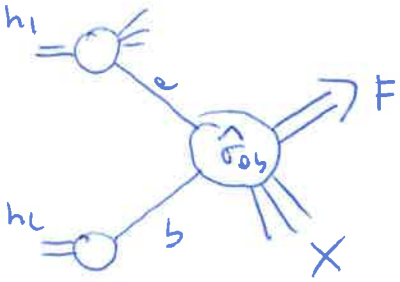
HARD PROCESSES AT HADRON COLLIDERS

The possibility to obtain theoretical predictions for hard scattering processes at hadron colliders rely on the so called FACTORIZATION THEOREM, which allow us to write the cross section to produce some final state F characterized by a hard scale Q as

partonic cross section

$$\sigma^F = \sum_{ab} \int_0^1 dx_1\, dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \hat{\sigma}^F_{ab}\left(x_1 P_1, x_2 P_2, \alpha_S(\mu_R^2), \frac{\mu_F^2}{Q^2}, \frac{\mu_R^2}{Q^2}\right)$$

parton distributions



$$h_1(P_1) + h_2(P_2) \to F + X$$

we are inclusive over additional QCD radiation

The partonic cross section can be computed in perturbation theory while the parton distributions are usually extracted from data.

μ_R^2 renormalization scale: it is the scale appearing in the renormalization procedure, at which the running coupling α_S is evaluated

μ_F^2 factorization scale: effectively it is the scales which characterize the factorization procedure. Radiation with transverse momenta below μ_F is absorbed into the PDFs

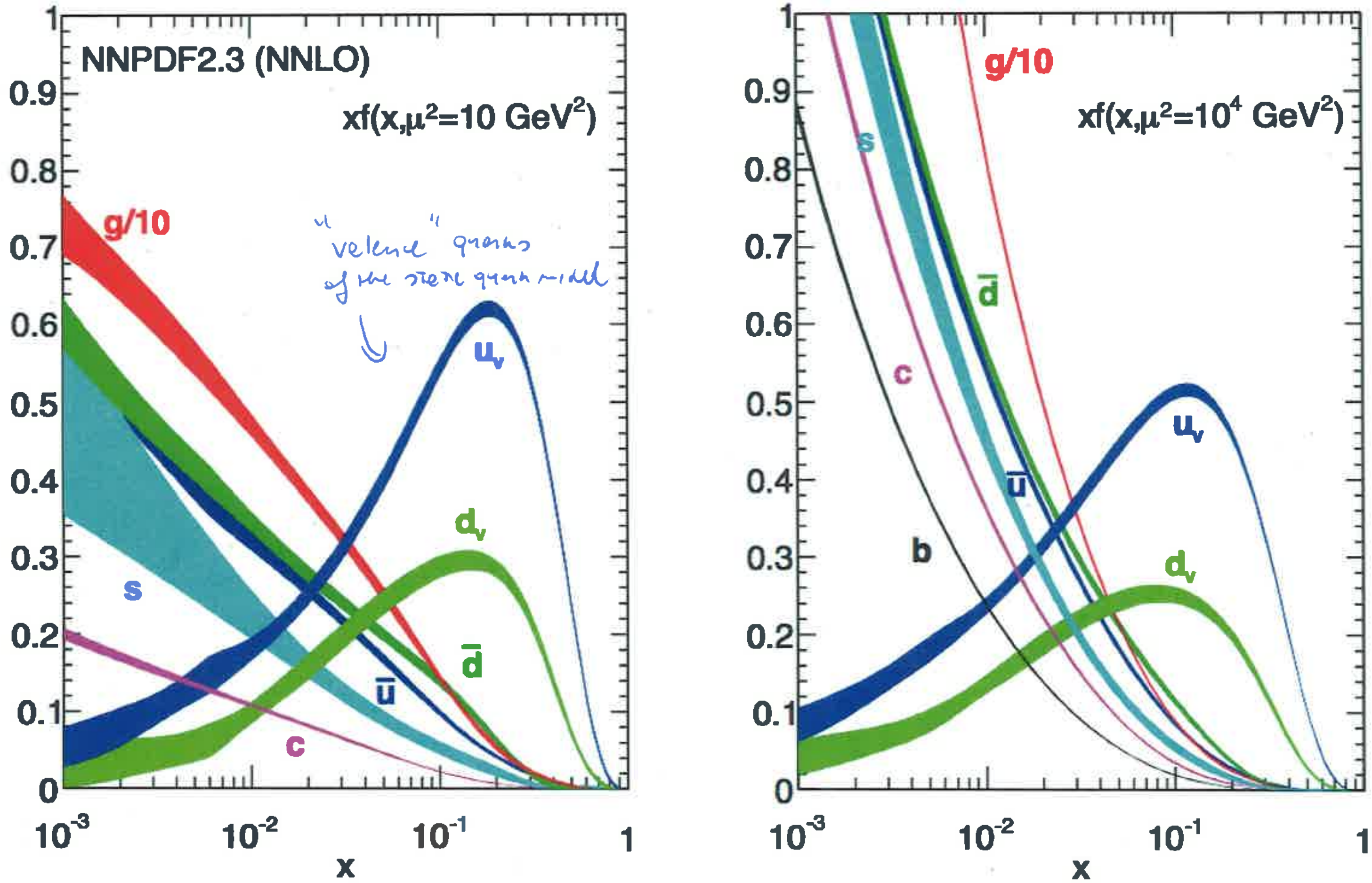
Usually one chooses $\mu_F \sim \mu_R \sim Q$: studying the effect of variations of the hadronic cross section with μ_F and μ_R can give us an idea of uncalculated higher order contributions

