Instanton effects in the EW theory and baryon and lepton number non conservation

We have seen that instantons provide tunneling between topologically distinct vacua in the Yang-Mills theory. Similar effects exist in the $SU(2)_L \otimes U(1)$ EW theory. To simplify the discussion, we can focus on the $SU(2)_L$ gauge group, forgetting for a moment about the $U(1)$ part (for example setting $\theta_W = 0$). We also consider massless fermions (we neglect the Higgs coupling to fermions), and suppose to have only one generation. The theory is invariant under the global $U(1)$ transformations associated to barion (B) and lepton (N_e) number conservation.

these assumptions make the discussion simpler but the results do not change significantly

$$\psi(x) \to e^{iB\theta}\,\psi(x)$$

$$\psi(x) \to e^{iN_e\theta}\,\psi(x)$$

It follows that the both the lepton

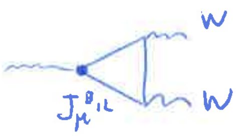
$$J^L_\mu = \bar{e}\gamma_\mu e + \bar{\nu}_e \gamma_\mu \nu_e$$

and the baryon current

$$J^B_\mu = \sum_{color}\left(\frac{1}{3}\bar{u}\gamma_\mu u + \frac{1}{3}\bar{d}\gamma_\mu d\right)$$

are classically conserved.

However, due to the anomaly, the conservation is broken at quantum level. Taking into account the factor $\frac{1}{3}$ of normalization and the fact that the lepton and quark doublet have the same $SU(2)$ quantum numbers, we conclude that



$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{g^2}{32\pi^2} F\tilde{F}$$

Note that, because of the cancellation of the electric charges in one generation, the electric current

$$J_\mu^{em} = -\bar{e}\gamma_\mu e + \sum_{\text{color}}\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d\right)$$

is conserved

$$\partial_\mu J^\mu_{em} = 0$$

as it should be.

The above discussion implies that baryon and lepton numbers are not separately conserved.

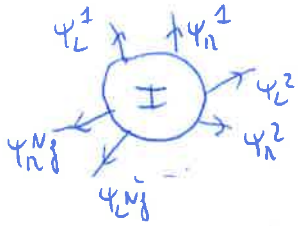
The difference $B-L$ is instead conserved.

We can repeat the same discussion done for QCD and conclude that

$$\Delta Q_B = \Delta Q_L = \frac{g^2}{32\pi^2}\int d^4x\, F\tilde{F} = \frac{g^2}{32\pi^2}\int d^4x\, \partial_\mu K^\mu$$

where K_μ is the Chern-Simons current.

We have seen in LECTURE 3, when talking of the Schwinger model on a circle, that the effect of a "large" (topologically non trivial) gauge transformation is to restructure the energy levels in the fermionic spectrum such that for a fermion we have $\Delta Q_5 = 2$. This means that, if there are N_f fermions, the effect of instantons is to produce an effective Lagrangian with $2N_F$ interacting fermions



$$\mathcal{L}_{eff} \sim \prod_i^{N_f} \psi_{L_i}\psi_{R_i}$$

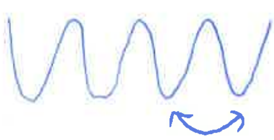
't Hooft vertex

which produces a $\Delta Q_5 = 2N_f$ violation.

(EACH INSTANTON EMITS OR ABSORBS TWO WEYL FERMIONS OF THE SAME CHIRALITY PER MASSLESS QUARK FLAVOR)

In our case we have

$$\Delta Q_B = \int d^4x\, \partial_\mu J^\mu_B = N_g \underset{\pm 1, \pm 2 \dots}{\underset{\shortparallel}{\Delta N_{CS}}}$$



$\Delta B = \Delta L = 1$

This means that the effective interaction to produce $\Delta B = \Delta L = 1$ for one generation is

$$\mathcal{L}_{eff} \sim \prod_{i=1}^{N_G}\left(q_{Li}\, q_{Li}\, q_{Li}\, \ell_{Li}\right)$$

With 3 generations, the simplest interaction involves 12 fermions!

Example: $q\bar{q} \to 7\bar{q} + 3\ell$

The strength for this interaction turns out to be

$$e^{-\frac{8\pi^2}{g^2}} = e^{-\frac{2\pi}{\alpha_W}} \sim 10^{-80}$$

$$g \approx 1.6 \cdot 10^{-101}\,(\mathrm{GeV})^{-14}$$

and it gives negligibly small cross sections.

