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# Muon physics at a neutrino factory*

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**Abstract**

A neutrino factory could deliver beams of low-energy muons thousand times more intense than available today. In this paper I will review the field of muon physics and discuss what experiments could benefit most from such high-intensity beams. Emphasis will be given on experiments in particle physics, i.e. the determination of the muon anomalous magnetic moment and searches for charged lepton flavour violation.

## 1. Introduction

Working group WG4, *Non-neutrino science at a neutrino factory* at the recent NuFact02 workshop [1] was devoted solely to the physics with stopped muon beams. Table 1 lists the various topics that were discussed in the areas normal $\mu$ decay, $\mu$ decays violating lepton flavour conservation, bound systems and $\mu$sr. Rather than trying to present a balanced review of the field, I will focus on topics that were not addressed (such as $g-2$) and on my own contributions in the field of forbidden decays. For a much more complete recent review see [2].

## 2. Muon $g-2$

The magnetic moment of a fundamental fermion relates to its spin through

$$\vec{\mu}_s = g\frac{e}{2m}\vec{S}. \tag{1}$$

In the Dirac theory $g = 2$. Higher order corrections result in

$$\vec{\mu}_s = (1+a)\frac{e}{m}\vec{S}, \tag{2}$$

with $a = (g-2)/2$ the so-called anomalous magnetic moment which accounts for various contributions within and possibly beyond the standard model:

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**Table 1.** Physics with low-energy muon beams. For projects discussed in WG4 the speakers are listed. Also shown are the improvement factors that may be expected at a future neutrino factory [2].

| | | | Gain | |
|---|---|---|---|---|
| | Lab/project | WG4 speaker | Soon | $\nu$-Factory |
| Fundamental properties: | | | | |
| $g-2$ | BNL/E821 | | 20 | 200 |
| e.d.m. | BNL/E821 loi | | $10^6$ | $10^7$ |
| Normal decay: | | | | |
| Muon lifetime | RAL | Tomono | 2 | 200 |
| | PSI | Malgeri | 20 | 200 |
| Michel parameters | TRIUMF/614 | Poutissou | 30 | |
| $e^+$ polarization | PSI | Fetscher | | |
| Lepton flavour violation: | | | | |
| Theoretical speculation | | Babu, Koike, Sato, Shimizu, Shimoyama | | |
| $\mu \to e\gamma$ | PSI/MEG | Signorelli | 500 | $10^4$ |
| $\mu^+ \to e^+e^+e^-$ | | | | $10^4$ |
| $\mu^+e^- \leftrightarrow \mu^-e^+$ | | | | $10^4$ |
| $\mu e$ conversion | PSI/SINDRUM II | van der Schaaf | 10–100 | |
| | BNL/MECO | Aoki | $5 \times 10^4$ | |
| | PRISM/PRIME | Kuno, Sato | | $10^6$ |
| Bound systems: | | | | |
| $\mu^+e^-$ (QED tests) | | | | |
| $\mu^- p$, $\mu^- A^*$ | | | | |
| ($g_p$, nuclear radii, moments) | | | | |
| $\mu^-$ catalyzed fusion | | Ishida | | |
| (Surface) $\mu$sr: | | | | |
| Slow muons | PSI | | | |
| | KEK | Matsuda | | |

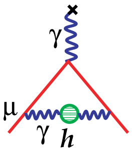

**Figure 1.** Lowest order hadronic contribution to the muon magnetic moment which is caused by hadronic vacuum polarization in the photon propagator of the lowest order electromagnetic correction (Schwinger term).

$$a = a^{\mathrm{QED}} + a^{\mathrm{hadronic}} + a^{\mathrm{electroweak}} + a^{\mathrm{non\text{-}SM}} \tag{3}$$

$$a^{\mathrm{QED}} = \alpha/2\pi + \mathrm{h.c.} \tag{4}$$

With the exception of $a^{\mathrm{QED}}$ all terms scale with the fermion mass squared. Whereas the magnetic moment of $e^{\pm}$ can be described by QED alone (with a precision of 20 ppb) for muons $a^{\mathrm{hadronic}}$ and $a^{\mathrm{electroweak}}$ have to be included [3].