F contains our "hard probe": if we deal with Higgs production we will have $F = H$ + an additional SM particle which depends on the production process.

PDFs are obtained by assuming a certain parametrization for the partons at a starting scale $Q_0 \sim 1\text{-}2$ GeV

$f_a(x, Q_0^2) = x^{\alpha_a} (1-x)^{\beta_a} P_a(x)$ $\Rightarrow$ evolve up to the desired scale Q^2 through DGLAP evolution equations and then FIT TO DATA

slowly varying function

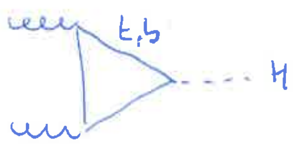


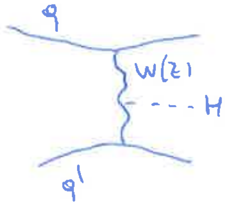
The probability of extracting a soft ($x \ll 1$) gluon from the proton is much higher: gluon initiated processes have an increasing role as the CM energy of the collider increases

HIGGS PRODUCTION AT HADRON COLLIDERS

As in e^+e^- collisions, at hadron colliders the Higgs production mechanisms all make use of the fact that the Higgs boson couples predominantly to heavy particles.

The main production channels are thus 1) gluon-gluon fusion 2) vector-boson fusion 3) associated production with a vector boson 4) associated production with a $t\bar{t}$ pair

1) 

2) 

3) 

4) 

We discuss the four production channels in turn.

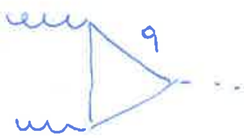
$\boxed{\text{GLUON FUSION}}$

Gluon fusion is the dominant production channel for Higgs boson production in the SM, due to the large probability to extract gluons from the incoming protons.

It is a process which already starts at one loop level in the Born approximation.

The LO cross section can be written as

$$\sigma_{LO} = \frac{\alpha_s^2}{\pi} \frac{m_H^2}{256 v^2} |A|^2 \delta(s - m_H^2)$$



where

$$A = A_{1/2} = \sum_q \tau_q \left(1 + (1-\tau_q) f(\tau_q)\right) \qquad \tau_q = \frac{4 m_q^2}{m_H^2}$$

and

$$f(\tau_q) = \begin{cases} \arcsin^2 \sqrt{1/\tau_q} & \tau_q \geq 1 \\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau_q}}{1-\sqrt{1-\tau_q}} - i\pi\right]^2 & \tau_q < 1 \end{cases}$$

