Digital Signal Processing for Germanium Detectors: Theory and Practice

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GERDA

Universität Zürich[UZH]

Contents

- Motivation: search for $0\nu\beta\beta$ decay with the GERDA experiment
- From a semiconductor to a semiconductor detector
- Noise sources and characteristics for a germanium detectors
- Analog and digital pulse shaping
- Energy resolution and its improvement
- Example(s): improvement of the charge integration
- Example: improvement of the energy resolution for GERDA Phase I

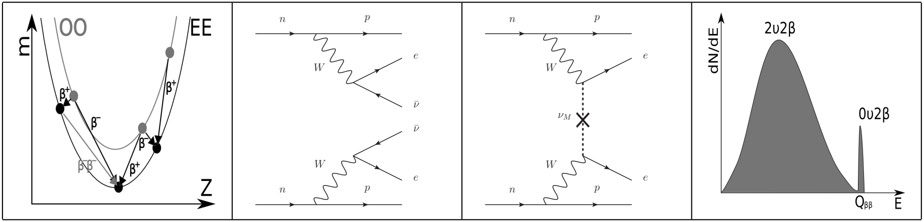
Motivation: Investigating the Neutrino nature

Open questions

- Is lepton number conservation violated?
- Is the neutrino a Majorana particle?
- What's the absolute neutrino mass scale?
- What's the neutrino mass hierarchy?

Possible answer: double beta decay

- Occurs in even-even isobars
- Measurable if single β decay energetically forbidden
- Rare process $\rightarrow$ ultra-low bkg required!



$2\nu\beta\beta$ decay

- Allowed in the SM, ΔL=0
- Signature: continuum from 0 to $Q_{\beta\beta}$
- Half life: $T^{2\nu}_{1/2} \sim (10^{18}\text{-}10^{24})$ yr
- $T^{2\nu}_{1/2}(^{76}\text{Ge}) = \left(1.926 \pm 0.095\right) \cdot 10^{21}$ yr
  ArXiV:1501.02345

$0\nu\beta\beta$ decay

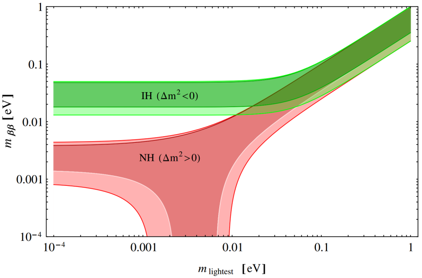
- Non-SM process, ΔL=2
- Possible only if neutrinos have Majorana mass component
- Signature: peak at $Q_{\beta\beta}$(^{76}Ge: 2039 keV)

The mass mechanism

- For light Majorana ν exchange:

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu}(Q,Z)\left|M^{0\nu}\right|^2 \langle m_{\beta\beta}\rangle^2$$

- $G^{0\nu}(Q,Z)$ = Phase Space integral
- $\left|M^{0\nu}\right|^2$ = nuclear matrix element
- $\langle m_{\beta\beta}\rangle^2 = \sum_i U_{ei}^2 m_i$ = effective ν mass
- U_{ei} = PMNS mixing matrix elements



Phys. Rev. D90 (2014) 033005

Experimental sensitivity:

- Number of signal events:

$$n_S = \frac{1}{T_{1/2}^{0\nu}} \cdot \frac{\ln 2 \cdot N_A}{m_A} \cdot f_{76} \cdot \varepsilon \cdot M \cdot t$$

- Number of background events:

$$n_B = BI \cdot \Delta E \cdot M \cdot t$$

where:
f = enrichment fraction
N_A = Avogadro number
m_A = atomic mass
ε = total efficiency
M = detector mass
t = live time
$M \cdot t$ = exposure
BI = Background Index
ΔE = Region Of Interest (ROI)

Why using germanium?

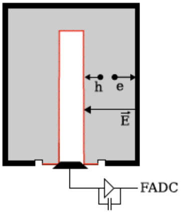
- High total efficiency: $\varepsilon \sim 0.75$
- Best energy resolution on the market: $\sim 1.5‰$ Full Width at Half Maximum (FWHM) at $Q_{\beta\beta}$
- Can be enriched to 86% in ^{76}Ge

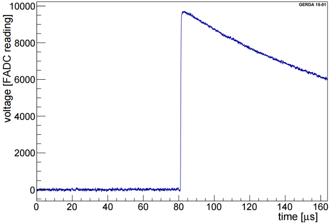
How to reduce the background?

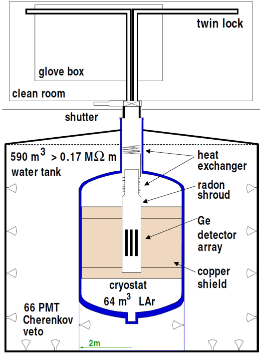
- Operate the experiment underground
- Use active veto for cosmic muons and external radiation
- Minimize radioactive contamination in the materials close to the detectors
- Current pulse is different for single site events (like $0\nu\beta\beta$ signal) versus multi-site events (like Compton scattered γ) or surface events
  $\rightarrow$ Pulse Shape Discrimination (PSD)

Ge detector readout

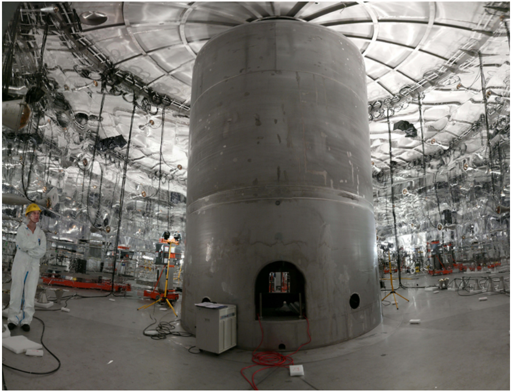
- Ge diode in reverse bias
  $\rightarrow$ measurement of ionization energy
- FADC allows offline analysis of recorded signals (energy, rise time, PSD parameters, ...)







- Located in Hall A at Laboratori Nazionali del Gran Sasso of INFN
- 3800 mwe overburden (μ flux $\sim 1\ \mathrm{m}^{-2}\mathrm{h}^{-1}$))
- Array of bare Ge detectors 86% enriched in ^{76}Ge directly inserted in liquid argon (LAr)



Why Liquid Argon + Water?