In the case of $a_\mu$, 98% of the non-electromagnetic contribution originates in the lowest order hadronic contribution $a_\mu(\mathrm{had;1})$, which is a correction to the Schwinger term (see figure 1). For this reason and since $a_\mu(\mathrm{had;1})$ gives the dominant contribution to the uncertainty

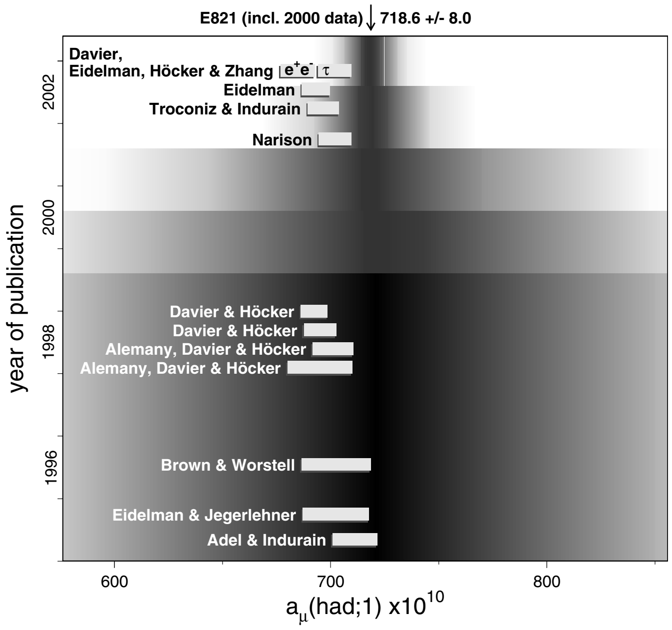

**Figure 2.** Calculated (bars) and measured (shaded area) values of $a_\mu$(had;1) the lowest order hadronic contribution to the muon anomalous moment as they became available in the past decade. Note the dramatic improvement in the experimental precision as achieved by BNL experiment E821.

in the prediction of $a_\mu$ it makes sense to express the results in terms of the lowest order hadronic term by subtracting all other standard model contributions from the measured value of $a_\mu$.

Figure 2 shows results for $a_\mu$(had;1) as they became available during the past decade. The present world average for the experimental value is completely dominated by the latest Brookhaven results [4]. Calculations of hadronic vacuum polarization rely heavily on experimental input, either from the cross sections of $e^+e^-$ annihilation into hadrons or from branching ratios of $\tau$ decay with hadronic final states. In a recent paper [5], Davier, Eidelman, Höcker and Zhang present a thorough analysis of new experimental data in both fields. Two results for $a_\mu$(had;1) are presented in the figure which differ by $2.1\sigma$. When excluding the $\tau$ data: $a_\mu(\text{had;1}) = 685(7) \times 10^{-10}$, i.e. 3.0 standard deviations below the measured value. Using the $\tau$ data: $a_\mu(\text{had;1}) = 702(6) \times 10^{-10}$, i.e. 1.6 standard deviations below the measured value. Although one might argue that the three pieces of experimental information disagree with each other and speculate about explanations outside the standard model, one should keep in mind the very complex procedures that are required in producing these results. See [5] for a discussion. In [2] we conclude that another order of magnitude (beyond the projected 0.4 ppm of E821) could be reached at a neutrino factory. It may be too early to decide how useful this would be.

## 3. Normal $\mu$ decay

### *3.1. $\mu$ lifetime*

Assuming a purely *V–A* structure of the weak interaction (see section 3.2) $\tau_\mu$ is directly related to the Fermi coupling constant:

$$\tau_\mu G_{\mathrm{F}}^2 m_\mu^5 = 192\pi^3. \qquad (5)$$

Presently three experiments (one at RAL, the other two at PSI; see contributions to WG4 [1]) are in progress which will improve our knowledge of $\tau_\mu$ by more than an order of magnitude. Still better measurements could be done at a neutrino factory. Since $G_\mathrm{F}$ enters the standard model calculations in the combination $G_F m_W^2$ further improvement seems not that interesting, however, unless we improve our knowledge of $m_W$ or $m_Z$ as well.