The limit $\tau_q \to 0$ corresponds to light quarks, and the cross section vanishes as $\tau_q \log \tau_q$.
The dominant contribution is given by heavy quarks, and in particular by the top quark.
In the standard model the bottom quark contributes about 10% of the cross section.
The limit $\tau_q \to \infty$ is called heavy top limit: in this limit we have

$$A_{1/2} \underset{\tau_q \to \infty}{\approx} \tau_q \left(1 - (1-\tau_q)\left(\frac{1}{\tau_q} + \frac{1}{3\tau_q^2}\right)\right) \to \frac{2}{3}$$

and we obtain

$$\sigma_{LO} \to \frac{\alpha_s^2}{\pi} \frac{m_H^2}{576 v^2} \delta(s - m_H^2)$$

The limit in which the heavy-quark mass becomes much larger than the Higgs mass corresponds to INTEGRATE OUT the heavy-quark field in the theory, and to shrink the quark loop to a point

This means that, effectively, the gluons become directly coupled to the Higgs through a new interaction, which is driven by an effective Lagrangian which is obtained by integrating out the heavy quark.

The explicit form of the effective Lagrangian can be obtained by observing that the Higgs boson couples to the trace of the energy-momentum tensor.
In an exactly scale invariant theory the dilatation transformation $x \to x e^{-\varepsilon}$ is a symmetry and the corresponding current is conserved $\partial_\mu D^\mu = 0 = \theta_\mu{}^\mu$, $\theta_\mu{}^\mu$ being the trace of the energy-momentum tensor. In practice the divergence of the dilatation current does not vanish for two reasons: first, there is an explicit mass term (we assume only the top quark to be massive).
Second, the renormalization procedure forces us to break scale invariance.

We can thus write

$$\partial_\mu D^\mu = \theta_\mu{}^\mu = (1+\gamma_m)\, m_t^0\, \bar{t}^0 t^0 + \frac{\beta(\alpha_s)}{2\alpha_s}\, G^a_{\mu\nu} G^{\mu\nu}_a$$

the first term corresponds to the explicit breaking (γ_m is the mass anomalous dimension) while the second term is the so called TRACE ANOMALY (its form can be understood by observing that under the scale transformation we have $\mu^2 \to e^{2\varepsilon}\mu^2$ and $\delta\alpha_s = \frac{\delta\mu^2}{\mu^2}\beta(\alpha_s) = 2\varepsilon\,\beta(\alpha_s)$ such that $\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\alpha_s}\delta\alpha_s \sim \frac{\beta(\alpha_s)}{\alpha_s} G^a_{\mu\nu} G^{\mu\nu}_a$)

We now observe that the matrix element $\langle 0|\theta_\mu{}^\mu|gg\rangle$ vanishes at zero momentum transfer

$$\lim_{Q^2\to 0} \langle 0|\theta_\mu{}^\mu|gg\rangle = 0$$

IWAZAKI, PRD15 (1977) 1172

Since when the Higgs has vanishing momentum it acts as a constant field we have

$$\lim_{p_H\to 0} \langle h|\theta_\mu{}^\mu h|gg\rangle = 0$$

we can exploit the previous expression for the $\theta_\mu{}^\mu$ to obtain

$$\mathcal{L}_{eff} = \frac{1}{2}\,\frac{\beta^t(\alpha_s)/\alpha_s}{1+\gamma_m}\, G^a_{\mu\nu} G^{\mu\nu}_a\, \frac{h}{v}$$

where the top contribution to the β function is obtained by taking the limit $p_H\to 0$ (which is indeed what we need to justify the effective field theory approach).

The approach we have used is very powerful, because it tells us that, to all orders in α_s, the coefficient in the effective Lagrangian is determined by the top contribution to the QCD β function and by the mass anomalous dimension.

We have

$$\beta_0 = \frac{11C_A - 2n_F}{12\pi} \qquad \beta_1 = \frac{17C_A^2 - 5C_A n_F - 3C_F n_F}{24\pi^2} \qquad \gamma_m = \frac{3}{2}C_F\frac{\alpha_s}{\pi} + O(\alpha_s^2)$$

$$\beta(\alpha_s) = \frac{d\alpha_s}{d\ln\mu^2} = -\beta_0\alpha_s^2 - \beta_1\alpha_s^3 + O(\alpha_s^4)$$