Material	^{208}Tl Activity [μBq/Kg]
Rock, concrete	3000000
Stainless steel	~ 5000
Cu (NOSV), Pb	< 20
Purified water	< 1
LN_2, LAr	~ 0

The two phases of GERDA

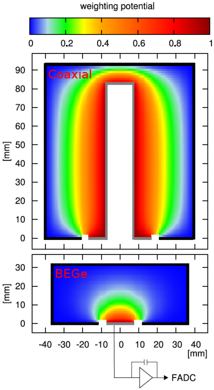
	Mass [kg]	Expected BI [counts/(keV·kg·yr)]	Live time [yr]	Expected $T^{0\nu}_{1/2}$ Sensitivity [yr]
Phase I	15	10^{-2}	1	$2.4 \cdot 10^{25}$
Phase II	35	10^{-3}	3	$1.4 \cdot 10^{26}$

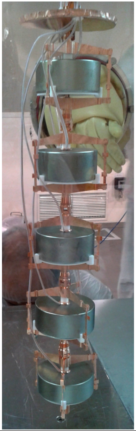
Coaxial detectors

- Inherited from HdM and IGEX experiments
- 2.4‰ FWHM at $Q_{\beta\beta}$ (1.7‰ reachable with better cables & improved signal shaping)
- Total enriched mass: 17.7 kg (analysis on 14.6 kg)

BEGe detectors (design for Phase II)

- BEGe = Broad Energy Germanium
- 1.6‰ FWHM at $Q_{\beta\beta}$ (1.2‰ reachable)
- Enhanced PSD
- $\sim$ 20 kg of BEGe's produced and tested in 2012
- 5 BEGe's inserted in GERDA in July 2012





Incoming particle

→ Photo-electric, Compton, pair-production

→ $e^-(e^+)$ in detector volume, E_{kin}

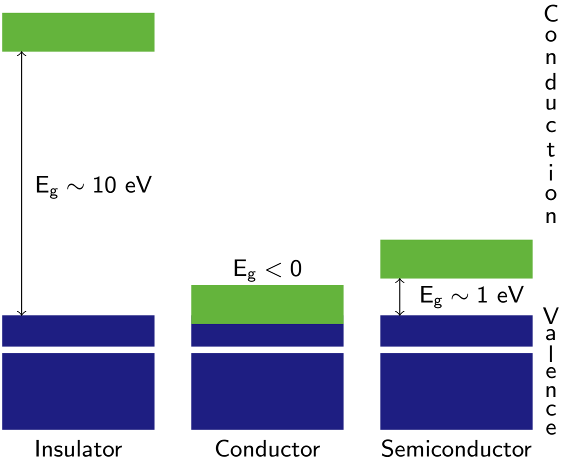
→ Ionization of other atoms in the detector

→ Charge collection

→ Pre-amplification + amplification

→ Electrical signal

→ Digitization



- Probability for an electron to jump to the conduction band (thermal excitation):

$$P(T) \propto T^{3/2} \exp\left(-\frac{E_g}{2kT}\right)$$

where:

$$E_g = \text{band gap}$$
$$k = \text{Boltzmann constant}$$
$$T = \text{temperature}$$

- Leakage current: background current induced by thermal motions of electrons into the conduction band
- Low temperature reduces the leakage current!

Suppose we have an electron jumping into the conduction band...

- We get a hole (positive charge) in the valence band
- If no external electric field is present, at some point the electron will fall down to the valence band: "charge recombination"
- If we put an external electric field, the electron (e) and the hole (h) migrate $\rightarrow$ need high enough electric field to avoid recombination!

A semiconductor detector is:

- a semiconductor with an electric field applied to collect the charge deposited by a particle

How many e-h pairs are produced in a particle-detector interaction?

- Let η be the average energy necessary for the creation of a e-h pair, then:

$$n = \frac{E_{absorbed}}{\eta}$$

- To improve energy resolution, we need to minimize η in order to maximize n

How do η and E_g depend on temperature?

- No theoretical models, only empirical parametrizations
- For Ge (F. E. Emergy and T. A. Rabson, Phys. Rev. 140 (1965) 2089-2093):

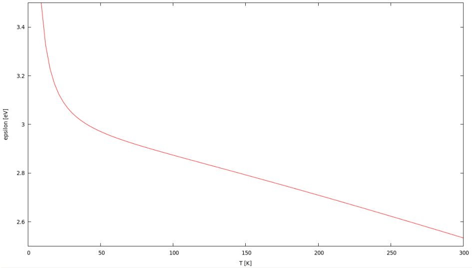
$$\eta(T) = 2.2 \cdot E_g(T) + 1.99 \cdot E_g^{3/2}(T) \cdot \exp\left(4.75 \frac{E_g(T)}{T}\right)$$

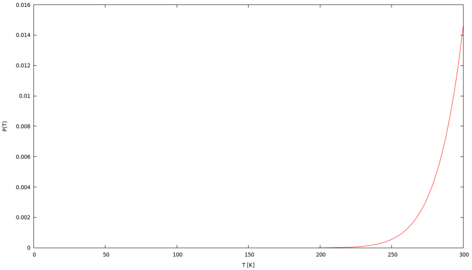
- For all semiconductors (Y. P. Varshni, Physica 34 (1967) 149-154):

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta}$$

- Typical values:

Material	$E_g(0)$ [eV]	α [eV/K]	β [K]
Si	1.1157	7.021	1108
Ge	0.7142	$4.561 \cdot 10^{-4}$	210





- Trade-off for germanium: operation at liquid nitrogen temperature (77 K)

	E_g [eV]	η [eV]
Si	1.106 (300 K)	3.62 (300 K)
Ge	0.67 (77 K)	2.96 (77K)

How fast are the charges collected?

- Drift velocity of electrons ad holes depends on the applied voltage:

	Mobility [$cm^2V^{-1}s^{-1}$]	
Material	electrons	holes
Si	1350	480
Ge	$3.6 \cdot 10^4$	$4.1 \cdot 10^4$

- Big detectors are possible with germanium!

