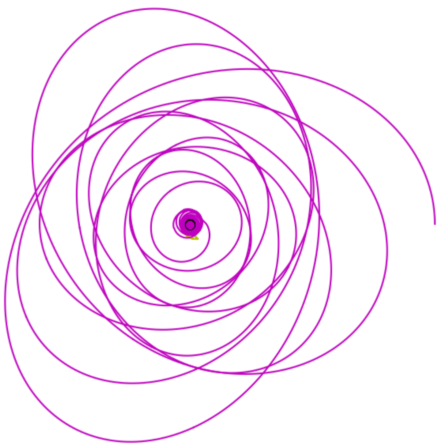
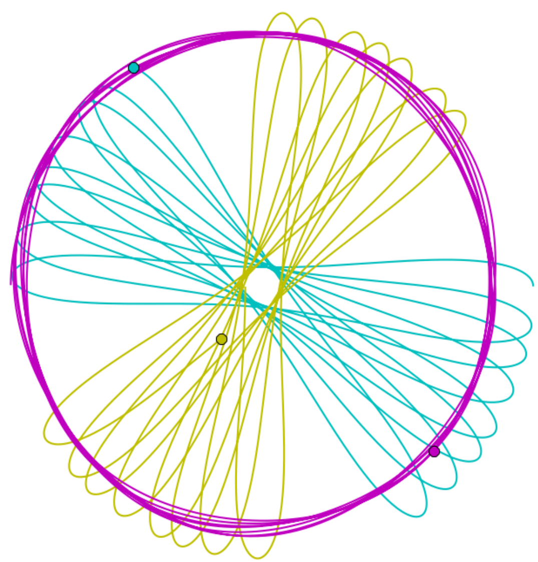
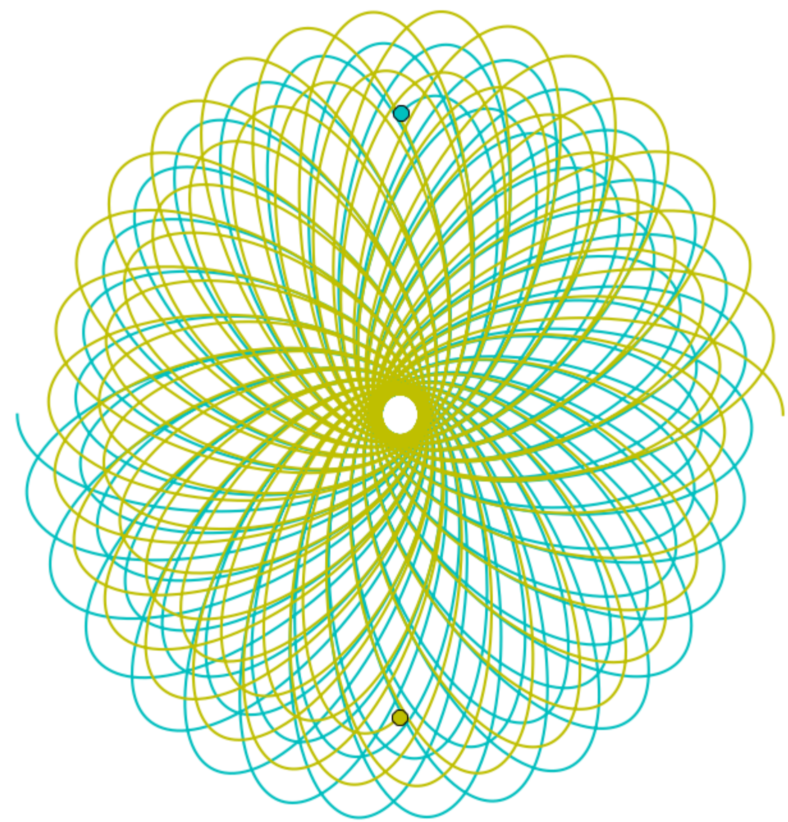
Post-Newtonian Dynamics of Massive Black Holes Triplets







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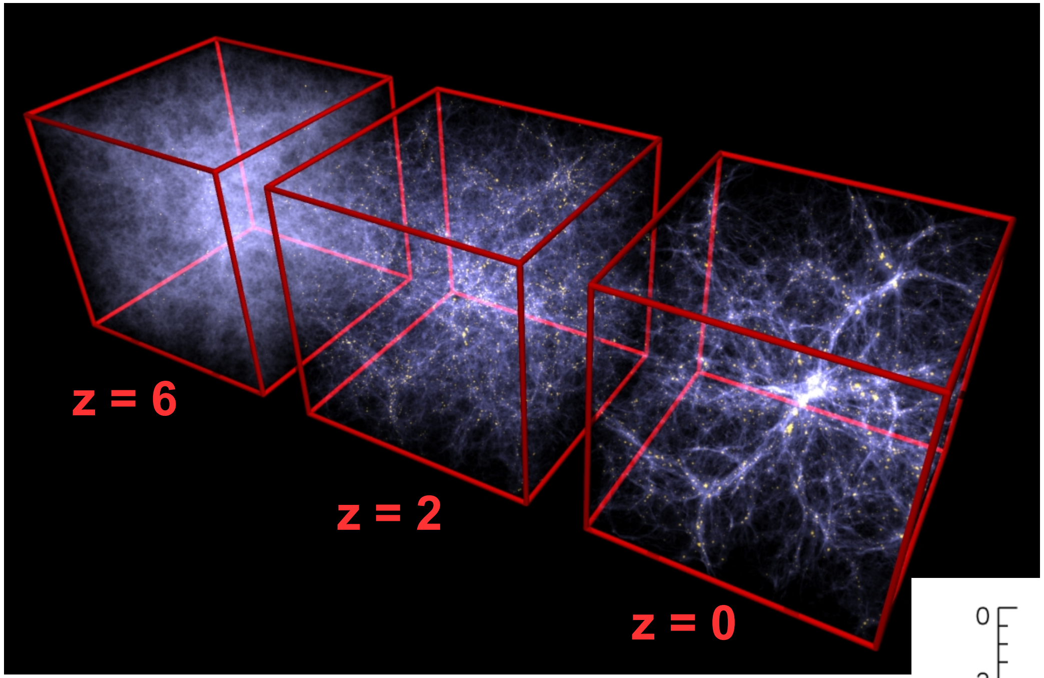
Francesco Haardt, Alberto Sesana, Enrico Barausse, Monica Colpi

Standard Cosmological Model

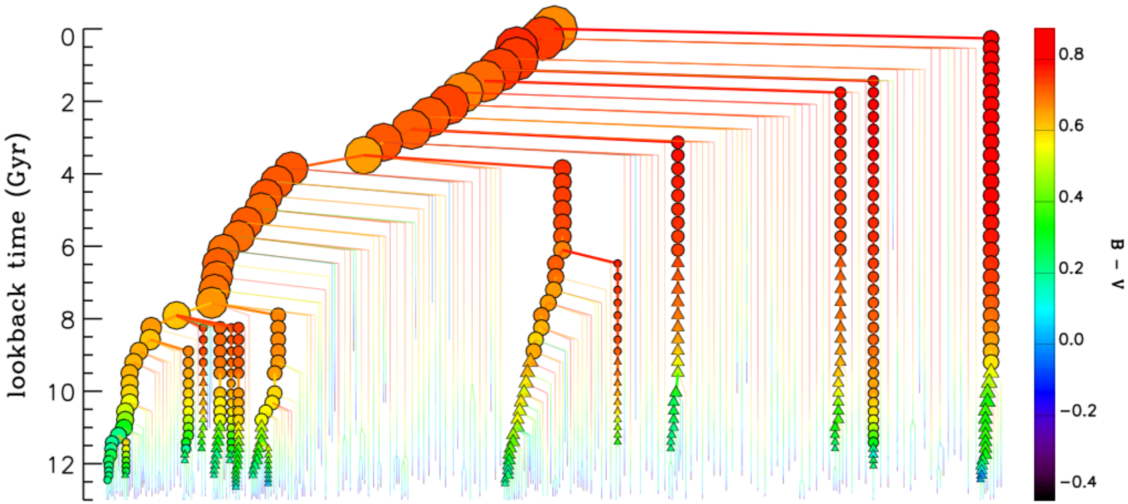
Hierarchical clustering of dark matter halos

Galaxies form in dark matter halos



Credit: Springel

Credit: De Lucia et al.



