^2 = 0$$ GELL-MANN OKUBO relation

$$\frac{2 m_K^2 - m_\pi^2}{m_\pi^2} = \frac{2 m_s}{m_u + m_d}$$ WEINBERG RATIO OF QUARK MASSES

To understand how the mesons interact with photons, W and Z bosons, it is useful to introduce EXTERNAL SOURCES with appropriate quantum numbers both into the full and EFT lagrangians. We can write

$$\mathcal{L} = \mathcal{L}^0_{QCD} + \bar{\psi}\gamma^\mu(v_\mu + a_\mu\gamma_5)\psi - \bar{\psi}(s + ip\gamma_5)\psi \equiv \mathcal{L}_0 + \mathcal{L}_1$$

The external fields v_μ, a_μ, s, p can be used to probe different aspects of the theory. We have seen already an example of application of this method in the case of the inclusion of quark masses ($s \leftrightarrow \mathcal{M}$)

The scalar and pseudoscalar fields s and p provide a convenient way to represent photons, W and Z bosons.

To construct the EFT lagrangian in the presence of these external sources, one can use the fact that $\mathcal{L}_{QCD}$ becomes invariant under LOCAL transformations provided that the external fields transform as GAUGE FIELDS

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$$\begin{cases} r_\mu = v_\mu + a_\mu \to U_R (v_\mu + a_\mu) U_R^+ - i(\partial_\mu U_R) U_R^+ \\ l_\mu = v_\mu - a_\mu \to U_L (v_\mu - a_\mu) U_L^+ - i(\partial_\mu U_L) U_L^+ \end{cases}$$

The effective lagrangian can be constructed by replacing ∂_μ with D_μ

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$$

The above procedure however has a complication. $\mathcal{L}_{QCD}$ and $\mathcal{L}_{eff}$ are invariant under local chiral transformations, but the generating functional

$$Z(v, a, s, p) = \int D\psi D\bar{\psi} DA_\mu \, e^{i\int d^4x (\mathcal{L}_0 + \mathcal{L}_1)}$$

has an ANOMALY due to the integration measure over the fermion fields.

$\Rightarrow$ Since the effective field theory does not involve the quark fields any more the invariance of the effective lagrangian leads to the invariance of the theory. To correct this mismatch we must add a term to $\mathcal{L}_{eff}$ to reproduce the change of the QCD generating functional

This term is called WESS-ZUMINO-WITTEN term

$$\mathcal{L}_{eff} = \mathcal{L}_{inv} + \mathcal{L}_{WZW}$$

compensates for the "lack" of the anomaly in the EFT

INTERACTION WITH BARIONS

To complete our discussion we have to include in the effective lagrangian the meson-barion interaction

Defining $u = U^{1/2}$ $\qquad u_\mu = i\{u^+(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^+\}$

(7)

we have

$$\mathcal{L}_{\pi N} = \bar{\Psi}\left(i\not{\partial} - m + \frac{g_A}{2}\not{u}\gamma_5\right)\Psi \qquad \underline{n_F = 2}$$

↑ expected from the σ-model

$$\mathcal{L}_{MB} = \mathrm{Tr}\left(\bar{B}(i\not{D}-m)B + \frac{D}{2}\bar{B}\gamma^\mu\{u_\mu, B\} + \frac{F}{2}\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\right) \qquad \underline{n_F = 3}$$

$g_A = F + D$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$