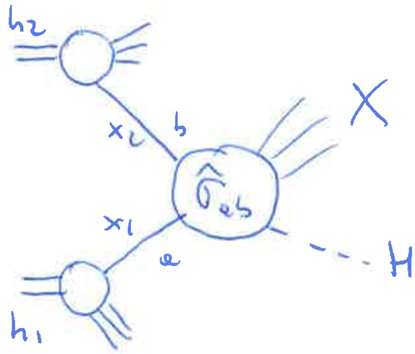
Before starting to discuss the calculation let us recall what the factorization theorem tell us for this process

$$\sigma(s, m_H^2) = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_{a/h_1}(x_1, \mu_F^2) \, f_b(x_2, \mu_F^2) \int_0^1 dz \, \delta\left(z - \frac{\tau}{x_1 x_2}\right)$$

$$\cdot \, \hat{\sigma}_{ab}\left(z; \alpha_s(\mu_R^2), m_H^2/\mu_R^2, m_H^2/\mu_F^2\right)$$

$$\tau = \frac{m_H^2}{S}$$



The partonic CM energy squared s is related to the hadronic one by the relation

$$s = x_1 x_2 S$$

The variable z give us a measure of the "inelasticity" of the process and it is defined as $m_H^2 = z s$.

Since at Born level there is no additional radiation, we have $z=1$

The partonic cross section can be computed as a perturbative expansion in α_s

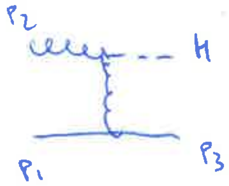
$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0)} + \left(\frac{\alpha_s}{\pi}\right) \hat{\sigma}_{ab}^{(1)} + \dots$$

Where the zero-order contribution, that we have already evaluated, is

$$\hat{\sigma}_{ab}^{(0)} = \frac{\alpha_s^2}{\pi} \frac{1}{576 v^2} \delta(1-z) \, \delta_{ag} \delta_{bg}$$

NLO corrections

We start the calculation from the qg channel, and there is only one Feynman diagram



$$\overline{|M_{qg\to qH}|^2} = -\frac{\alpha_s^3}{18\pi v^2}\,\frac{1}{N(1-\varepsilon)}\,\frac{s^2+u^2-\varepsilon(s+u)^2}{\underset{\uparrow\ \text{collinear singularity}}{t}}$$

$$s=(p_1+p_2)^2 \qquad t=(p_1-p_3)^2 \qquad u=(p_2-p_3)^2$$

To compute the cross section we have to integrate the matrix element on the two particle phase space, which can be expressed as

$$\sigma_{qg} = \frac{1}{2s}\int \overline{|M_{qg\to qH}|^2}\, d\phi_2 \qquad\qquad d\phi_2 = \frac{1}{8\pi}\left(\frac{4\pi}{s}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\, z^{\varepsilon}(1-z)^{1-2\varepsilon}\, y^{-\varepsilon}(1-y)^{-\varepsilon}\, dy$$

$$y=\frac{1+\cos\theta_3}{2} \qquad z=\frac{m_H^2}{s}$$

By doing the phase space integral we obtain

$$\sigma_{qg} = (4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon)\left(\frac{\mu^2}{m_H^2}\right)^{\varepsilon}\frac{\alpha_s}{2\pi}\,\sigma_{LO}(\varepsilon)\left[\cancel{-\frac{1}{\varepsilon}P_{gq}(z)} + P_{gq}(z)\log\frac{(1-z)^2}{z} + \frac{4}{3}z - 2\frac{(1-z)^2}{z}\right]$$

↑ collinear pole

The collinear pole, proportional to the $q\to g$ DGLAP splitting function $P_{gq}(z)=C_F\frac{1+(1-z)^2}{z}$ has to be cancelled with the FACTORIZATION COUNTERTERM

In this way the collinear singularity is reabsorbed in the quark PDF

The function $\sigma_{LO}(\varepsilon)$ is the lowest order cross section computed in $D=4-2\varepsilon$ and has the form $\sigma_{LO}=\frac{\alpha_s^2}{\pi}\,\frac{z}{576 v^2}\,\frac{1}{1-\varepsilon}\,\delta(1-z)$

We note that after the subtraction of the collinear singularity, there is a logarithmic term $\log\frac{(1-z)^2}{z}$ which remains, which is still controlled by P_{gq}

This term corresponds to the left over of the collinear singularity, which has been integrated from m_H^2 to $q^2_{T\,MAX}\sim\frac{(1-z)^2}{z}m_H^2$ which is the maximum transverse momentum allowed by kinematics.