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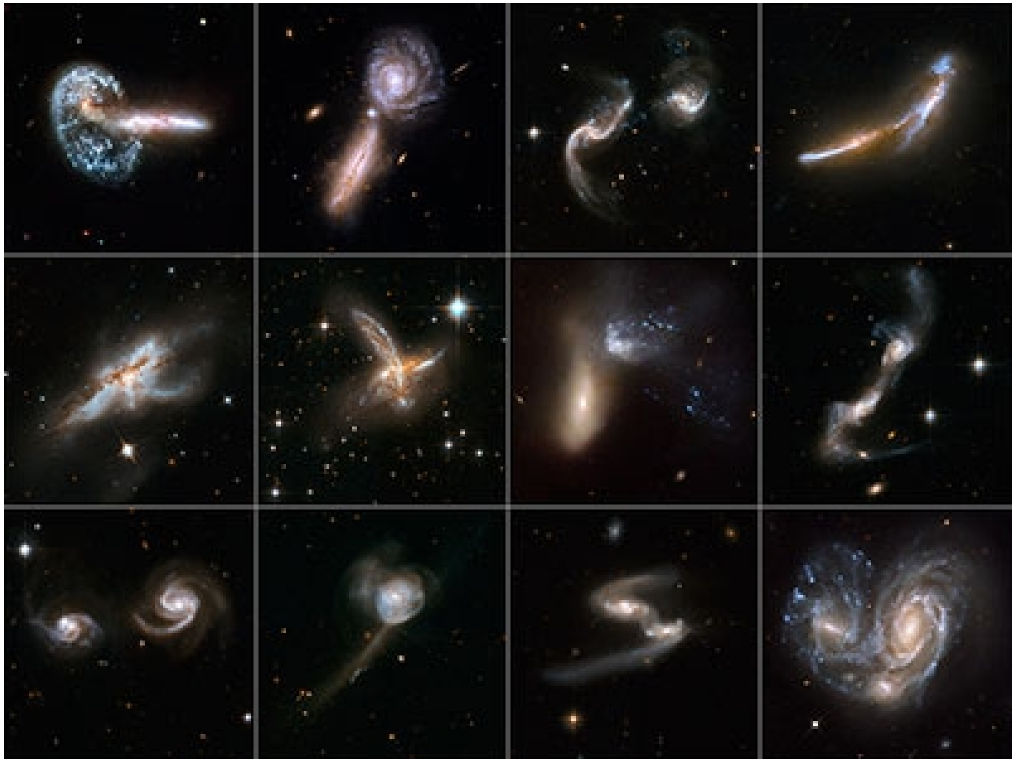
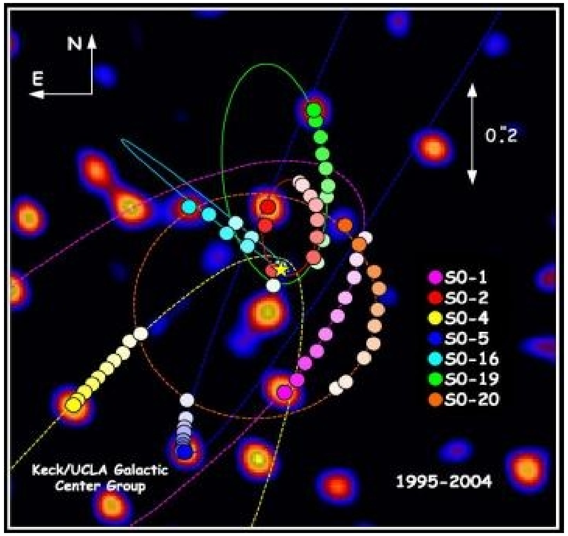


Galaxy merger

Galaxies host MBHs



Strong electromagnetic and dynamical activity characterizes central regions of galaxies

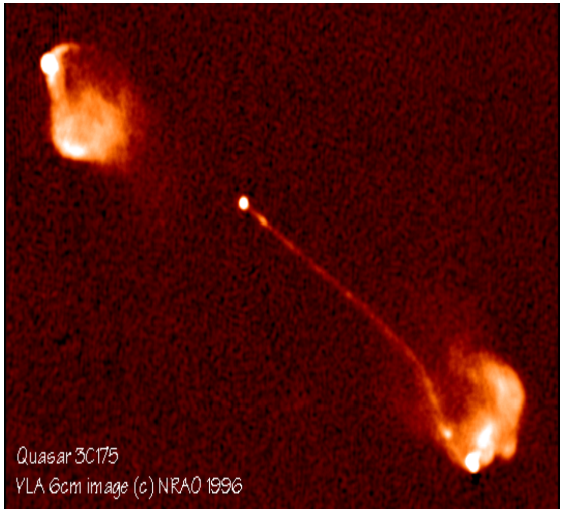
The triggers seem to be tiny objects (with respect to the whole galaxy) with mass range

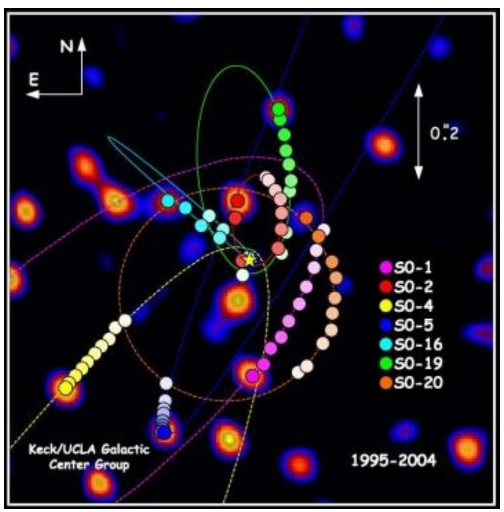
$$M \simeq 10^6 - 10^{10}\ \mathrm{M}_\odot$$

Compact objects

Best candidates are Massive Black Holes (MBHs)

Galaxies host MBH





Galaxies merge

Formation of MBHs binaries

Source of GW?

Standard scenario

MBHBs evolution essentially consists of 3 steps

1) Dynamical friction

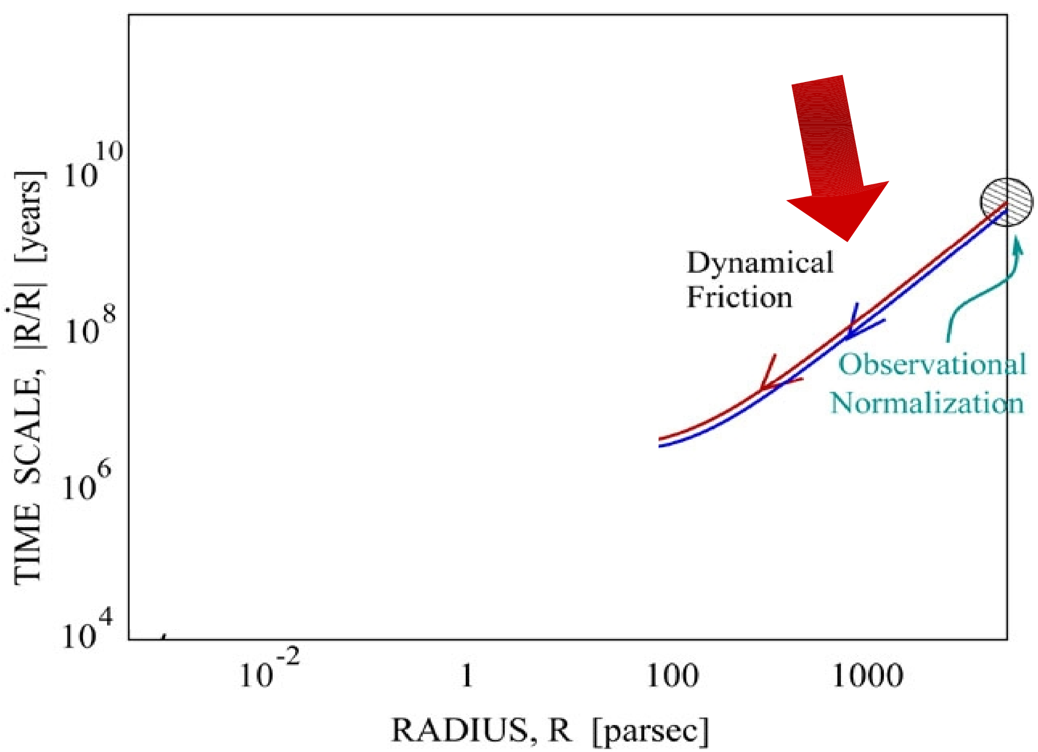
2) Close encounter binary-stars (hardening)