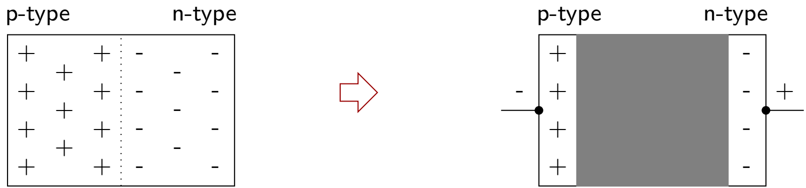
"We live in a real world. Ideal germanium crystals do NOT exist.*"

Possible impurities in the crystal lattice (Ge is 4-valent):

- Acceptors, e.g. boron with 3 valence electrons → p-type crystal
- Donors, e.g. 5-valent arsenic or 1-valent Lithium → n-type crystal

Doping makes you win!†

- Insert acceptors on one side and donors on the other → "compensated" germanium
- Apply a voltage to attract e and h to the opposite sides (reverse biased junction) → the central region is "depleted"



- Once a bias voltage is applied, the Ge detectors behaves as a capacitor!

* Old Indian saying of unkwnown origin.
† Old secret bequeathed among several generations of Tour de France winners

Given:

k = dielectric constant $\simeq 16.2$

ε_0 = space permittivity $= 8.85 \cdot 10^{-15}$ F/mm

h = height for coaxial detector $\rightarrow$ assume 80 mm

$r_1(r_2)$ = inner (outer) diameter for coaxial detector $\rightarrow$ assume 5 (40) mm

d = height for cylindrical planar detector $\rightarrow$ assume 35 mm

r = diameter for cylindrical planar detector $\rightarrow$ assume 35 mm

For a true-coaxial detector:

$$C_d = k\varepsilon_0 \frac{2\pi h}{\ln\left(\frac{r_2}{r_1}\right)} \sim 34 \text{ pF}$$

For a planar (cylindrical) detector:

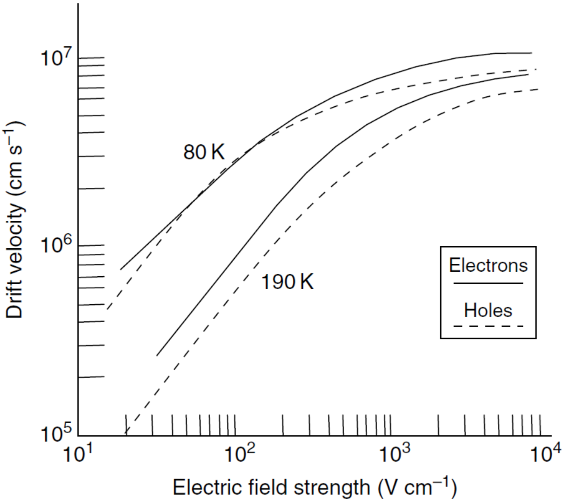
$$C_d = k\varepsilon_0 \frac{\pi r^2}{h} \sim 16 \text{ pF}$$

Why do we care about the detector capacitance?

- Wait a few slides and you'll see!

What is the charge collection time, given a bias voltage V_b?

- For both e and h we can define the mobility as: $\mu = \frac{v_d}{E}$
  where: v_d = drift velocity
  E = electric field



Example (planar detector):

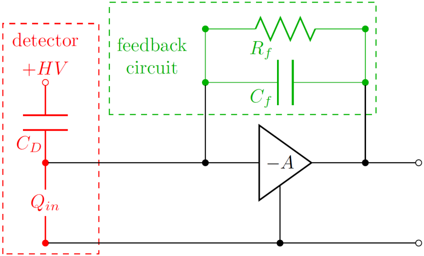
- $V_b = 4$ kV
- $d = 4$ cm
- $E = 1000$ V/cm
- From the plot:
  $v_d \sim 7 \cdot 10^6$ cm/s
- Suppose the charge (e/h) travels 3 cm:

$$t_c = \frac{3 \text{ cm}}{v_d} \sim 0.5\ \mu\text{s}$$

- Collected charge $\propto$ deposited energy
- Goal of readout electronics: transfer the collected charge to the ADC (MCA/FADC) with the smallest possible alteration

Solution: charge-sensitive preamplifier

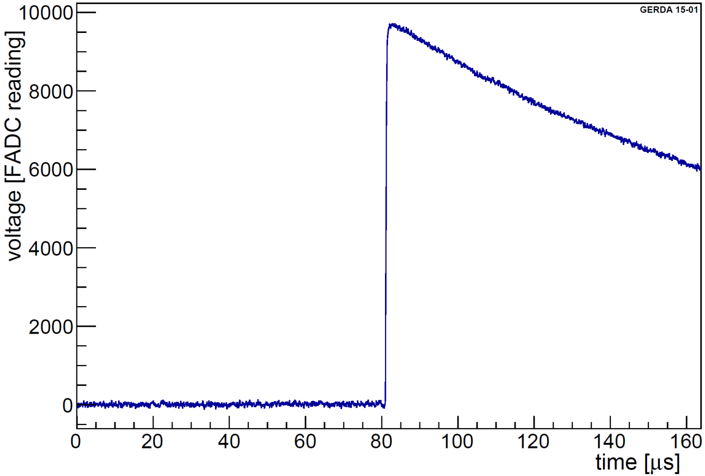
- High impedance load for detector
- Low impedance source for the amplifier (if any)
- Gain independent of detector capacitance



- Junction gate field-effect transistor (JFET) coupled to feedback circuit
- Capacitor C_f integrates charge from detector
- Resistor R_f discharges the capacitor not to saturate the dynamic range of the ADC
- Charge pulse will have an exponential decay with $\tau = R_f C_f$

How does a waveform look like?