### 3.2. *V–A tests*

The decay $\mu^+ \to e^+\nu_e\overline{\nu}_\mu$ of polarized muons at rest can be described in terms of the Michel parameters:

$$\Gamma \propto x^2 \left(3 - 3x + \tfrac{2}{3}\rho(4x-3)\right) + P_\mu \xi z \left(1 - x + \tfrac{2}{3}\delta(4x-3)\right) \tag{6}$$

where $x \equiv 2p_e/m_\mu c$ and $z \equiv \cos\theta$ with $\theta$ the angle between the $\mu$ spin and the $e^+$ direction. Experiment gives $\rho = 0.7519(26)$, $P_\mu\xi = 1.0027(85)$, $\delta = 0.7486(38)$ in good agreement with the $V{-}A$ values $\rho = 3/4$, $P_\mu\xi = 1$ and $\delta = 3/4$.

The ongoing TRIUMF experiment 614 (TWIST) will reduce the errors in all three parameters by 1–2 orders of magnitude in the next few years. The experiment is not limited by beam intensity but rather by beam quality. Of crucial importance is an almost 100% beam polarization which is achieved by selecting muons from $\pi^+$ decay in a 25 $\mu$m skin of the production target. For this reason these measurements will not be on top of the list at a neutrino factory.

## 4. Rare $\mu$ decays

$\nu$-Oscillations directly lead to finite rates for rare muon decays. Second-order charged weak interactions result, however, in negligible contributions to the branching ratios since they are strongly suppressed dynamically:

$$B(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \sum_i \left| V_{\mu i}^* V_{ei} \frac{m_{\nu_i}^2}{M_W^2} \right|^2 . \tag{7}$$

Note that the corresponding mechanism in the quark sector leads to $b \to s\gamma$ with a branching ratio of $O(10^{-4})$ due to the large top mass.

The observation of charged lepton number violation would thus be an unambiguous sign of new physics and indeed, a number of standard model extensions are probed sensitively.

Table 2 lists the present limits on charged lepton flavour violation. Best constraints come from the forbidden $\mu$ and $K$ decays where dedicated experiments have been performed. One should keep in mind, however, that couplings to the third generation could be enhanced in which case the $\tau$ limits start to become interesting as well.

### 4.1. *$\mu e$ conversion*

Neutrino-less $\mu e$ conversion in muonic atoms offers some of the most sensitive tests of lepton flavour conservation. For conversions leaving the nucleus in its ground state the nucleons act coherently, which boosts the conversion probability relative to the rate of nuclear muon capture which is the dominant competing process except for light nuclei. For the same reason transitions to the ground state are enhanced relative to other final states which are expected to occur with a probability below 10% for all nuclear systems. Experiments have been performed on a variety of nuclei (see table 2). Many authors have studied the nuclear physics aspects of the process, unfortunately with conflicting results [24, 25].