We now go on with the calculation in the gg channel.

The amplitude squared is

$$|M_{gg\to gH}|^2 = \frac{\alpha_s^3}{v^2}\left(\frac{32}{3\pi}\right)\left\{\frac{m_H^8+s^4+t^4+u^4}{stu}(1-2\varepsilon)+\frac{\varepsilon}{2}\frac{(m_H^4+s^2+t^2+u^2)^2}{stu}\right\}$$

which has to be multiplied by a factor $\frac{1}{4}\cdot\frac{1}{(N^2-1)^2}\frac{1}{(1-\varepsilon)^2}$ to average over colour and spin.

By doing the integral over the two particle phase space we get

$$\hat\sigma_{gg}^{real} = \frac{1}{576\pi^2}\frac{\alpha_s^3}{v^2}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\varepsilon}\Gamma(1+\varepsilon)\,z^{-\varepsilon}\left(1-\frac{\pi^2\varepsilon^2}{3}\right)(1-z)^{-1-2\varepsilon}\times$$

$$\times\left[-\frac{3}{\varepsilon}\left(1+z^4+(1-z)^4\right)-\frac{11}{2}(1-z)^4-6(1-z+z^2)^2-6\varepsilon\right]$$

As in the qg channel, the integration produces a pole in $\frac{1}{\varepsilon}$, however, in this channel we also get a factor $(1-z)^{-1-2\varepsilon}$. This term cannot be expanded at $\varepsilon\to 0$ because it would lead to singularities as $z\to 1$. The limit $z\to 1$ indeed probes the SOFT REGION, in which the radiation recoiling against the Higgs boson is forced to be soft ($p_1+p_2=p_3+p_4 \Rightarrow (p_1+p_2-p_3)^2=m_H^2 \Rightarrow s-2p_3(p_1+p_2)=m_H^2$

$\Rightarrow 1-z=\frac{2p_3(p_1+p_2)}{s} \Rightarrow 1-z\to 0$ forces p_3 to be soft)

The term $(1-z)^{-1-2\varepsilon}$ has to be treated as a DISTRIBUTION : let's see how it acts onto a test function $f(z)$.

$$\int_0^1(1-z)^{-1-2\varepsilon}f(z) = \int_0^1(1-z)^{-1-2\varepsilon}\left[f(z)-f(1)+f(1)\right] = f(1)\left(-\frac{1}{2\varepsilon}\right)+\int_0^1\frac{f(z)-f(1)}{1-z}$$

$$-2\varepsilon\int_0^1\frac{f(z)-f(1)}{1-z}\log(1-z) \quad +O(\varepsilon^2)$$

$$\Rightarrow (1-z)^{-1-2\varepsilon} = -\frac{1}{2\varepsilon}\delta(1-z)+\left(\frac{1}{1-z}\right)_+ -2\varepsilon\left(\frac{\log(1-z)}{1-z}\right)_+ +O(\varepsilon^2)$$

We now must consider the virtual correction. The one-loop contribution is

$$\overline{\mathcal{M}^{(1)}_{UN}\,\mathcal{M}^{(0)*}+c.c.} = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,C_A\left(-\frac{2}{\epsilon^2}+\frac{11}{3}+\pi^2\right)\overline{|\mathcal{M}^{(0)}|^2}$$

where $\mathcal{M}^{(1)}_{UN}$ is the UNRENORMALIZED amplitude

The UV renormalization is accounted for by subtracting the UV pole as

$$\overline{\mathcal{M}^{(1)}\mathcal{M}^{(0)*}+c.c.} = \overline{\mathcal{M}^{(1)}_{UN}\mathcal{M}^{(0)*}+c.c.} - \frac{\alpha_s}{\epsilon}\,2\,(4\pi)^{\epsilon}\,\Gamma(1+\epsilon)\,\beta_0\,\overline{|\mathcal{M}^{(0)}|^2}$$