- Flat baseline before the charge collection
- Rise time $\sim 0.5\mu s$
- Exponentially decaying tail



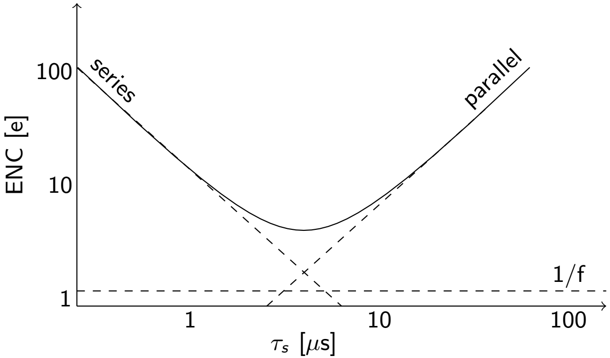
- The ENC is the number of electrons which would need to be collected in order to obtain a signal with the amplitude of the electronic noise RMS
- In general (QUOTE GATTI MANFREDI AND/OR ZAC PAPER):

$$ENC^2 = \alpha \frac{2kT}{g_m \tau_s} C_T^2 + \beta A_f C_T^2 + \gamma \left(e(I_G + I_L) + \frac{2kT}{R_f} \right) \tau_s$$

$$
\begin{aligned}
k &= \text{Boltzmann constant } = 1.38 \cdot 10^{-23} \text{ J/K} \\
T &= \text{Operational temperature } = 77 \text{ K for LN} \\
g_m &= \text{JFET trasconductance } \simeq 5 \text{ mA/V for } \textsc{Gerda} \\
C_T &= \text{Total capacitance } = C_D + C_i + C_f \\
C_D &\sim 1(30) \text{ pF for BEGe (coaxial) detectors} \\
C_i &= \text{Preamplifier input capacitance } \sim 10 \text{ pF} \\
C_f &= \text{Feedback capacitance } = 0.3 \text{ pF (for } \textsc{Gerda}) \\
A_f &= \text{1/f noise term } \sim 10^{-12}\text{-}10^{-14} \text{ V}^2 \text{ (difficult to calculate, better measure it)} \\
I_g &= \text{Gate current } \sim 1 \text{ pA } \rightarrow \text{ negligible} \\
I_L &= \text{Leakage current} \sim O(100) \text{ pF} \\
R_f &= \text{Feedback resistance } = 500 \text{ M}\Omega \text{ (for } \textsc{Gerda}) \\
\alpha, \beta, \gamma &= \text{Constants depending of } O(1) \text{ on filter shape and electrical components} \\
\tau_s &= \text{Shaping time of the considered filter. Typically } O(10)\ \mu\text{s}
\end{aligned}
$$

What do we learn from this long formula?

- Operating at low temperature helps a lot!
- The series noise (first term) is $\propto 1/\tau_S$, while the parallel noise (third term) is $\propto \tau_s$. Hence, τ_s can be optimized.
- Must pay attention to the total capacitance!
- Mechanical movements can alter C_i, inducing microphonic noise $\rightarrow$ better put the preamplifier close to the detector



Equivalent Noise Charge: What is the Contribution of Each Term?

Series noise

$$\frac{2kT}{g_m \tau_s} C_T^2 = \frac{2 \cdot 1.38 \cdot 10^{-23} \frac{\mathrm{J}}{\mathrm{K}} \cdot 77\ \mathrm{K}}{5 \cdot 10^{-3} \frac{\mathrm{A}}{\mathrm{V}} \cdot 10^{-5}\mathrm{s}} C_T^2 = 4.3 \cdot 10^{-15}\mathrm{V}^2 \cdot C_T^2$$

$$\text{BEGe:} \quad \sim 4.3 \cdot 10^{-36}\mathrm{C}^2 = 166\ \mathrm{e}^2$$

$$\text{coaxial:} \quad \sim 6.9 \cdot 10^{-35}\mathrm{C}^2 = 2680\ \mathrm{e}^2$$

Parallel noise

$$\left(\mathrm{e}(I_G + I_L) + \frac{2kT}{R_f}\right) \cdot \tau_s = \left(\mathrm{e} \cdot 100\ \mathrm{pA} + \frac{2 \cdot 1.38 \cdot 10^{-23} \frac{\mathrm{J}}{\mathrm{K}} \cdot 77\ \mathrm{K}}{5 \cdot 10^{8}\Omega}\right) \cdot 10^{-5}\ \mathrm{s}$$

$$= \left(\mathrm{e} \cdot 10^{-10} \frac{\mathrm{C}}{\mathrm{s}} + 4.3 \cdot 10^{-30}\ \frac{\mathrm{C}^2}{\mathrm{s}}\right) \cdot 10^{-5}\ \mathrm{s}$$

$$= 7916\ \mathrm{e}^2$$

1/f noise

Assuming $A_f = 10^{-14}\ \mathrm{V}^2$:

BEGe: $A_f C_T^2 = 10^{-14}\ \mathrm{V}^2 \cdot 10^{-22}\ \frac{\mathrm{C}^2}{\mathrm{V}^2} \simeq 39\ \mathrm{e}^2$

coaxial: $A_f C_T^2 = 624\ \mathrm{e}^2$

Assuming $A_f = 10^{-12}\ \mathrm{V}^2$:

BEGe: $A_f C_T^2 \simeq 3900\ \mathrm{e}^2$

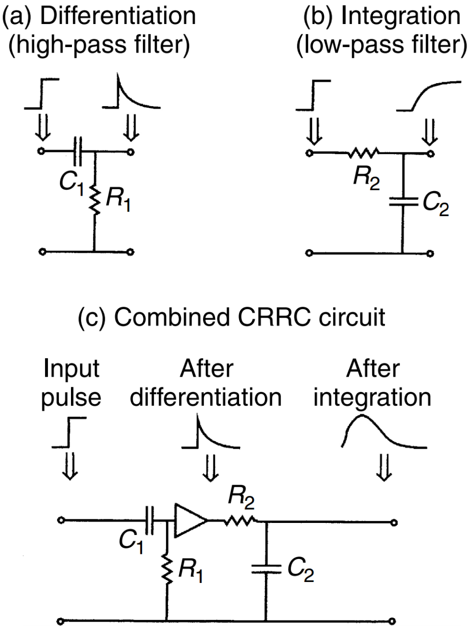
coaxial: $A_f C_T^2 = 62400\ \mathrm{e}^2$

What is pulse shaping?