3) Gravitational Waves emission

Standard scenario

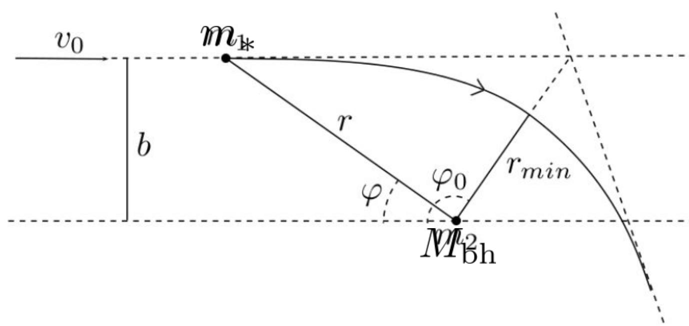
1) Dynamical friction:

MBHs sink to the center of the merger remnant



Begelman, Blanford, Rees, 1980

Cumulative effect of hyperbolic encounters with stars



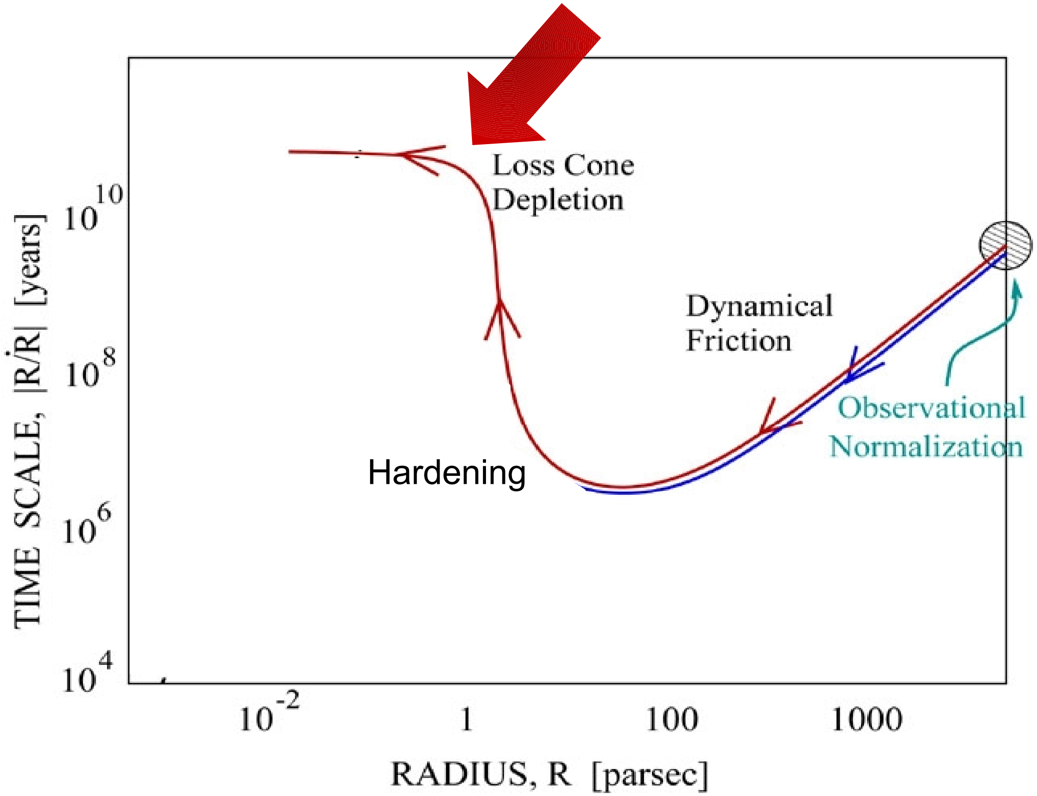
$$\vec{F}_{\rm dyn} \approx -C\frac{G^2 M_{\rm bh}^2 \rho}{v_{M_{\rm bh}}^3}\vec{v}_{M_{\rm bh}}$$

where $C \approx f\left(\frac{v_{M_{\rm bh}}}{\sigma}\right)$

Standard scenario

2) Close encounters binary-stars:

3-body interaction - MBHB hardening



Begelman, Blanford, Rees, 1980

$$\boxed{\begin{aligned}\frac{da}{dt} &= -\frac{a^2 G\rho}{\sigma}H \\ \frac{de}{dt} &= \frac{aG\rho}{\sigma}HK\end{aligned}}$$

Quinlan 1996; Sesana et al. 2006

Standard scenario

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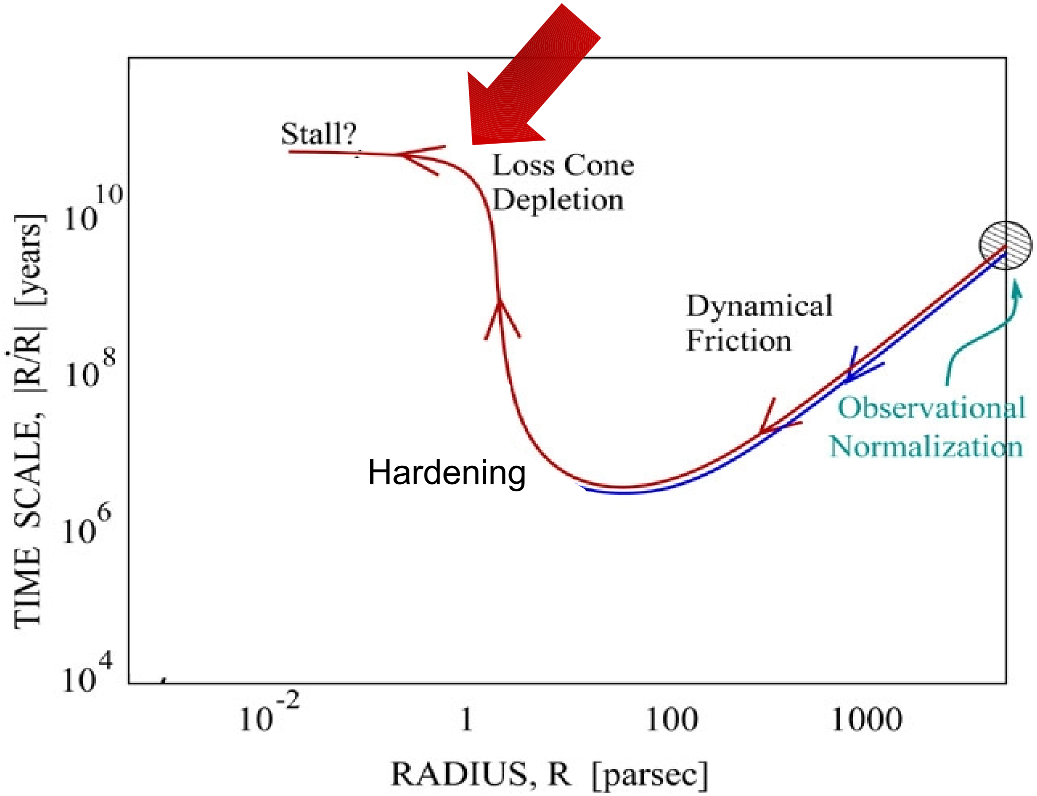