**Table 2.** Upper limits on branching ratios of particle decay modes that do not conserve lepton flavour.

| Decay | Upper limit | Exp./Lab. | Decay | Upper limit | Exp./Lab. |
|---|---|---|---|---|---|
| $\mu^+ \to e^+\gamma$ | $1.2 \times 10^{-11}$ [6] | MEGA | $\tau \to 2\mu e$ | $1.8 \times 10^{-6}$ [17] | |
| $\mu^+ \to e^+e^+e^-$ | $1.0 \times 10^{-12}$ [7] | SINDRUM | $\tau \to \mu 2e$ | $1.5 \times 10^{-6}$ [17] | |
| $\mu^+e^- \leftrightarrow \mu^-e^+$ | $8.3 \times 10^{-11}$ [8] | SINDRUM | $\tau \to 3e$ | $2.9 \times 10^{-6}$ [17] | |
| $\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$ | $6.1 \times 10^{-13}$ [9] | SINDRUM | $K^+ \to \pi^+\mu e$ | $2.8 \times 10^{-11}$ [18] | BNL |
| $\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}^*$ | $3.6 \times 10^{-11}$ [10] | SINDRUM | $K_L^o \to \mu e$ | $4.7 \times 10^{-12}$ [19] | BNL |
| $\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$ | $4.6 \times 10^{-11}$ [11] | SINDRUM | $K_L^o \to \pi^0\mu e$ | $4.4 \times 10^{-10}$ [20] | Fermi |
| $\mu^-\mathrm{Au} \to e^-\mathrm{Au}$ | $1.9 \times 10^{-11}$ [12] | SINDRUM | $B^o \to \mu e$ | $1.2 \times 10^{-7}$ [21] | BaBar |
| $\tau \to e\gamma$ | $2.7 \times 10^{-6}$ [13] | CLEO | $B^o \to \tau e$ | $5.3 \times 10^{-4}$ [22] | CLEO |
| $\tau \to \mu\gamma$ | $1.1 \times 10^{-6}$ [14] | CLEO | $B^o \to \tau\mu$ | $8.3 \times 10^{-4}$ [22] | |
| | $1.0 \times 10^{-6}$ [15] | Belle | $Z^0 \to e\mu$ | $1.7 \times 10^{-6}$ [23] | OPAL |
| | $2.0 \times 10^{-6}$ [16] | BaBar | $Z^0 \to e\tau$ | $9.8 \times 10^{-6}$ [23] | |
| $\tau \to 3\mu$ | $1.9 \times 10^{-6}$ [17] | CLEO | $Z^0 \to \mu\tau$ | $1.2 \times 10^{-5}$ [23] | |

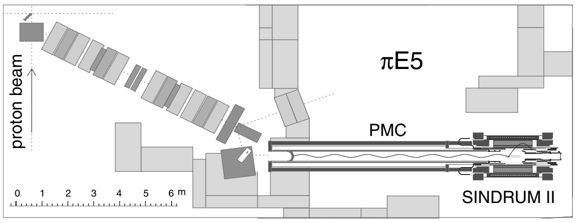

**Figure 3.** The SINDRUM II spectrometer at the $\pi$E5 beam line at PSI. $\pi^-$ and $\mu^-$ leaving the production target with momenta around 53 MeV/$c$ are focused on a degrader at the entrance of a long superconducting solenoid. Whereas most $\mu^-$ reach the gold target practically all $\pi^-$ stop in the moderator.

Experimentally coherent $\mu e$ conversion offers many advantages over $\mu \to e\gamma$. The electron is emitted at the kinematic endpoint of muon decay in an orbit which constitutes the only intrinsic background. Since the decay rate drops steeply above $m_\mu c^2/2$ the set-up may have a large geometrical acceptance and still the detectors can be protected against the vast majority of decay and capture events. This background scales with the energy resolution to the 5th power and a resolution around 1% is sufficient to keep it below $10^{-16}$.

Other potential background involves either beam particles ($\mu^-$, $\pi^-$, $e^-$) or cosmic rays. Capture gammas from $\mu^-$ and $\pi^-$ produce electrons mostly through $e^+e^-$ pair production inside the target. Beam-related background can be suppressed by beam pulsing, a beam veto counter or beam purity.

#### *4.1.1.* SINDRUM II.

In the year 2000 data were taken on gold. During an effective measuring period of 75 days $4.3 \times 10^{13} \mu^-$ stopped in the target (see figure 3).