2 powers of α_s!

and $\beta_0 = \frac{11}{12\pi}C_A - \frac{n_F}{6\pi}$

we thus get

$$\overline{\mathcal{M}^{(1)}\mathcal{M}^{(0)*}+c.c.} = \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{6}{\epsilon^2}-\frac{4\pi\beta_0}{\epsilon}+11+2\pi^2\right)\overline{|\mathcal{M}^{(0)}|^2}$$

The factorization counterterm to cancel the initial state collinear singularities is

$$\sigma_{CT} = \sigma_0(\epsilon)\,\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{m_H^2}\right)^{\epsilon}\Gamma(1+\epsilon)\,\frac{2}{\epsilon}\,P_{gg}(z)$$

with $P_{gg}(z) = 2C_A\left[\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)\right]+2\pi\beta_0\,\delta(1-z)$

Summing everything all the singularities cancel out and we obtain

$$\sigma_{gg} = \frac{\alpha_s}{\pi}\sigma_0\left[\left(\frac{11}{2}+\pi^2\right)\delta(1-z)+12\left(\frac{\log(1-z)}{1-z}\right)_+ + P^{reg}_{gg}(z)\,\ln\frac{(1-z)^2}{z}-\frac{6\ln z}{1-z}-\frac{11}{2}\frac{(1-z)^3}{z}\right]$$

contains a factor z

where $P^{reg}_{gg} = 6\left(\frac{1-z}{z}-1+z(1-z)\right) = 6\left[\left(\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)\right)-\frac{1}{(1-z)_+}\right]$

is the P_{gg} splitting function after having subtracted the soft singularity.

The final contribution to be considered is the one of the $q\bar{q}$ channel,
where only one diagram contributes

This diagram produces no divergences $\qquad \overline{|M|^2} = \frac{16}{9}\frac{\alpha_s^3}{\pi v^2}\frac{u^2+t^2-\varepsilon(u+t)^2}{s}$

$$\Rightarrow \quad \sigma_{q\bar{q}} = \frac{32}{27}\,\sigma_0\,\frac{\alpha_s}{\pi}\,\frac{(1-z)^3}{z}$$

contains a factor z

The impact of the $q\bar{q}$ channel is very small, since the q density is much suppressed with respect to the gluon density.