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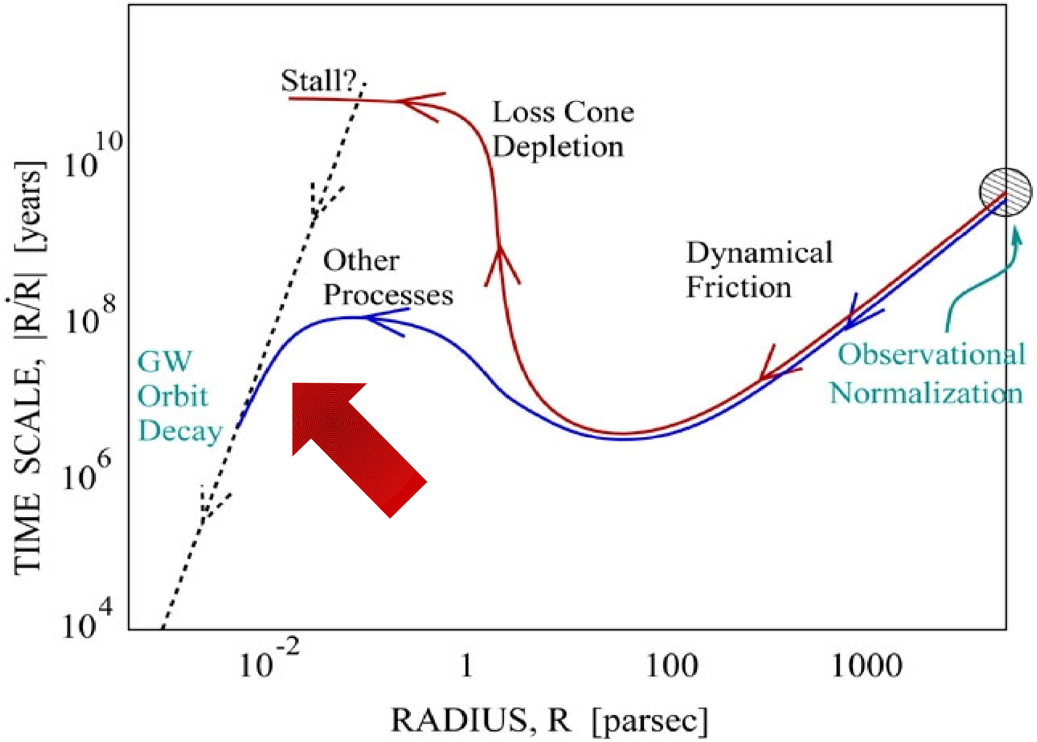
$$\boxed{\begin{aligned} \frac{da}{dt} &= -\frac{a^2 G\rho}{\sigma} H \\ \frac{de}{dt} &= \frac{aG\rho}{\sigma} HK \end{aligned}}$$

Quinlan 1996; Sesana et al. 2006

No more efficient without stars!

Standard scenario

3) Gravitational waves: MBHB merging



Begelman, Blanford, Rees, 1980

But only below

$$a_{\rm GW} \approx 0.002\ f(e)^{1/4} \frac{q^{1/4}}{(1+q)^{1/2}} \left(\frac{M}{10^6\ {\rm M}_\odot}\right)^{3/4} {\rm pc}$$

$$f(e) = \left[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right](1-e^2)^{-7/2}$$

Other processes are needed to fill the gap!

Standard scenario

Last Parsec Problem

…avitational waves:

…IB merging

But only below

$$\ldots 002\ f(e)^{1/4} \frac{q^{1/4}}{(1+q)^{1/2}} \left(\frac{M}{10^6\ \mathrm{M}_\odot}\right)^{3/4} \mathrm{pc}$$

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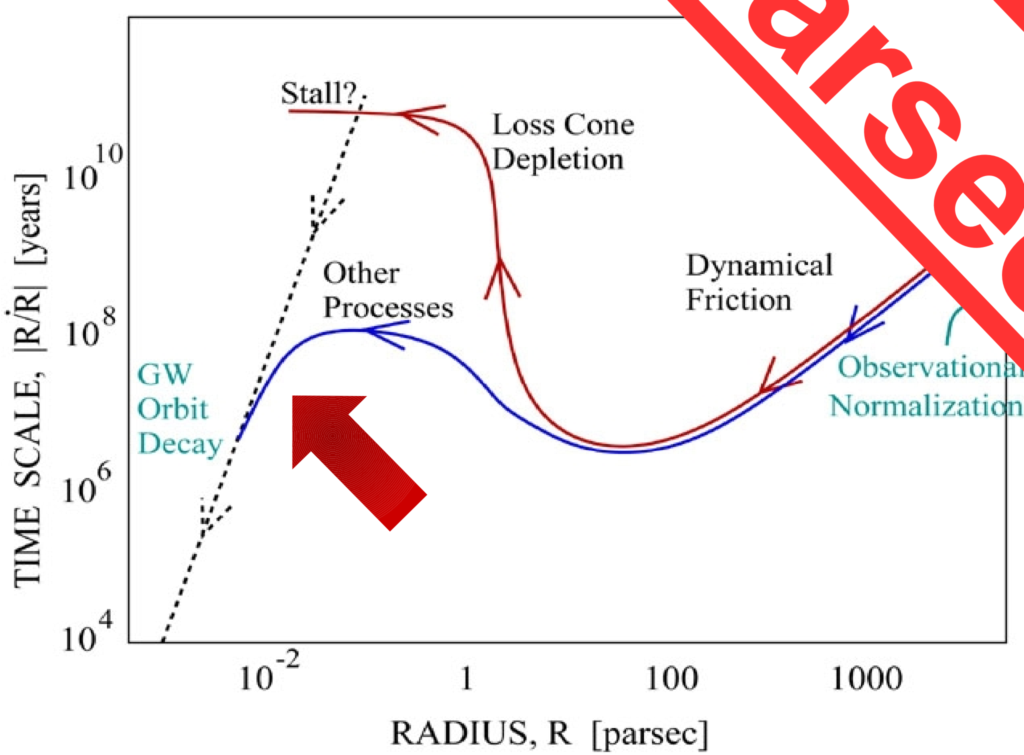


Begelman, Blanford, Rees, 1980

Ot… …ses are
neede… …gap!

Possible wayouts

- Gas rich galaxies
- Triaxiality of merger remnant
- 3 (or more?) - body interaction

Not mutually exclusive!

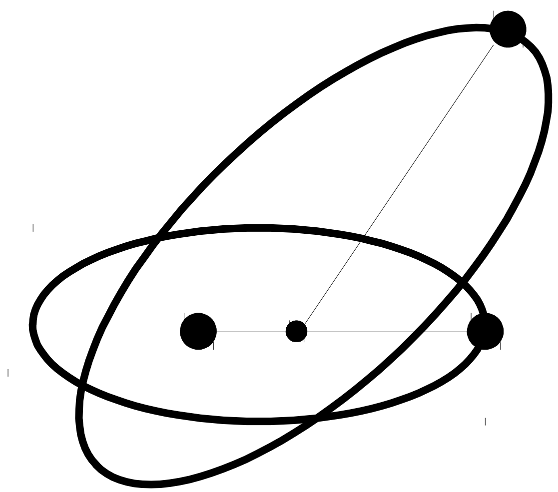
Possible wayouts

- Gas rich galaxies
- Triaxiality of merger remnant
- 3 (or more?) - body interaction

➡ If dense environnments can yield successive galaxy mergers

➡ If every galaxy host a MBH

Possible wayouts



- Gas rich galaxies
- Triaxiality of merger remnant
- 3 (or more?) - body interaction

➡ If dense environnments can yield successive galaxy mergers

➡ If every galaxy host a MBH

Formation of triplets of MBHs

Our Goal

Simulate MBH triplets in galactic nuclei with astrophysically and cosmologically motivated initial conditions

Code method:

C++ implementation of Bulirsch-Stoer (BS) algorithm for resolution of Ordinary Differential Equations (ODE)

Physics:

3-body Newtonian dynamics + GR corrections up to 2.5PN
+ interaction with stellar environment, i.e.