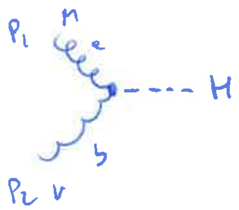
⇒ the top quark contribution comes from the M_F term

$$\beta_0^t = -\frac{1}{6\pi} \qquad \beta_1^t = -\frac{5C_A+3C_F}{24\pi^2}$$

$$\beta^t(\alpha_s) = \frac{\alpha_s^2}{6\pi}\left(1+\frac{5C_A+3C_F}{4\pi}\alpha_s+\cdots\right) = \frac{\alpha_s^2}{6\pi}\left(1+\frac{19}{4\pi}\alpha_s+\cdots\right)$$

$$\frac{1}{2}\,\frac{\beta^t(\alpha_s)/\alpha_s}{1+\gamma_m} = \frac{\alpha_s}{12\pi}\left(1+\frac{11}{4}\frac{\alpha_s}{\pi}+\cdots\right)$$

It turns out that the large m_{top} approximation is very good ⇒ we can use this approximation for our calculations. The effective ggH vertex is



$$i\,\frac{\alpha_s}{3\pi v}\,\delta^{ab}\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)$$

The corresponding scattering amplitude is

$$\mathcal{M} = i\,\frac{\alpha_s}{3\pi v}\,\delta^{ab}\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)\varepsilon_{1\mu}\varepsilon_{2\nu}$$

$$|\mathcal{M}|^2 = \frac{\alpha_s^2}{9\pi^2v^2}(N_c^2-1)\left(g^{\mu\nu}p_1p_2 - p_1^\nu p_2^\mu\right)\varepsilon_{1\mu}\varepsilon_{\nu 2}\left(g^{\rho\sigma}p_1p_2 - p_1^\sigma p_2^\rho\right)\varepsilon^*_{1\rho}\varepsilon^*_{2\sigma}$$

$= $ summing over the polarizations $= \dfrac{\alpha_s^2}{9\pi^2v^2}(N_c^2-1)\,\dfrac{1}{2}m_H^4$

$$\sigma = \frac{1}{2s}\overline{|M_0|^2}\,\cancel{(2\pi)^4\,\delta^4(p_1+p_2-p_H)}\,\frac{\cancel{d^4p_H}}{\cancel{(2\pi)^4}}\,2\pi\,\delta(s-m_H^2)$$

$$= \frac{1}{2s}\,\frac{|M_0|^2}{(N_c^2-1)^2}\,2\pi\,\delta(s-m_H^2) = \frac{\alpha_s^2}{\pi}\,\frac{m_H^2}{576v^2}\,\delta(s-m_H^2)$$

and the result coincides with what obtained in the heavy-top limit

We now want to compute QCD radiative corrections to this process. These corrections were first evaluated exactly by Djouadi, Graudenz, Spira and Zerwas already in 1995.

We will consider the calculation in the effective field theory approach.

One has to consider REAL CORRECTIONS

and VIRTUAL CORRECTIONS

These corrections are affected by different kinds of singularities: UV singularities are dealt with through the renormalization procedure, and are reabsorbed into the redefinition of the QCD coupling α_S.

IR singularities affect both virtual and real corrections. They are due to the emission of SOFT and/or COLLINEAR particles. Separately, real and virtual contributions are IR divergent, and it is only in the sum that the singularities cancel out. More precisely, it is soft and final state collinear singularities that cancel. Initial state collinear singularities do not cancel and they must be reabsorbed into the PDFs. Dealing with IR and UV divergences requires a REGULARIZATION: we use dimensional regularization, by working in $D=4-2\varepsilon$ dimensions, and in particular, in the CDR scheme, in which there are 2 independent polarizations for massless quarks and $2(1-\varepsilon)$ for gluons. The subtraction of both UV and collinear poles is done in the $\overline{MS}$ scheme, which works as follows.

The poles appear as singularities in $\frac{1}{\varepsilon}$ that, in practice, come in the form

$$\Gamma(1+\varepsilon) \simeq 1-\gamma_E\varepsilon$$

$$\frac{1}{\varepsilon}(4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon) \simeq \frac{1}{\varepsilon} - \gamma_E + \log(4\pi)$$

$\Rightarrow$ WE SIMPLY SUBTRACT THESE ADDITIONAL CONSTANTS TOGETHER WITH THE $1/\varepsilon$ POLES