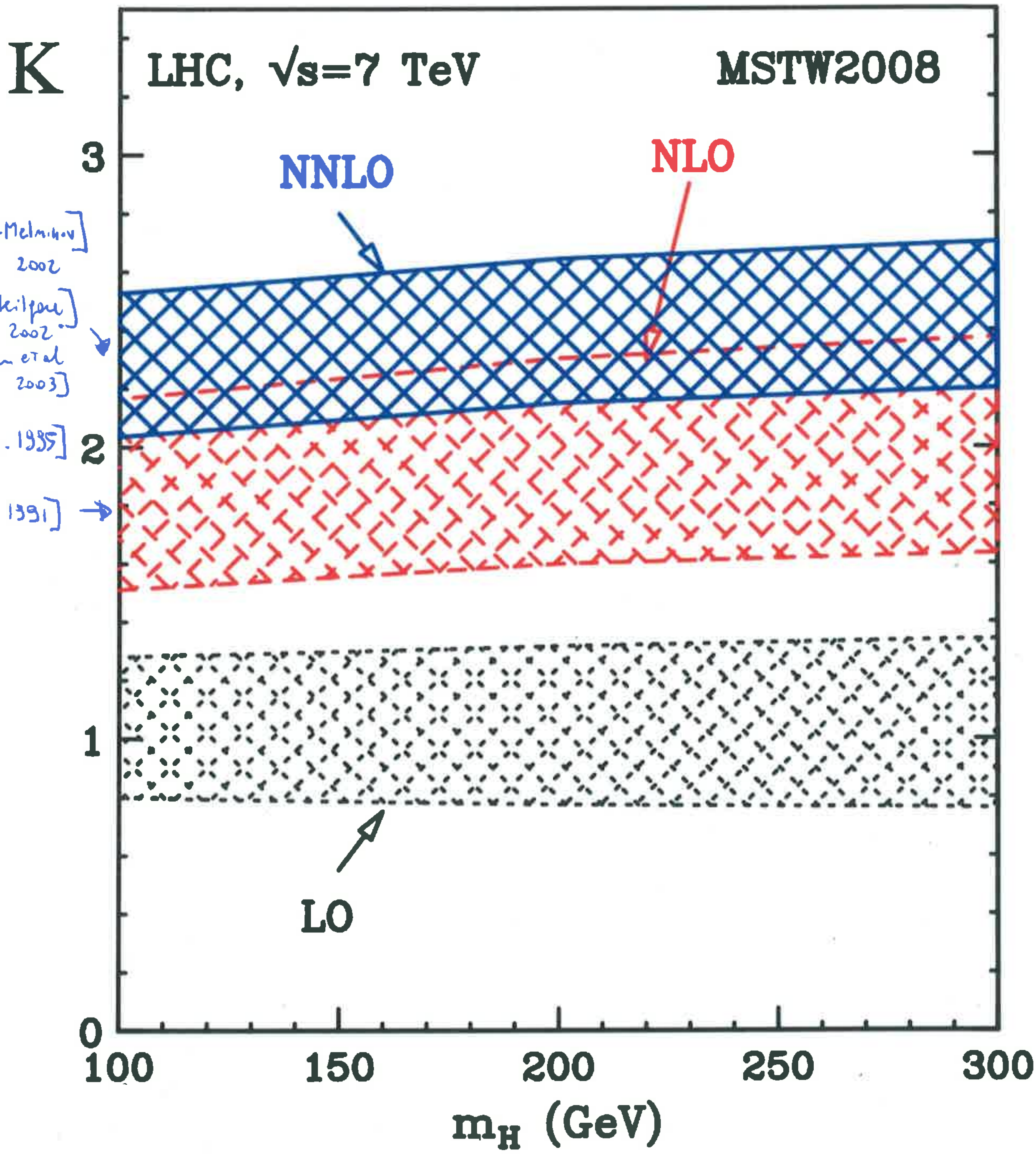
The impact of NLO corrections is very large, and increase the LO result by about 100% at LHC energies. The impact of QCD radiative corrections can be presented by using K-factors