- Pulse shaping is the process of changing of the signal waveform to get a "better" signal shape
- Goal: enhancing the signal-to-noise ration to get a more precise energy estimation
- Analog pulse shaping: set of RC (differentiation, high-pass) and (RC) (integration, low-pass) filters
- Digital pulse shaping: equivalent of analog shaping, but performed via software on digitized waveforms $\rightarrow$ need to use a FADC
- Filter defined by:
  1) shape
  2) shaping time (τ_s), additional parameters

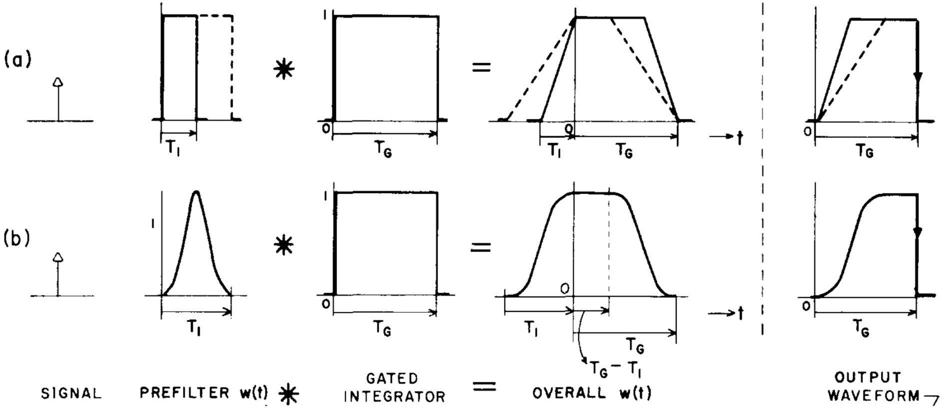
Semi-Gaussian shaping

- R_1C_1-$(C_2R_2)^n$ with $n \geq 2$
- Optimal resolution obtained with $R_1C_1 = R_2C_2$
- Shaping time: $\tau_s = RC$ [μs]
- Typical shaping times: 1-20 μs



Trapezoidal shaping

- Convolution of two squared filters of same (different) duration[‡]
- Circuit implementation not so trivial



‡V. Radeka, Nucl. Instrum. Methods 99 (1972) 525-539.

Advantages with respect to analog shaping

- Infinite number of filters available $\rightarrow$ space to creativity
- Waveform digitization allows to reprocess data in a second time
  $\rightarrow$ possible to improve energy resolution and recover bad-quality data

How does it work?

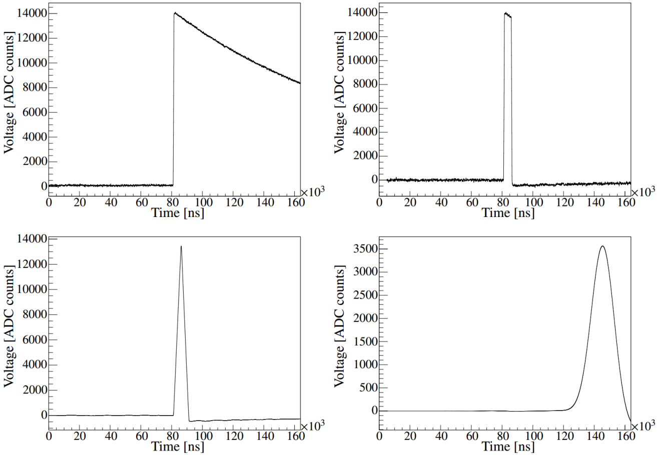
- Substitute filtering circuits with equivalent digital filters
- Perform the convolution of the waveform with the digital filter

How to improve energy resolution or other physical quantities?

- Play with filter shape
- Optimize filter parameters

Semi-Gaussian shaping

- RC $\rightarrow$ delayed differentiation: $x_0[t] \rightarrow x_1[t] - x_0[t - \tau_s]$
- $(\mathrm{CR})^n \rightarrow$ Moving Average: $x_i[t] \rightarrow x_{i+1}[t] = \frac{1}{\tau_s} \sum_{t'=t-\tau_s}^{t} x_i[t'] \qquad i = 1, \ldots, n$
- Pro: stable, robust, fast
- Con: sensible to low-frequency noise



How to define energy resolution?

- FWHM: Full Width at Half Maximum (in keV) of a gamma line in the energy spectrum
- For a Gaussian peak: FWHM $= 2.355\ \sigma$
- FWHM(E) $= \sqrt{w_i^2 + w_e^2 + w_p^2 + w_c^2}$
- $w_i =$ intrinsic width of the gamma line. $w_c << 0.1$ eV $\rightarrow$ negligible
- $w_e =$ electronic noise contribution
- $w_p =$ charge production term
- $w_c =$ charge collection and integration term

Electronic noise

- $w_e = 2.355 \cdot \frac{\eta}{e} ENC^2$
- Series noise: $w_{e,series} \sim 0.1(0.4)$ keV for BEGe (semi-coaxial) detectors
- Parallel noise: $w_{e,parallel} \sim 0.6$ keV
- 1/f noise: from 0 to several keV, depending on the situation
- In total: $w_e \geq 0.65(0.75)$ keV for BEGe (semi-coaxial) detectors
- All quoted numbers depend on filter shape and shaping time!
- Once fixed the detector + electronics system, we can still play with the shaping filter to optimize the energy resolution

Charge production

- $\eta = 2.96$ eV $=$ average energy necessary for the creation of a e-h pair
- Given a deposited energy E, we expect $N = \frac{E}{\eta}$ e-h pairs. But η is just an average...
- Assume the e-h pair creation obeys to Poisson statistics. Then: $\sigma_N = \sqrt{N} = \sqrt{\frac{E}{N}}$
- The uncertainty on the absorbed energy is: $\sigma_E = \eta \cdot \sqrt{N} = \sqrt{\eta \cdot E}$
- The corresponding contribution to FWHM in keV is: $w_p = 2.355 \cdot \sqrt{\eta \cdot E}$
  For the ^{60}Co line at 1333 ke: $w_p = 4.68$ keV, but experimentally is $O(2)$ keV...
- Poisson statistics applies to independent events, but the e-h creation in the crystal lattice is not!
- Solution: introduce an additional "Fano" factor[§]:

$$F = \frac{\sigma_{N,exp}}{\sigma_{N,Poisson}} \simeq 0.11 \qquad \text{for Ge}$$

- Corrected formulation of w_p:

$$w_p = 2.355 \cdot \sqrt{\eta F E} \rightarrow 1.55 \text{ keV at } 1333 \text{ keV}$$

- w_p is an irreducible term. No way to improve it!