- bulge potential (spherically symmetric)
- stellar hardening

Sketch of Triplet Evolution

IC: stalled binary + third body

Hierarchical triplet
(two separate binaries)

$i > i_{\rm crit}$

$i < i_{\rm crit}$

Kozai-Lidov
mechanism

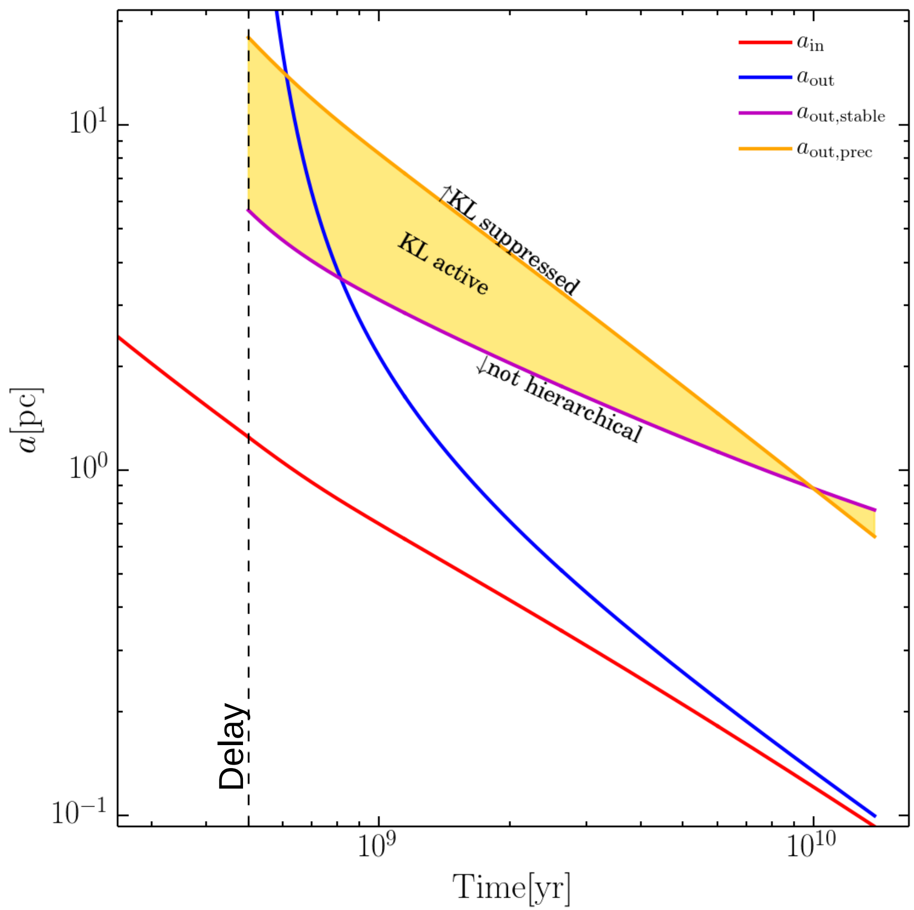
Growth of inner binary
eccentricity

Tiny perturbation
experienced by
inner binary

Hardening

Strong chaotic
interaction

Merger ⇔ Exchange ⇔ Ejection



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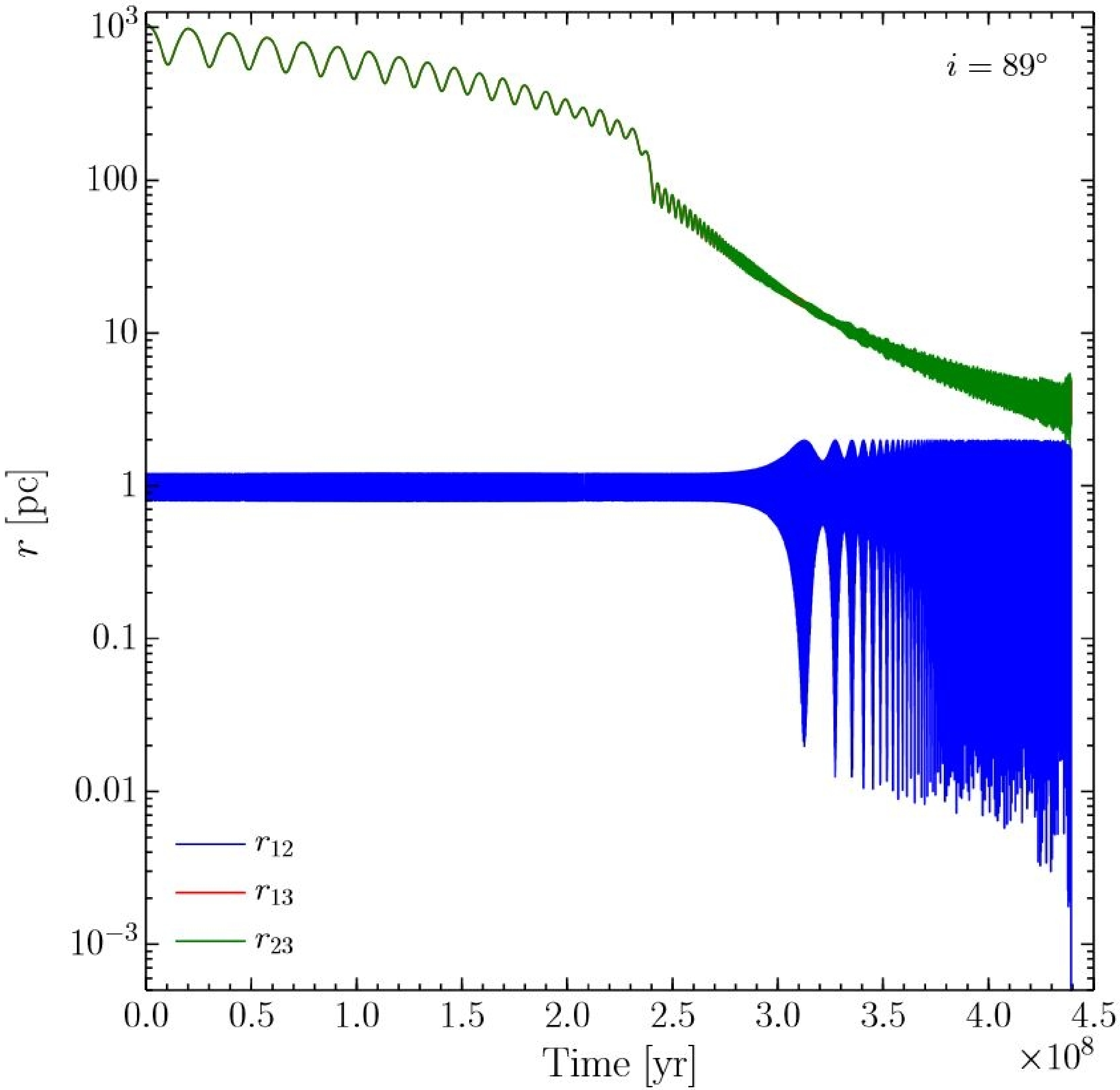


$m_1 = 10^8\ \mathrm{M}_\odot$

$m_2 = 3 \times 10^7\ \mathrm{M}_\odot$

$m_3 = 5 \times 10^7\ \mathrm{M}_\odot$

$a_{\rm in} = 1$ pc

$e_{\rm in} = 0.2$

$a_{\rm out} \simeq 1$ kpc

$e_{\rm out} = 0.3$

Sketch of Triplet Evolution

IC: stalled binary + third body → Hierarchical triplet (two separate binaries)