Figure 4 illustrates the procedures of the event selection. Cosmic background is recognized by the occurrence of additional signals in the various detector elements. After selection of decay electrons originating in the target ten events remain with energies above 92 MeV. As is seen from the figure, eight of these events are forward peaked with a well-defined time correlation with the cyclotron rf signal. A similar distribution is observed for decay positrons. These events are explained by radiative $\pi^-$ capture in the moderator (see figure 3) followed by $\gamma \to e^+e^-$ pair production. The resulting electrons and positrons may

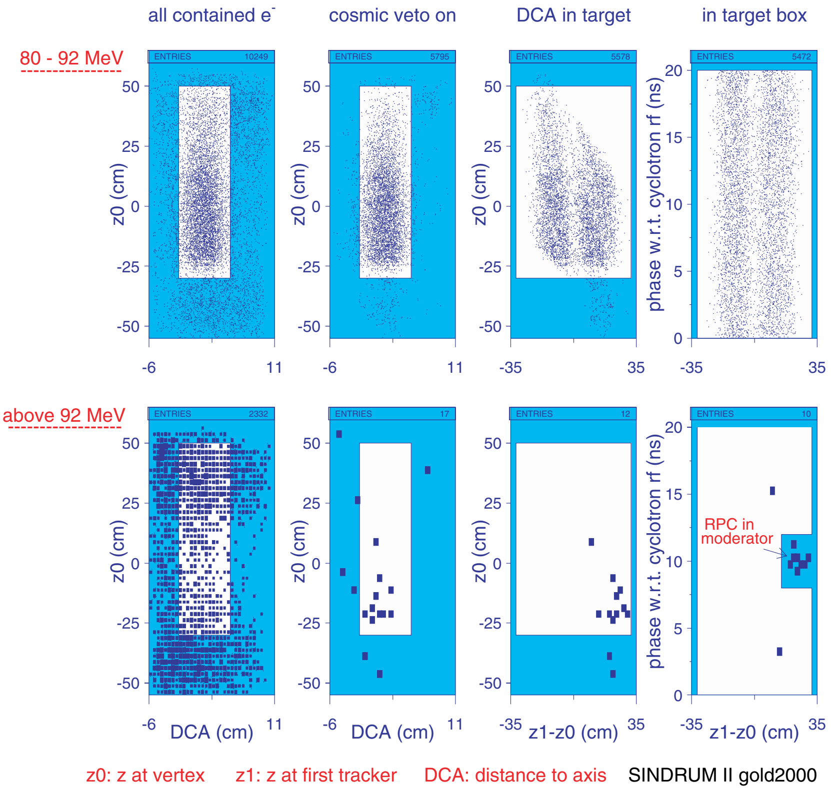

**Figure 4.** Distributions of z0 and DCA (track coordinates at origin), z1 (at tracker entrance) and phase with respect to 50 MHz cyclotron rf at various stages of the event selection for two regions of $e^-$ energy. 92 MeV is the effective endpoint energy for $\mu^-$ decay in muonic gold.

reach the target where they can scatter into the acceptance of the spectrometer. See [1] for a discussion of the remaining two events.

*4.1.2. Future $\mu e$ searches.* Two projects were discussed in WG4:

- The MECO experiment [26] (figure 5), BNL E940, aims at a single-event sensitivity of $2 \times 10^{-17}$ for conversion on aluminium.
- PRIME, a search at the proposed phase-space rotated PRISM beam at the JPS facility in Japan. This beam (see WG4 proceedings) would allow very thin targets and result in negligible pion contamination with no need to rely on beam pulsing.

### 4.2. $\mu \to e\gamma$

This decay mode gives the best constraint in many models. A new PSI experiment aims at an increase in sensitivity by 2–3 orders of magnitude (see WG4 programme). The main limitation in searches for $\mu \to e\gamma$ is the background of accidental $e\gamma$ coincidences and for this reason beam intensities have to be reduced now already. Of course the higher intensities available at a neutrino factory (assuming a DC subsurface $\mu^+$ beam could be produced) could be used to improve the beam quality and reduce the target thickness which presently limits the resolution in $e\gamma$ opening angle. See [2] for a discussion of the future prospects.