[§] U. Fano, Phys. Rev. 72 (1947) 26-29
B. G. Lowe, Nucl. Instrum. Methods A 399 (1997) 354-364

Charge collection and integration: possible problems

1. Presence of strong crystal imperfections can cause charge trapping
   $\rightarrow$ not all the charge is collected
   Solution: almost none
2. Too low bias voltage can turn small crystal imperfections to big ones
   Solution: higher V_b, if possible
3. A too short shaping filter might not fully integrate the charge
   Solution: increase τ_s and/or use a filter with a flat top for all the duration of charge collection
4. $\tau = RC$ short with respect to charge collection time
   Solution: pole-zero cancellation

How does w_c depend on energy?

- Difficult to model, but empirically: $w_c = 2.355\sqrt{c^2 E^2}$

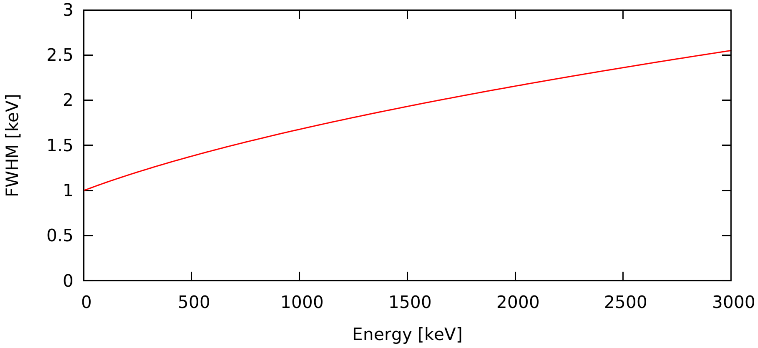
What's the effect on the spectrum?

- In all cases we underestimate the energy by some variable amount $\rightarrow$ Low-energy tails

P.S.: in case the charge collection is fine (points 1, 2) but the filter does not fully integrate the charge (points 3, 4), we talk about "ballistic deficit"

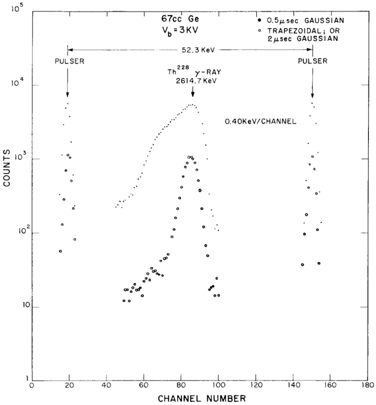
To summarize:

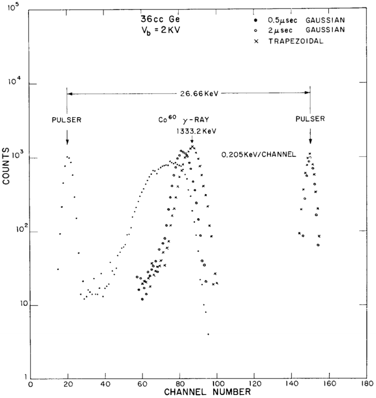
$$FWHM = 2.355\sqrt{\frac{\eta^2}{e^2}ENC^2 + \eta F \cdot E + c^2 E^2}$$



Method 1: Use of a filter with flat-top¶

- Fully integrate the charge by using a flat filter for the whole duration of the charge collection
- Pro: very easy to implement
- Con: Sensible to low-frequency noise





¶V. Radeka, Nucl. Instrum. Methods 99 (1972) 525-539

Method 2: Goulding-Landis[‖]

- Energy correction based on the delay in peak time of the shaped signal:

$$\frac{\Delta S}{S_0} = \left(\frac{\Delta\tau_p}{\tau_p}\right)^k \qquad \text{where: } \Delta S = \text{signal amplitude deficit}$$

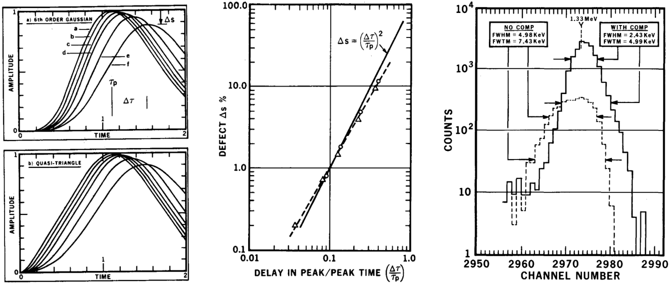
$$S_0 = \text{peak amplitude for signal with zero risetime}$$

$$\Delta\tau_p = \text{peak delay of the shaped signal}$$

$$\tau_p = \text{peaking time of signal with zero risetime}$$

$$k = \text{empirical} \quad \in [2;3]$$

- Partially corrects for energy loss due to charge trapping, too!



‖ F. S. Goulding and D. A. Landis, IEEE Trans. Nucl. Sci. 35 (1988) 119-124

Method 3: Hinshaw**

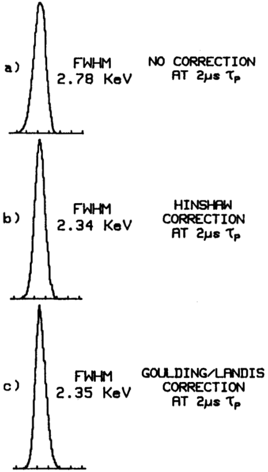
- Use two shaper: a quasi-triangle and a quasi-triangle + RC differentiation. This has a shorter peaking time, hence a larger ballistic deficit. Measure the difference in deficit and correct for it:

$$\begin{aligned} T_{P_1}, T_{P_2} &= \text{peaking times } (T_{P_2} > T_{P_1}) \\ \Delta A_1, \Delta A_2 &= \text{deficits} \\ T_r &= \text{input signal risetime} \\ \Delta T_{P_1}, \Delta T_{P_2} &= \text{delays in peaking time} \end{aligned}$$

$$\left.\begin{array}{l} \dfrac{\Delta A}{A} = \left(\dfrac{\Delta T_P}{T_P}\right)^2 \\ \Delta T_P \propto T_R \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \dfrac{\Delta A_1}{A_1} = k\left(\dfrac{\Delta T_R}{T_{P_1}}\right)^2 \\ \dfrac{\Delta A_2}{A_2} = k\left(\dfrac{\Delta T_R}{T_{P_2}}\right)^2 \end{array}\right.$$

$$\Delta m := (A - \Delta A_2) - (A - \Delta A_1) = kAT_R^2\left(\frac{1}{T_{P_1}^2} - \frac{1}{T_{P_2}^2}\right)$$

$$\Delta A_2 = \frac{\Delta m}{R^2 - 1} \qquad \text{with } R = \frac{T_{P_2}}{T_{P_1}}$$



