$SU(2)_L \otimes U(1)_Y$: A BRIEF RECAP

- the gauge principle

$$\mathcal{L} = \bar{\psi}(i\not\partial - m)\psi$$

the Dirac Lagrangian is invariant under GLOBAL phase transformations $\psi \to \psi e^{i\theta}$: $U(1)$ symmetry

BUT : global symmetries are not fundamental symmetries of nature:
two observers separated by a SPACELIKE distance are forced to change the phase of the field in the same way $\to$ Violation of causality

When we promote the phase transformation to local the Dirac Lagrangian is not any more invariant : we can make it local by coupling it with a $U(1)$ gauge field.

$$\mathcal{L} \to \bar{\psi}(i\not D - m)\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$D_\mu = \partial_\mu - ieA_\mu$$

In this way the Lagrangian is GAUGE INVARIANT under local $U(1)$ transformations $\psi \to \psi e^{ie f(x)}$ $\quad A_\mu \to A_\mu + \partial_\mu f$
since the effect of the derivative term is compensated by the transformation of A_μ

- $SU(2)_L \otimes U(1)_Y$ symmetry

We consider the gauge sector of the SM with one family

$$E_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$\mathcal{L} = \bar{E}_L i\not D E_L + \bar{e}_R i\not D e_R + \bar{Q}_L i\not D Q_L + \bar{u}_R i\not D u_R + \bar{d}_R i\not D d_R$$

4 generators
$\to$ 4 gauge fields

g	W_i^μ	$SU(2)_L$
g'	B^μ	$U(1)_Y$

$\uparrow$ 2 couplings

weak hypercharge

$$D_\mu = \partial_\mu + ig\frac{\tau_i}{2}W_\mu^i + ig'\frac{Y}{2}B_\mu$$

$\uparrow$ present only for doublets

How do we fix Y ?

We do it by defining the electric charge as $\quad Q = T_3 + \frac{Y}{2}$

$T_3 = \pm \frac{1}{2}$ for doublets

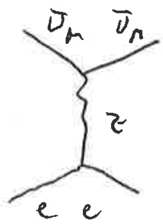
$= 0$ for singlets

for example $Q(e) = -1 \Rightarrow Y = -1$ for the lepton doublet

$Y(e_R) = -2$

COMMENTS

1) $SU(2)_L \otimes U(1)_Y$ theory predicts the existence of neutral current interactions (mediated by the Z boson). They were indeed discovered in 1973 in the GARGAMELLE bubble chamber $\quad \bar{\nu}_\mu e \to \bar{\nu}_\mu e$



2) Charged current interactions are the only known interactions able to change flavor $\quad e^- \to \nu_e$, $u \to d$, etc

$\Rightarrow$ no flavor changing neutral currents at tree level

(also at one loop we have strong cancellations)

3) Contrary to QED and QCD, $SU(2)_L \otimes U(1)_Y$ theory is a chiral theory

$\Rightarrow$ left handed and right handed fields couple differently to the gauge group

4) Right-handed neutrinos are completely decoupled: since $Q = T_3 + \frac{Y}{2}$ and they are neutral, they are singlets both for $SU(2)_L$ and $U(1)_Y$

HIGGS SECTOR

The naive introduction of a vector boson mass term in the lagrangian breaks gauge invariance

$$\mathcal{L}_M = -\frac{M^2}{2} A_\mu A^\mu$$

is not invariant for $A_\mu \to A_\mu - \partial_\mu \alpha$

Moreover, the EW theory is a CHIRAL THEORY : left handed and right handed fields transform differently under the gauge group.

Let us consider a naive mass term for a fermion

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

Since left and right handed fields transform differently this cannot be invariant

$\Rightarrow$ not only mass terms for the vector bosons are forbidden by gauge invariance, but also for the fermions

The solution is provided by SPONTANEOUS SYMMETRY BREAKING (SSB)

In short : the lagrangian is still gauge invariant but the symmetry is broken by vacuum

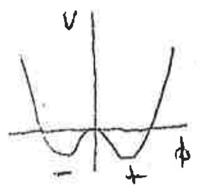
Quantum implementation of symmetries (Coleman theorem)

Q — $Q|0\rangle = 0$ WIGNER REALIZATION : states are constructed according to the irreducible representations of the symmetry group

Q — $Q|0\rangle \neq 0$ GOLDSTONE REALIZATION : the symmetry is broken by the vacuum and thus it is not realized in the particle spectrum

THE IMPLEMENTATION OF A SYMMETRY DEPENDS ON THE BEHAVIOR OF THE VACUUM

SSB is typical of systems of infinite volume



double well potential $\phi \to -\phi$ symmetry

$$\langle + | H | + \rangle = \langle - | H | - \rangle = a$$

$$\langle + | H | - \rangle = \langle - | H | + \rangle = b$$

vacua are $\dfrac{|+\rangle \pm |-\rangle}{\sqrt{2}}$ symmetric up to sign

with eigenvalues $E_{1,2} = a \pm b$

$\Rightarrow$ NO SSB !