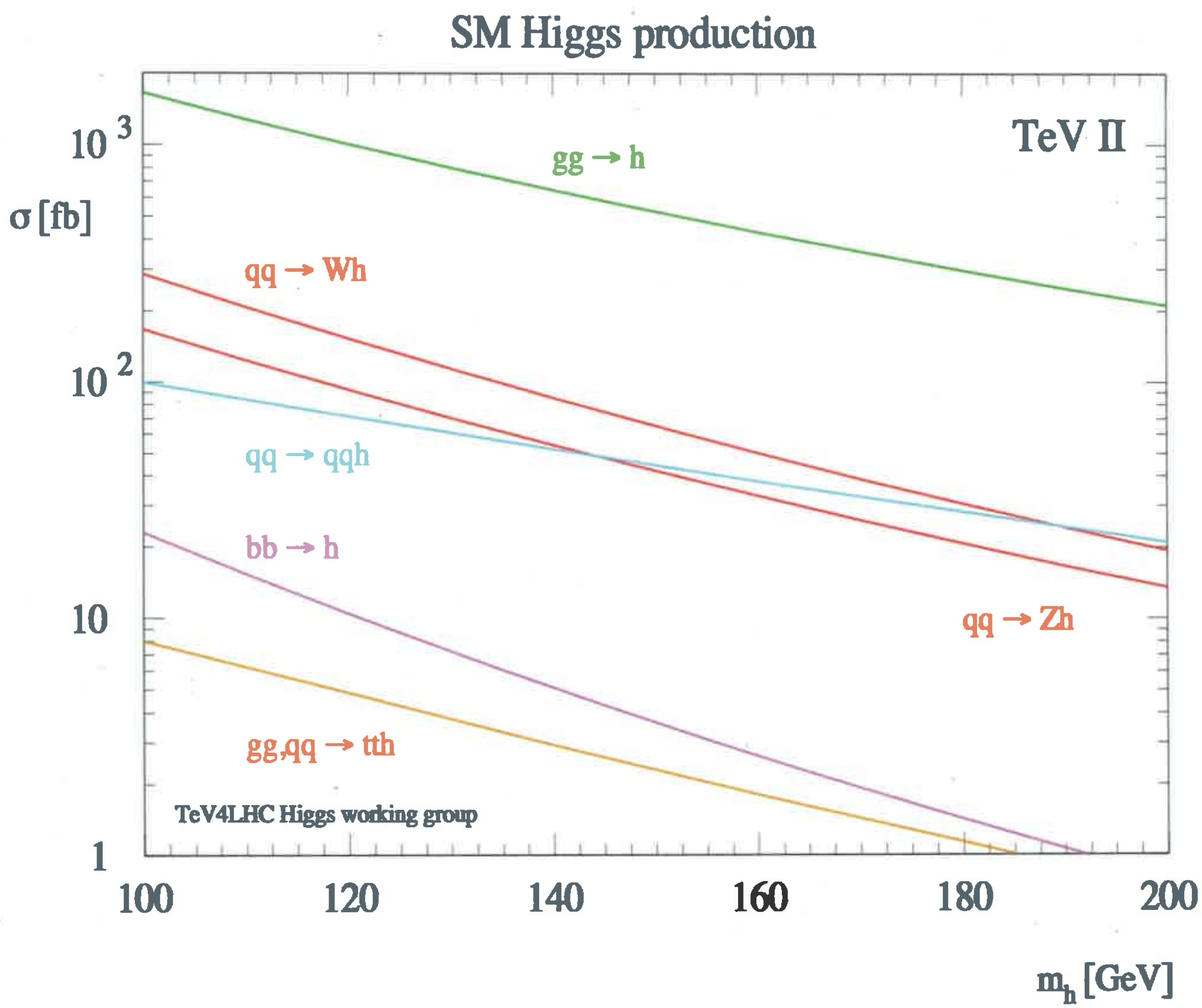
$$K_{NLO} = \frac{\sigma_{NLO}(\mu_F,\mu_R)}{\sigma_{LO}(\mu_F=\mu_R=m_H)} \qquad K_{NNLO} = \frac{\sigma_{NNLO}(\mu_F,\mu_R)}{\sigma_{LO}(\mu_F=\mu_R=m_H)}$$

The following plot is obtained by varying $\frac{1}{2}m_H < \mu_F,\mu_R < 2m_H$ with $\frac{1}{2} < \frac{\mu_F}{\mu_R} < 2$
This choice, which is to some extent arbitrary defines a way to estimate perturbative uncertainties. We see that the LO and NLO bands do not overlap, thus implying that this procedure can only give a LOWER LIMIT of the true perturbative uncertainty. However the NNLO result overlaps with the NLO one, thus suggesting that perturbation theory is under control.