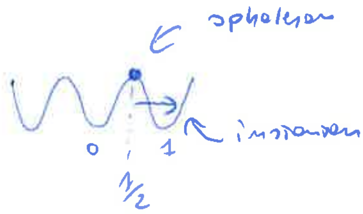
Let us now go back to the vacuum structure

We can ask ourselves what is the height of the potential barrier. The height of the maximum must have the dimension of a mass.

In QCD, since the IR limit is not tractable quasiclassically, it is not possible to quantify the height of the barrier, which is however $O(\Lambda_{QCD})$

But in the EW theory, with SSB and the Higgs mechanism, the vacuum expectation value of the Higgs field provides a new scale and the question of evaluating the height of the barrier is well posed.

<u>Sphalerons</u> (Manton 1983; Klinkhamer, Manton 1984)



The SM equations possess a static finite energy solution called "sphaleron" (from greek σφαλερός, "weak, dangerous", "ready to fall")

Since the vacuum is the only stable, static configuration, the sphaleron must be <u>unstable</u>!

The winding number for this configuration is $N_{CS} = \frac{1}{2}$. One can compute the energy associated to this configuration. The result is

$$E = \frac{2m_W}{\alpha_W}\,\mathcal{E}\left(\frac{m_H}{m_W}\right)$$

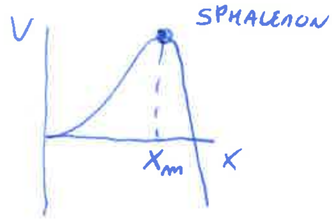
where $\mathcal{E}$ is a slowly varying function of m_H/m_W which can be approximated by

$$\mathcal{E}(x) = 1.58 + 0.32x - 0.05x^2$$

setting $x = \frac{m_H}{m_W} = 1.55$ we get $E \approx 10\,\mathrm{TeV}$

We now discuss the possibility that thermal fluctuations allow to go over the energy barrier. To do so let us go back to the Schroedinger equation in one dimension:

$$\left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}+V(x)\right)\psi = E\psi$$



The approximate semiclassical solution is

$$\psi \sim e^{\,i\int\sqrt{2M(E-V)}\,dx}$$

In the classically allowed regions, where $E>V$, the wave function oscillates, while under the barrier it decays exponentially.

$$\psi \sim e^{-\int\sqrt{2M(V-E)}\,dx}$$

Let us now consider a particle in equilibrium with a thermal bath of temperature T. The particle can acquire energy from the thermal bath, and if its energy exceeds the maximum, it can escape from the potential well.

The total probability to escape is given by

$$P \sim \sum_E \exp\left\{-\frac{E}{T} - 2\theta\int\sqrt{2M(V-E)}\right\}$$

where $\theta=1$ if $E<V(x_m)$ and $\theta=0$ if $E>V(x_m)$

To estimate this sum we can use the saddle point approximation. If we take the derivative of the exponent with respect to E we get

$$\frac{1}{T} = 2\int\sqrt{\frac{M}{2(V-E)}}\,dx = 2\int\frac{dx}{\dot{x}} = \int 2d\tau \qquad \dot{x} \text{ Euclidean velocity}$$

↳ period of oscillation in the inverted potential $-V$

⇒ For a given temperature T

· the dominant contribution to the escape probability gives the periodic Euclidean trajectory in the potential $-V$

We can consider two limiting cases. If T is very small, the main contribution to the escape probability comes from the instanton solution, that is, the tunneling probability. At very high temperature, $T \gg \frac{V(x_m)}{S_I}$, the period of oscillation tends to zero, hence the dominant trajectory comes close to the top of the potential V and the energy is $V(x_m)$. This unstable solution corresponds to the sphaleron. In this case the escape probability is $P \sim e^{-\frac{E_{sph}}{T}}$ when $E_{sph} = V(x_m)$

In this case there is no exponential suppression and the particle quickly leaves the basin.

Sakharov conditions

The baryon asymmetry can be dynamically generated in the decay of heavy particles if the following 3 conditions are satisfied:

- **Baryon number violation**

  If baryon number is conserved, no baryon asymmetry can be dynamically generated

- **C or CP violation**

  If C or CP are conserved, then the baryon-number violating process $\Gamma(X \to Y + B)$ would have the same rate as $\Gamma(\bar{X} \to \bar{Y} + \bar{B})$
  $\Rightarrow$ no net effect

- **Departure from thermal equilibrium**

  In thermal equilibrium, the production rate of baryons is equal to their destruction rate

  $\Gamma(X \to Y + B) = \Gamma(Y + B \to X) \Rightarrow$ no net effect