$b \sim e^{-cV}$ tunneling probability is exponentially suppressed when $V \to \infty$

Goldstone theorem

Every time a continuous symmetry is spontaneously broken a massless scalar particle appears

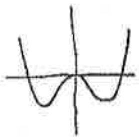
$\Rightarrow$ one for each broken generator

example: $\mathcal{L} = \partial_\mu \phi^+ \partial^\mu \phi - V(\phi^+\phi)$ $\qquad V(\phi^+\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ $\qquad$ U(1) symmetry

ϕ complex scalar field

U(1) invariance

$\lambda > 0$ potential bounded from below



$\mu > 0 \Rightarrow$ perturbation theory can be developed as usual around $\phi = 0$

$\hookrightarrow \dfrac{\partial V}{\partial |\phi|^2} = 0 \quad \Rightarrow \mu^2 + 2\lambda|\phi|^2 = 0$

$$|\phi|^2 = -\frac{\mu^2}{2\lambda}$$

The potential is constant for $\phi_0 = \sqrt{\dfrac{-\mu^2}{2\lambda}}\, e^{i\theta} \equiv \dfrac{v}{\sqrt{2}} e^{i\theta}$

$$v^2 = \frac{-\mu^2}{\lambda}$$

Define $\phi = \dfrac{v + \sigma + i\eta}{\sqrt{2}}$ $\qquad$ σ describes "radial" fluctuations around the minimum

η rotations along the minimum

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma - i\partial_\mu\eta)(\partial^\mu\sigma + i\partial^\mu\eta) - \frac{1}{2}\mu^2\left((v+\sigma)^2 + \eta^2\right) - \frac{\lambda}{4}\left((v+\sigma)^2 + \eta^2\right)^2$$

$$= \frac{1}{2}(\partial_\mu\sigma)^2 - \lambda\sigma^2 v^2$$

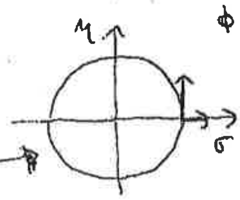
$$+ \frac{1}{2}(\partial_\mu\eta)^2 \qquad - \lambda v\sigma(\eta^2 + \sigma^2) - \frac{\lambda}{4}(\eta^2 + \sigma^2)^2 + \text{const}$$

NOTE: linear terms in σ and quadratic terms in η vanish

$\Rightarrow$ scalar field σ with mass $2\lambda v^2$

massless particle $\eta \Rightarrow$ <u>GOLDSTONE BOSON</u>

ϕ complex plane



minimum of the potential

σ "feels" the potential

η describes fluctuations along the minimum

Higgs mechanism

Let us "promote" the U(1) symmetry to local

$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi$$

$$\phi = \frac{1}{\sqrt{2}}(v+\sigma+i\eta)$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\lambda v^2)\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(ev)^2A^2$$

mass term for the gauge field! *

$$+ \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) \quad - evA^\mu\partial_\mu\eta$$

mixing term A-η

how to deal with this? (analogous term in σ cancels out with c.c.)

Let us count the degrees of freedom:

At the beginning we have $2 + 2$ dof (2 for ϕ and 2 for massless gauge field)

At the end $2 + 3$?

massive gauge field

η can be eliminated by using gauge invariance!

$$\begin{cases} \phi \to \phi\, e^{ie f(x)} \\ \phi^\dagger \to \phi^\dagger e^{-ie f(x)} \\ A_\mu \to A_\mu + \partial_\mu f \end{cases}$$

we can find a gauge transformation such that

$$\phi = \frac{1}{\sqrt{2}}(v+\sigma) \qquad \text{UNITARY GAUGE}$$

it displays only the physical fields of the theory

Sometimes we say that the A field has eaten up the η field and become heavy!

Higgs mechanism in the $SU(2) \otimes U(1)$ theory

Let us introduce a complex scalar doublet ϕ

$$D_\mu \phi = \left(\partial_\mu + ig \frac{\tau_i}{2} W_{i\mu} + ig' \frac{Y}{2} B_\mu\right)\phi \qquad \Rightarrow \text{ we will fix } Y \text{ later}$$

$$\mathcal{L}_{SB} = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi^\dagger \phi) \qquad V(\phi^\dagger\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$\mu^2 < 0 \Rightarrow |\phi_0|^2 = -\frac{\mu^2}{2\lambda}$ choose vacuum configuration $\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ $\qquad v = \sqrt{\frac{-\mu^2}{\lambda}}$

Y is assigned imposing that the charge generator Q annihilates the vacuum

$$Q\phi_0 = 0 \qquad \left[\begin{pmatrix} 1/2 & \\ & -1/2 \end{pmatrix} + Y \begin{pmatrix} \frac12 & \\ & \frac12 \end{pmatrix}\right]\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0 \quad \Rightarrow \quad \boxed{Y = 1}$$

The vacuum must be electrically neutral ($U(1)$ not spontaneously broken)