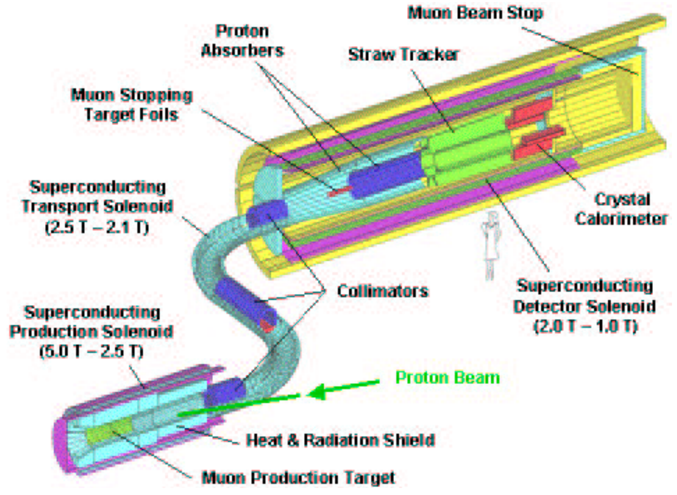

**Figure 5.** MECO setup. Pions are produced by 8 GeV$/c$ protons on a W target. Negatively charged particles of 60–120 MeV$/c$ are transported to the experimental target by a curved solenoid. Prompt background is removed by measuring in a time interval 0.7–1.3 $\mu$s after the proton bunch.

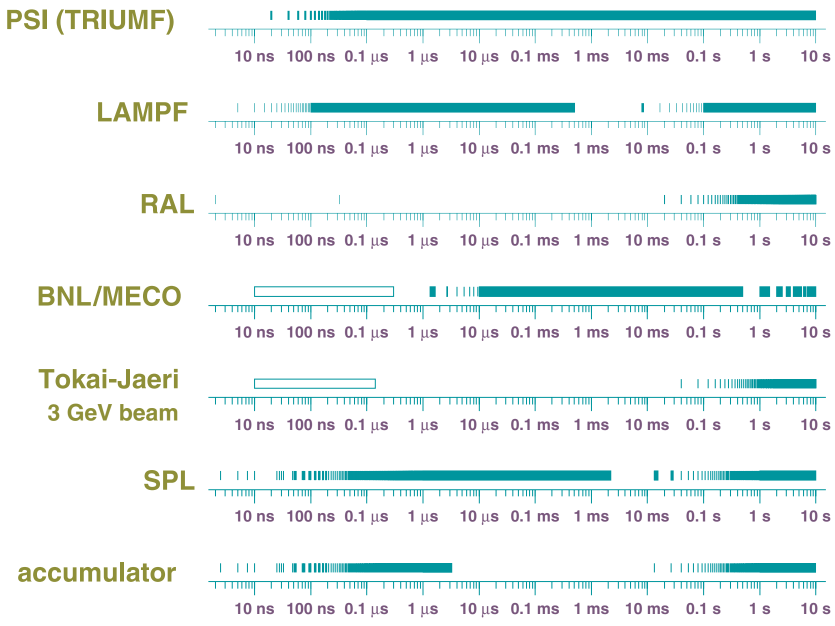

**Figure 6.** Time structures of various existing and planned proton accelerators used to produce muon beams. *SPL* and *Accumulator* are part of the CERN neutrino factory complex.

## 5. Beam requirements

The beam requirements may vary considerably for the various experiments. Whereas most experiments use stopped muons the $g-2$ experiment needs 3.1 GeV beams of both polarities. Required time structures may differ too. Searches for $\mu \to e\gamma$ need DC $\mu^+$ beams that can be stopped in targets of $<10$ mg cm$^{-2}$. Other experiments need pulsed beams with bunches that are narrow on the scale of $\tau_\mu$ and separation intervals of several $\tau_\mu$. Other parameters that may be crucial are beam contaminations ($e^\pm, \pi^\pm$) and beam extinction rate in between the bunches. Figure 6 shows examples of time structures of proton facilities used or planned to produce muon beams. Since neutrino beams may have, or even require, very low duty cycles it is not at all obvious that neutrino factories would deliver beams useful for stopped muon experiments. It seems time now to investigate in more detail how the requirements of the muon community could be met at the various options for a neutrino factory.

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