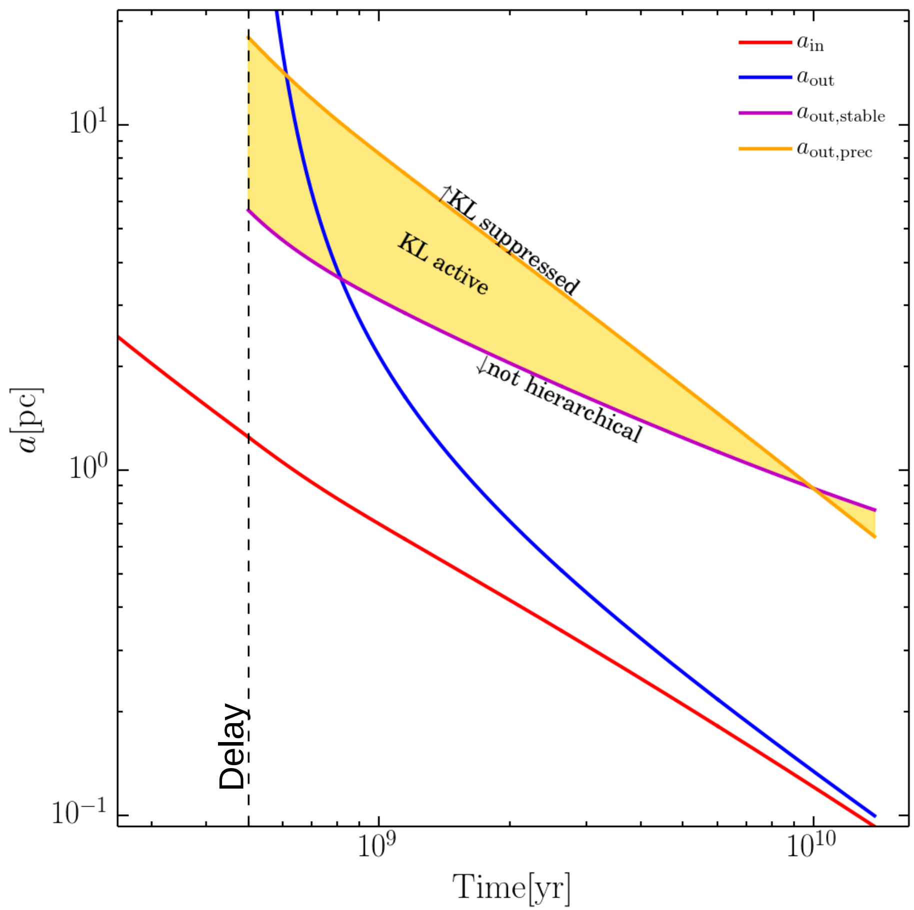
$i > i_{\rm crit}$ → Kozai-Lidov mechanism → Growth of inner binary eccentricity

$i < i_{\rm crit}$ → Tiny perturbation experienced by inner binary

Hardening

Strong chaotic interaction

Merger ↔ Exchange ↔ Ejection



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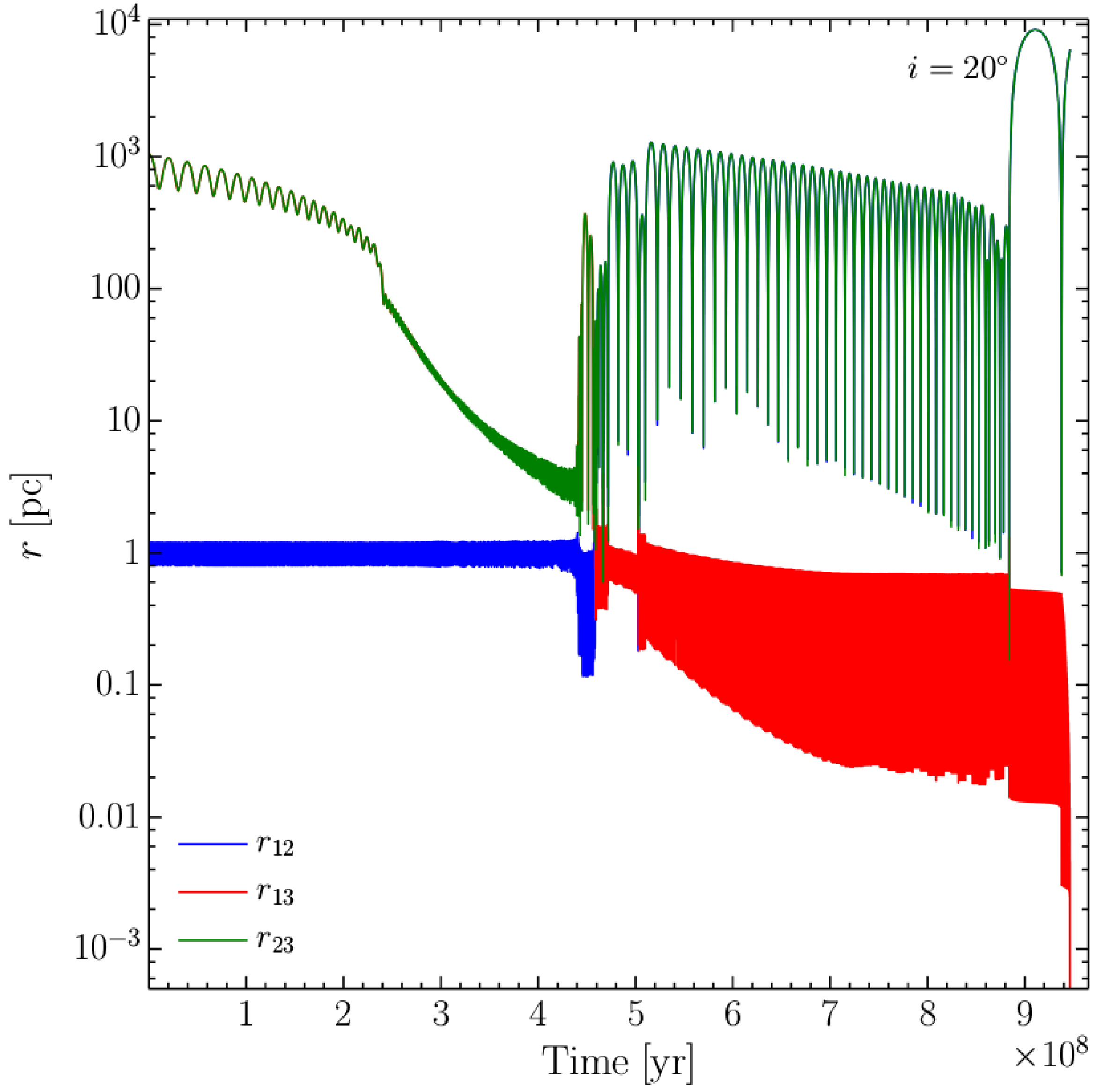

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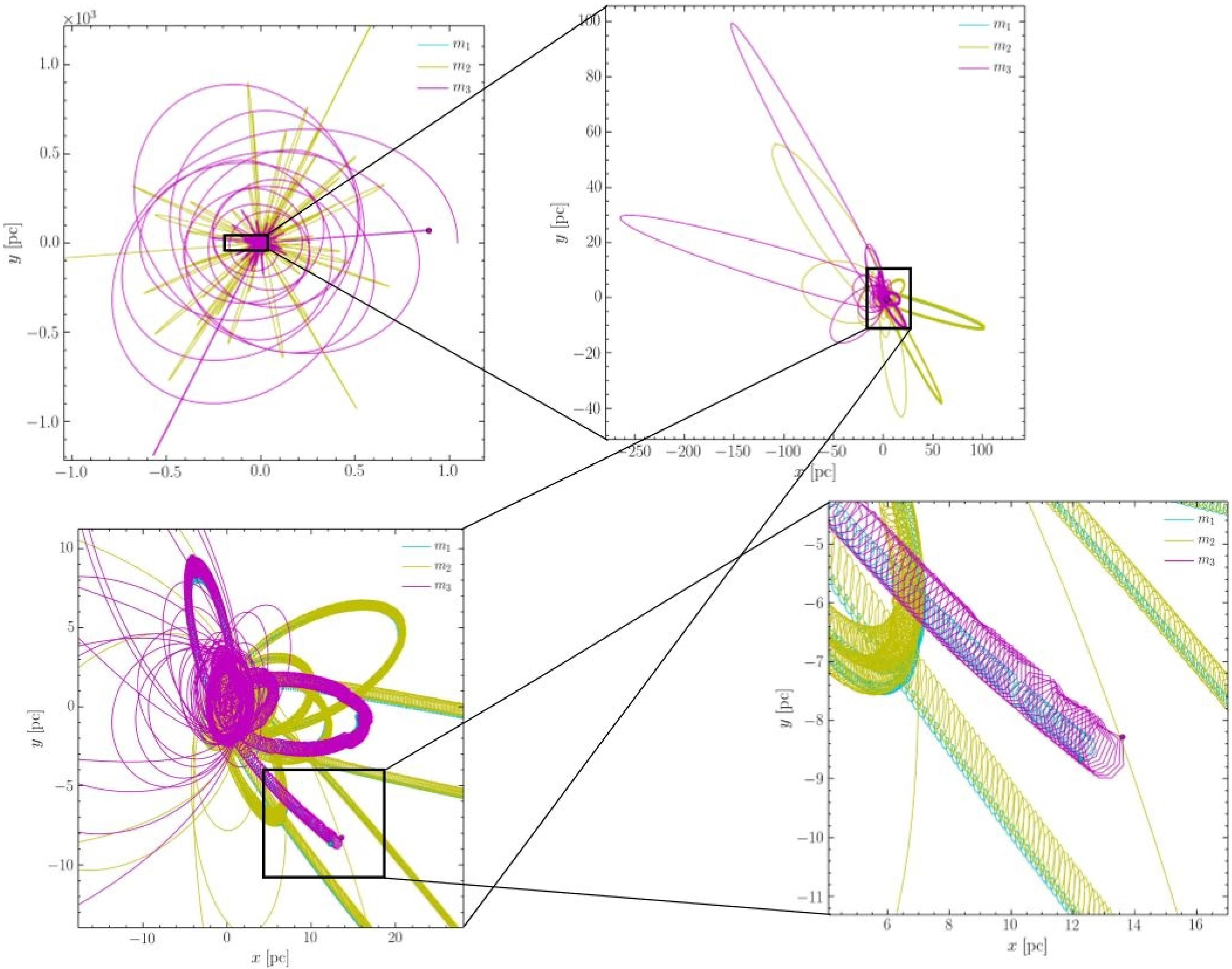