In general $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\eta_2 \\ v + \sigma + i\eta_3 \end{pmatrix}$ in unitary gauge $\eta_i = 0$ $\qquad \{\tau_i, \tau_j\} = 2\delta_{ij}$

$\tau_i^2 = 1$

Let us analyze the mass terms for W and Z

$$\mathcal{L}_m = \frac12 (0 \;\; v)\left(g \frac{\tau_i}{2} W_i^\mu + g' \frac12 B^\mu\right)\left(g \frac{\tau_j}{2} W_{j\mu} + \frac12 g' B_\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac12 (0 \;\; v)\left(g^2 \frac14 (W_1^2 + W_2^2 + W_3^2) + \frac14 g'^2 B^2\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$+ \frac12 (0 \;\; v)\left(\frac14 g^2 \left(W_1 W_2 \cancel{(\tau_1\tau_2 + \tau_2\tau_1)}\right) + \dots\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$+ \frac12 (0 \;\; v)\left(2 \frac{gg'}{4} W_3 B \, \tau_3\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac12 v^2 \left[\frac{g^2}{4}(W_1^2 + W_2^2) + \frac14 (g W_3 - g' B)^2\right] = \frac12 v^2 \left[\frac{g^2}{4}(W W^+ + h.c.) + \frac14 \frac{g^2}{\cos^2\theta} Z^2\right]$$

$$\boxed{M_W^2 = g^2 \frac{v^2}{4}} \qquad \boxed{M_Z^2 = \frac{g^2}{\cos^2\theta}\frac{v^2}{4} = \frac{v^2}{4}(g^2 + g'^2)}$$

$$\begin{cases} W_3 = \cos\theta \, Z + \sin\theta \, A \\ B = \cos\theta \, A - \sin\theta \, Z \end{cases}$$

$$\Downarrow$$

$$\begin{cases} Z = \cos\theta \, W_3 - \sin\theta \, B \\ A = \sin\theta \, W_3 + \cos\theta \, B \end{cases}$$

$$\boxed{M_Z^2 = \frac{M_W^2}{\cos^2\theta}} \qquad \boxed{\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta} = 1}$$

$\hookrightarrow$ it is a consequence of the fact that we have chosen a Higgs doublet

for triplets $\rho \neq 1$!

Higgs couplings to W, Z, γ

from the structure of the mass terms $\frac{1}{2}(0\ v)\left(g\frac{\tau_i}{2}W_i^\mu + g'\frac{1}{2}B^\mu\right)\left(g\frac{\tau_j}{2}W_{j\mu} + g'\frac{1}{2}B_\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}$

we can also infer the couplings to W, Z, γ

NO σγγ VERTEX!

Fermion masses

Fermion mass terms are forbidden by gauge invariance ⇒ they can be obtained through Yukawa couplings to ϕ

Example: electron mass term

$$\mathcal{L}_e = -\lambda_e \underbrace{\bar{E}_L}_{Y=1}\ \underbrace{\phi}_{Y=1}\ \underbrace{e_R}_{Y=-2} + h.c. \qquad e_R \longrightarrow \text{singlet under } SU(2)_L$$

⇒ $Y_{TOT} = 0$ as it should be!

Note that ϕ can only give mass to down type fermions

⇒ introduce $\tilde{\phi} = i\tau_2\phi^*$ $\qquad Y(\tilde{\phi}) = -1$

$$Q = T_3 + \frac{Y}{2}$$

$$Y(Q_L) = \frac{1}{3}$$

$$Y(u_R) = \frac{4}{3}$$

$$\mathcal{L}_u = -\lambda_u \underbrace{\bar{Q}_L}_{-\frac{1}{3}}\ \underbrace{\tilde{\phi}}_{-1}\ \underbrace{u_R}_{\frac{4}{3}} + h.c.$$

In this way (MINIMAL SOLUTION) we can give mass to all fermions with one Higgs doublet

In principle we could introduce one Higgs doublet for down and one for up fermions

(or even one for up quarks, one for down quarks, one for charged leptons)

SUMMARY : EW LAGRANGIAN

$$\mathcal{L} = \bar{E}_L i \not{D} E_L + \bar{e}_R i \not{D} e_R$$

$$+ \bar{Q}_L i \not{D} Q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R$$ ← fermion interactions with gauge fields

$$-\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$ ← gauge part

$$+ (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$ ← symmetry breaking part

$$- \lambda_e \bar{E}_L \Phi e_R + h.c. \quad - \lambda_u \bar{Q}_L \tilde{\Phi} u_R + h.c. \quad - \lambda_d \bar{Q}_L \Phi d_R + h.c.$$

↑ Yukawa interaction

$$E_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L \qquad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

In this way we have constructed the EW lagrangian for a single family of quarks and leptons

Note that right-handed neutrinos are SINGLET under $SU(2)_L \otimes U(1)_Y$

Experimentally we know that three families exist

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$ ⇒ WHAT ABOUT FLAVOUR MIXING ?

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

Generally speaking gauge and mass eigenstates can be different from each other

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