** F. S. Goulding, D. A. Landis and S. M. Hinshaw, IEEE Trans. Nucl. Sci. 37 (1990) 417-423

GERDA energy reconstruction

- Full traces digitized with FADC
- Digital pseudo-Gaussian filter (25×5 μs moving average)
- Same filter parameters for all detectors and all Phase I data

Possible improvements

- Stability of energy scale
- "Intrinsic" energy resolution of calibration data
- "Effective" energy resolution of physics data at $Q_{\beta\beta}$

Strategy

- Develop a new digital shaping filter tuned on the experimental noise figure $\rightarrow$ Enhanced noise whitening, less sensitive to 1/f noise
- Correct preamplifier response function
- Tune the filter separately for each detector
- Split the Phase I data in different data sets, according to the detector configurations and the noise conditions

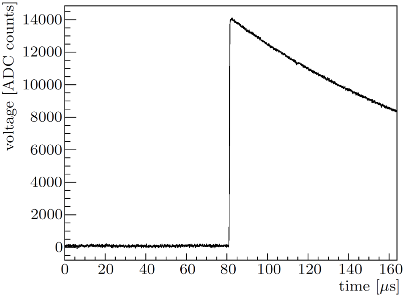
The ZAC filter

- Sinh-like cusp $\rightarrow$ optimal shaping filter for δ-like traces of finite length
- Central flat top (FT) $\rightarrow$ maximize charge integration
- Total zero-area $\rightarrow$ filter out 1/f noise
- Baseline subtraction best performed with parabolic filters

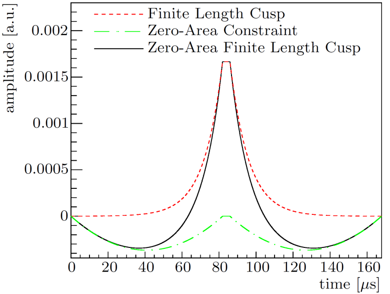
$$ZAC(t) = \begin{cases} \sinh\left(\frac{t}{\tau_s}\right) + A\left[\left(t - \frac{L}{2}\right)^2 - \frac{L}{2}^2\right] & 0 < t < L \\ \sinh\left(\frac{L}{\tau_s}\right) & L < t < L + FT \\ \sinh\left(\frac{2L+FT-t}{\tau_s}\right) + A\left[\left(\frac{3}{2}L + FT - t\right)^2 - \left(\frac{L}{2}\right)^2\right] & L + FT < t < 2L + FT \end{cases}$$

Final filter

- Deconvolution of the preamplifier response function: $f_\tau = \{1, -\exp\left(-\Delta t/\tau\right)\}$
- Final filter through convolution of ZAC with f_τ: $FF(t) = ZAC(t) * f_\tau(t)$

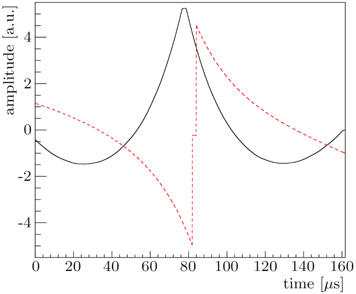


Original waveform



ZAC filter

Final filter FF (dashed red) and filtered waveform (black)



Optimization of the ZAC filter

- Phase I data divided in 5 periods according to detector configuration
- Filter optimization performed for 2-3 calibration runs of each period
- Scan parameter space, fit ^{208}Tl peak at 2614.5 keV, compute FWHM

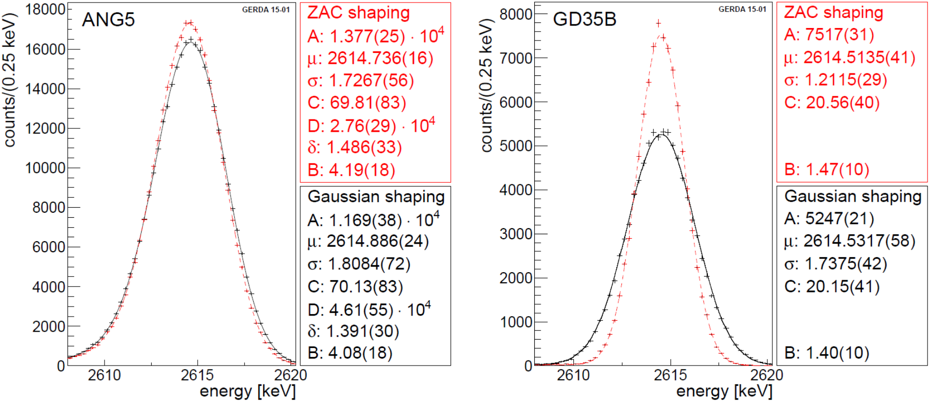
$$f(E) = A\exp\left(-\frac{(E-\mu)^2}{2\sigma^2}\right) + B + \frac{C}{2}\mathrm{erfc}\left(\frac{E-\mu}{\sqrt{2}\sigma}\right) + \frac{D}{2}\exp\left(\frac{E-\mu}{\delta}\right)\mathrm{erfc}\left(\frac{E-\mu}{\sqrt{2}\sigma} + \frac{\sigma}{\sqrt{2}\delta}\right)$$

- The optimal parameters are stable within each period

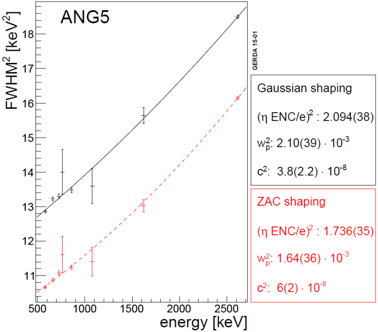
Reprocessing of calibration and physics Phase I data