Sketch of Triplet Evolution

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$i > i_{\rm crit}$

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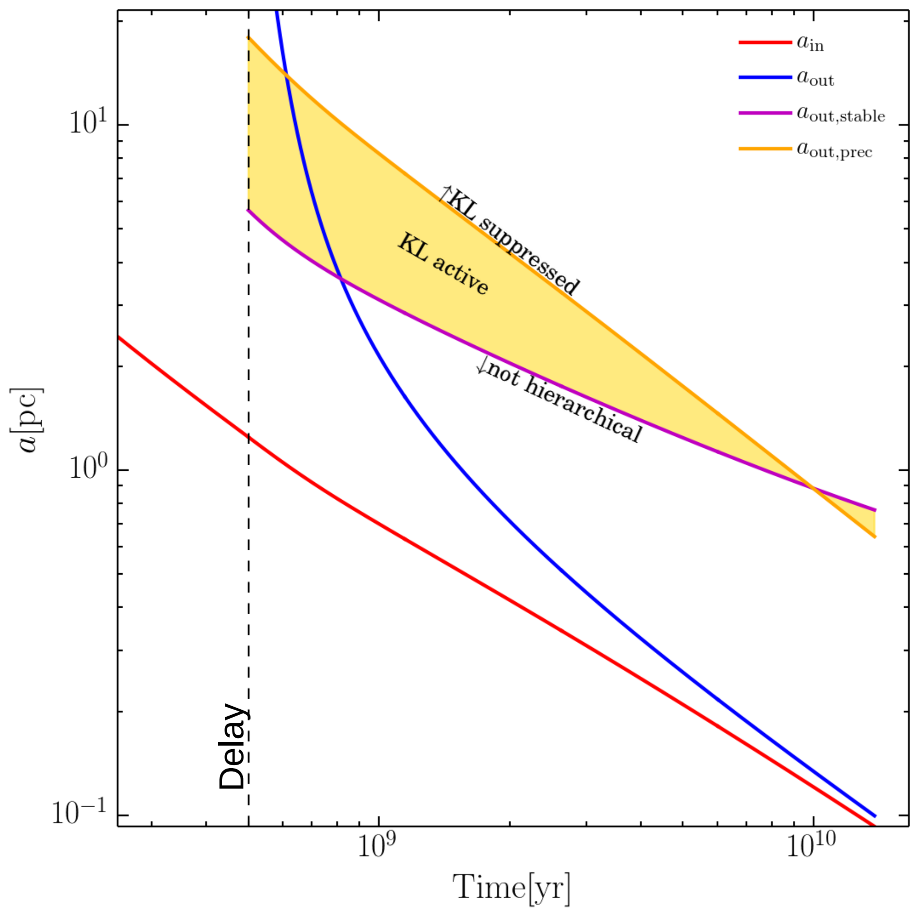

Kozai-Lidov
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Growth of inner binary
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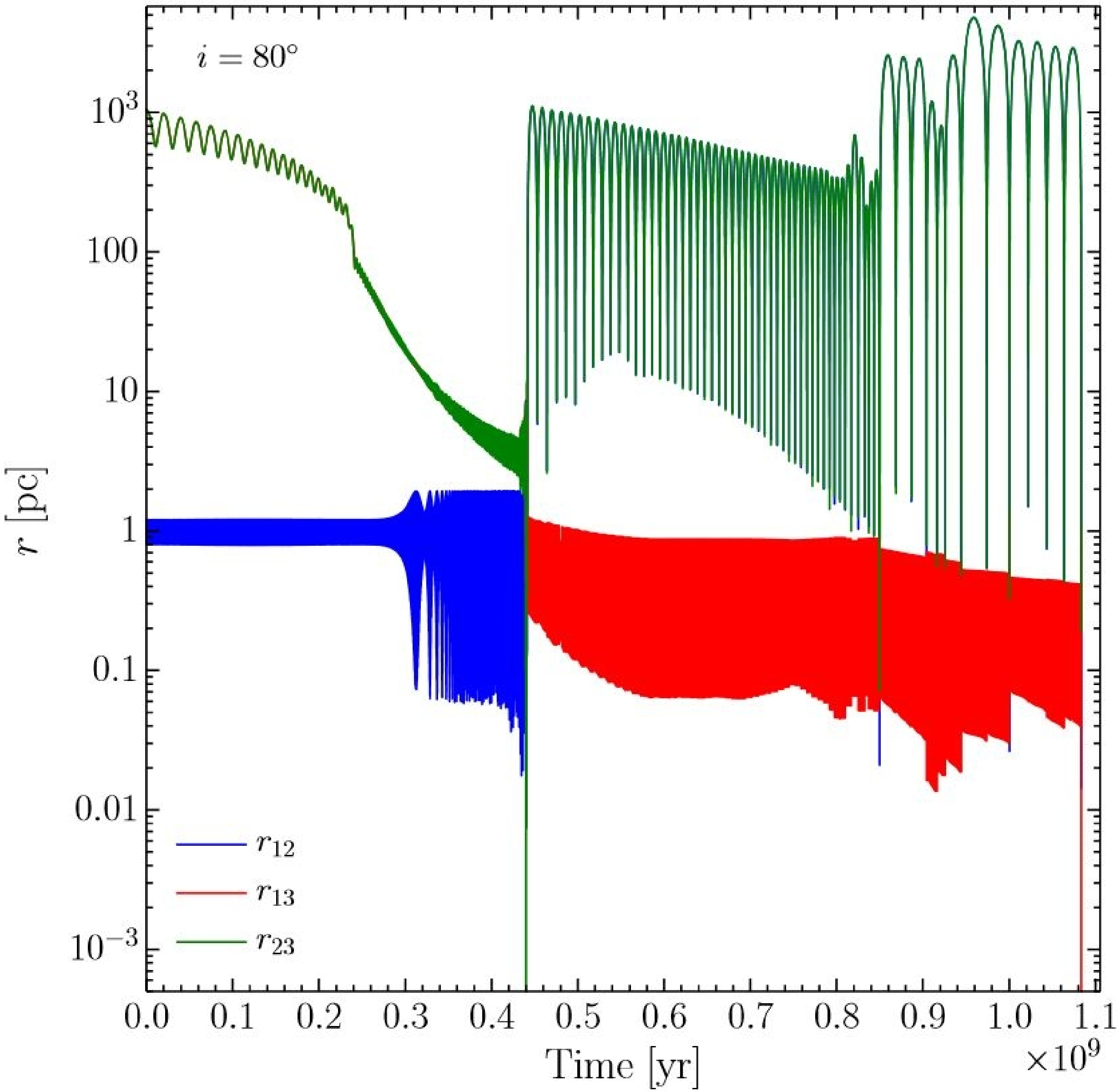