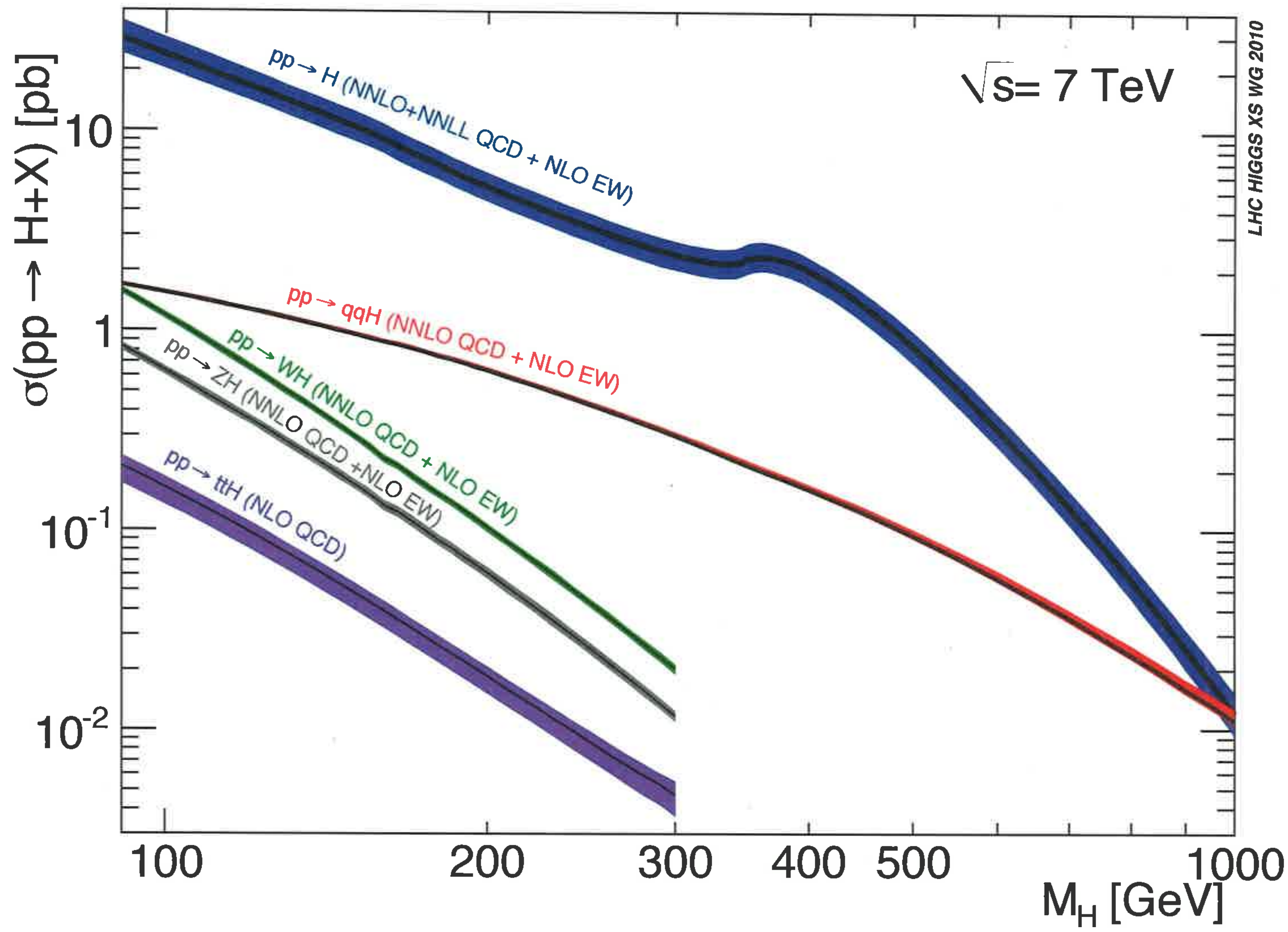
The recent computation of N^3LO corrections [Anastasiou et al. 2015] shows that indeed higher order corrections are rather small.

[Anastasiou-Melnikov 2002]
[Harlander-Kilgore 2002]
[Van Neerven et al 2003]
[Spira et al. 1995]
[Dawson, 1991]
$m_t \to \infty$





SM Higgs production
TeV II
σ [fb]
10 3
10 2
10
1
100
120
140
160
180
200
m_h [GeV]
gg → h
qq → Wh
qq → qqh
bb → h
qq → Zh
gg,qq → tth
TeV4LHC Higgs working group



σ(pp → H+X) [pb]
10
1
10-1
10-2
pp→ H (NNLO+NNLL QCD + NLO EW)
pp→ qqH (NNLO QCD + NLO EW)
pp→ WH (NNLO QCD + NLO EW)
pp→ ZH (NNLO QCD +NLO EW)
pp→ ttH (NLO QCD)
√s= 7 TeV
LHC HIGGS XS WG 2010
100
200
300
400
500
1000
MH [GeV]