Homework 1

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Exercise 1. Lagrangian point L1. In celestial mechanics, the Lagrangian points are positions in an orbital configuration of two large bodies where a small object affected only by gravity can maintain its position relative to the two large bodies. The Lagrange points mark positions where the combined gravitational pull of the two large masses provides precisely the centripetal force required to orbit with them. There are five such points (see figure 1), labeled L1 to L5, all in the orbital plane of the two large bodies.

Consider the two body system formed by the Earth and the Sun. A satellite is launched from Earth, and reach L1 (between the Earth and the Sun) in a stable orbit.
a) What is the distance between the Earth and L1 ?
b) What is the speed (scalar quantity) of the satellite (in the Sun reference system) ?
c) What is the ratio between the Earth angular velocity and the satellite angular velocity (in the Sun reference system) ?
d) What is the ratio between the Earth areal velocity and the satellite areal velocity (in the Sun reference system) ?
e) What is the minimum energy necessary to reach L1 ?

Consider all trajectories to be circular. Ignore effect like the gravitational pull of the Moon and of Jupiter. Write all the solution as function of as a function of M_{Earth}, M_{Sun}, $M_{Satellite}$, the Earth-Sun distance r and the L1 distance from Earth R.
Hint: L1 is *always* between the Earth and the Sun
Hint: Because the mass of the Earth is about 3 $\times 10^{-6}$ solar masses, L1 is very close to the Earth compared to the the Earth-Sun distance d_{AU} . In your calculation may be helpful to use the following approximation:

$$\frac{1}{(1-x)^3} \sim 1+3x \quad \frac{1}{(1-x)} \sim 1+x \quad for \quad x \ll 1 \tag{1}$$

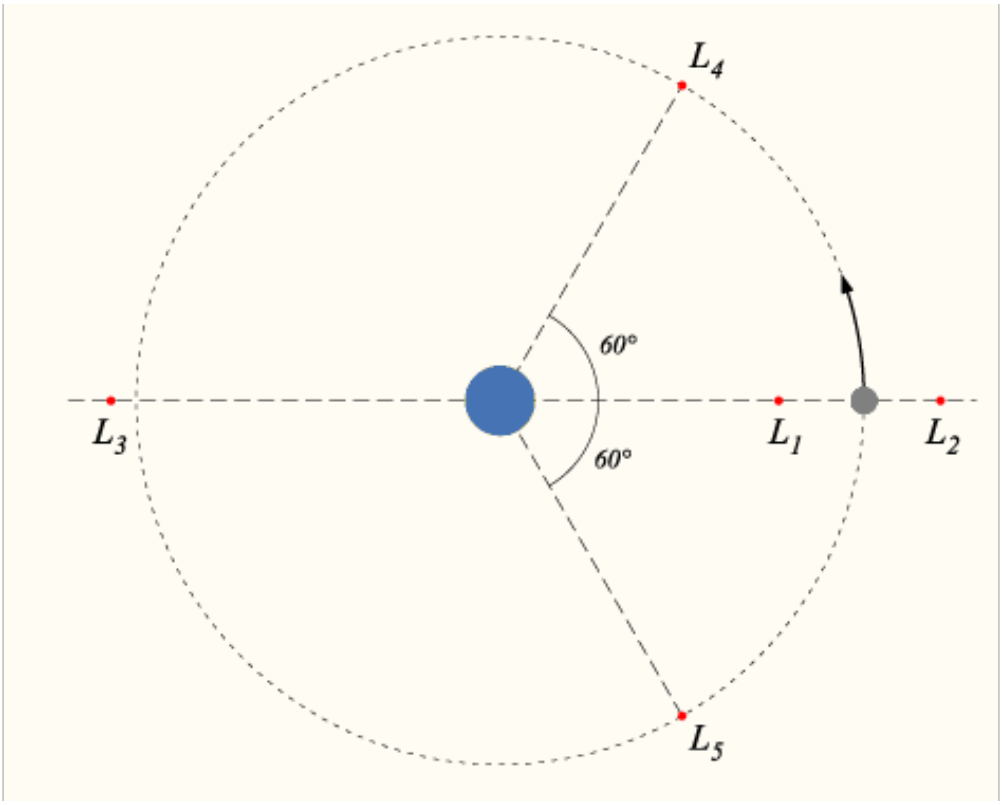

Figure 1: The five Lagrangian points around a two body system