- Create tier2 (uncalibrated spectra) of calibration data using optimized ZAC filter
  $\rightarrow$ Extract calibration curves, produce stability plots (e.g. FWHM vs time)
- Create tier3 (calibrated spectra) of calibration data
  $\rightarrow$ Further stability plots (deviations from literature, ...)
- Produce tier2 and tier3 of physics data using optimized ZAC filter

Comparison of the 2614.5 keV Peak

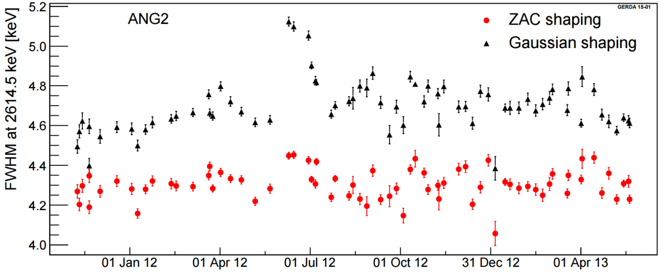


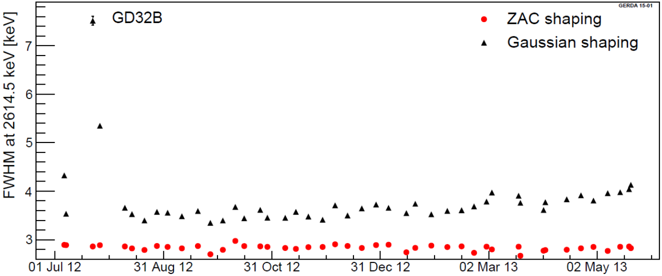
- All Phase I calibration spectra summed-up, same events considered in both cases
- Energy resolution improved in all cases
- Low-energy tail reduced thanks to better charge integration



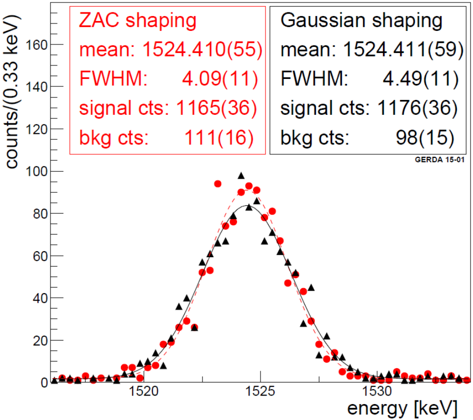
Detector	FWHM at 2614.5 keV Gaussian	ZAC	Improvement [keV]
ANG2	4.712(3)	4.314(3)	0.398(4)
ANG3	4.658(3)	4.390(3)	0.268(4)
ANG4	4.458(3)	4.151(3)	0.307(4)
ANG5	4.323(3)	4.022(3)	0.301(4)
RG1	4.595(4)	4.365(4)	0.230(6)
RG2	5.036(5)	4.707(4)	0.329(6)
GD32B	2.816(4)	2.699(3)	0.117(5)
GD32C	2.833(3)	2.702(3)	0.131(4)
GD32D	2.959(4)	2.807(3)	0.152(5)
GD35B	3.700(5)	2.836(3)	0.864(6)

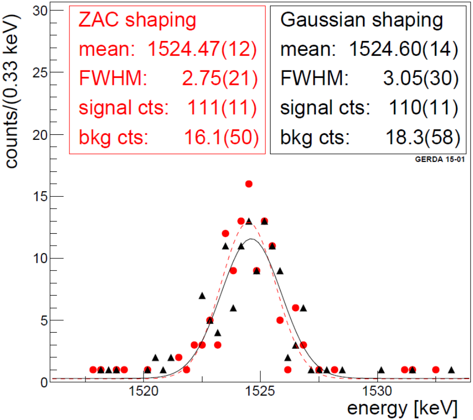
- Greatest improvement obtained on ENC^2
- Average improvement in FWHM at 2614.5 keV on all Phase I calibration data is 0.30 keV for coaxial and 0.13 keV for BEGes (GD35B excluded)
- Higher improvement for GD35B due to better treatment of low-frequency disturbance by the ZAC filter



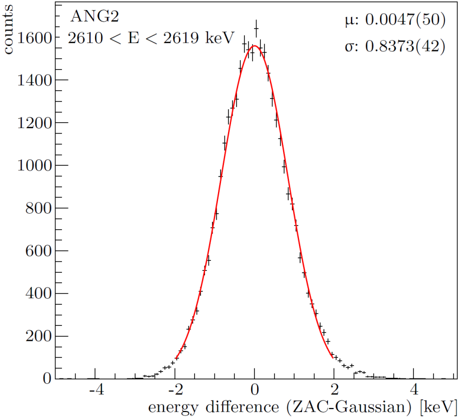


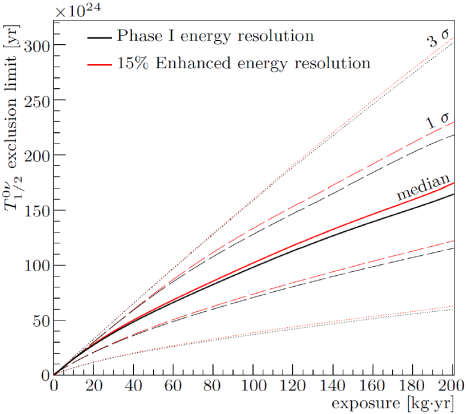
- ZAC filter insensitive to microphonic disturbance of ANG2 (June 2012)
- FWHM brought to nominal for GD35B for all Phase I duration





- ^{42}K peak at 1524.6 keV is the only spectral line in the physics spectrum
- Improvement of $\sim$ 0.4 keV, about 0.1 keV larger than expected for calibration data due to higher precision in the estimation of the calibration curves and lower sensitivity to time evolution of microphonics during physics run
- FWHM improvement at $Q_{\beta\beta}$ estimated to be $\sim$ 0.5 keV for both coaxial and BEGe detectors





- No surprise in the event-by-event energy difference (verified on physics data, too)
- Phase II $0\nu\beta\beta$ median sensitivity increased by $\sim 5\%$
- Same recipe for filter optimization will be used in Phase II
- Reprocessed Phase I data will be combined with Phase II data for $0\nu\beta\beta$ decay analysis
- GERDA collaboration paper accepted by Eur. Phys. J. C (ArXiV:1502.0392)