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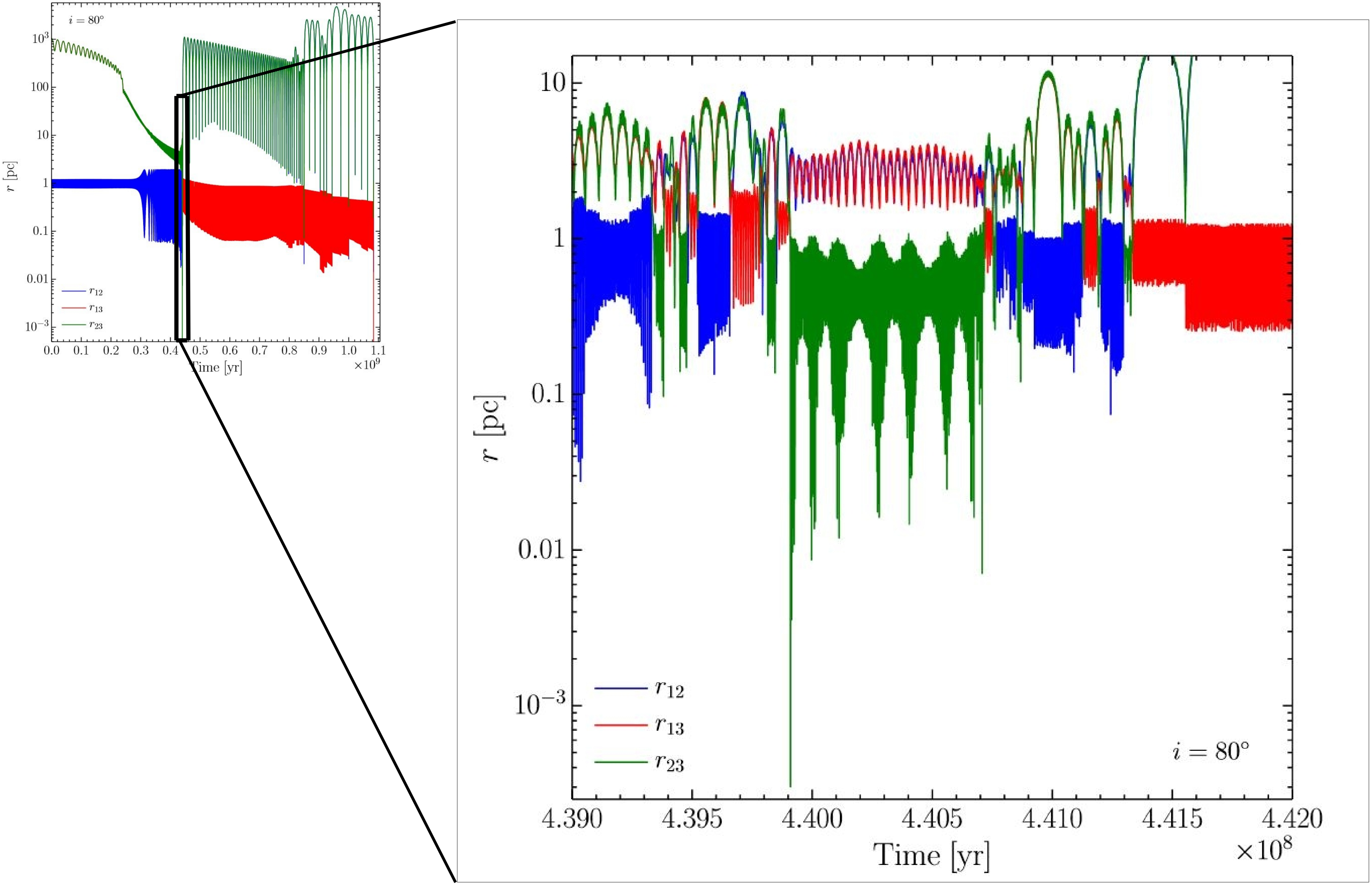


Project - Roadmap

Definition of the parameter space with astrophysically and cosmologically motivated initial conditions

- Mass (MBHs and stellar), inclination, eccentricity
- Third body arrival delay (inner semi-major axis, ejected stellar mass)

Project - Roadmap

Definition of the parameter space with astrophysically and cosmologically motivated initial conditions

↓

Systematic survey of the parameter space through simulations

~10.000 simulations

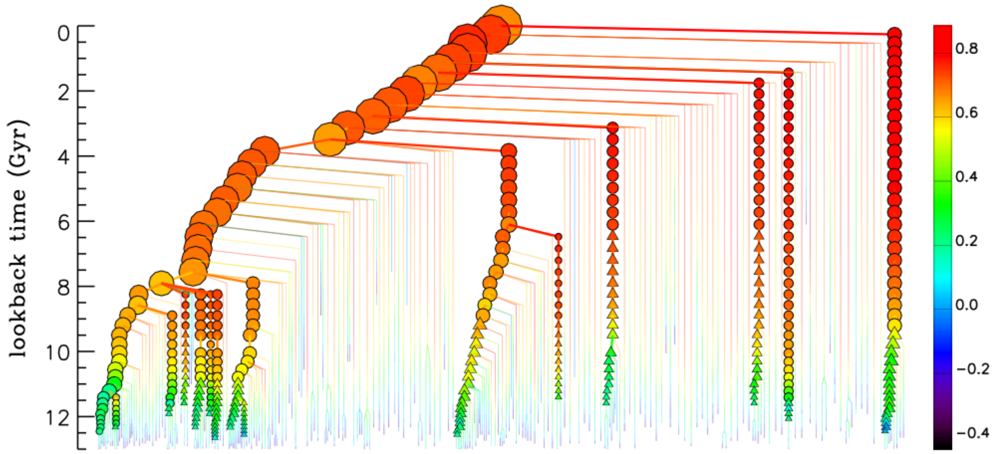
1/2 month/s (hopefully)

Project - Roadmap

Definition of the parameter space with astrophysically and cosmologically motivated initial conditions

Systematic survey of the parameter space through simulations

Coupling the results to a cosmological merger tree + SAM



Find out which region of the parameter space have statistical significance

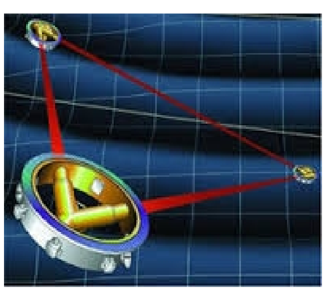
Project - Roadmap

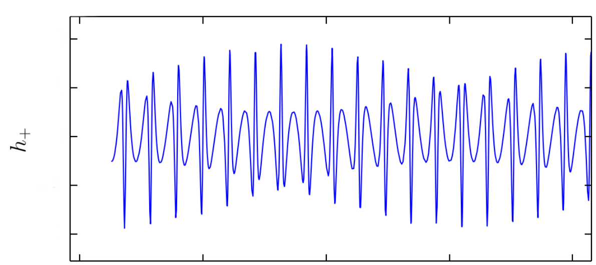
Definition of the parameter space with astrophysically and cosmologically motivated initial conditions

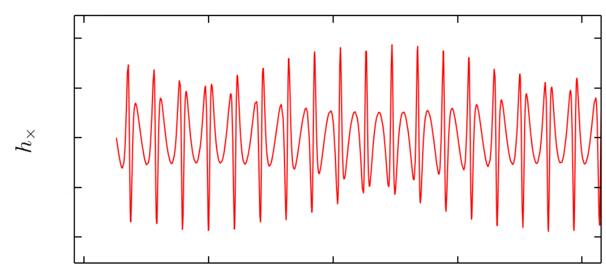
Systematic survey of the parameter space through simulations

Coupling the results to a cosmological merger tree + SAM

Waveforms template of GW emission for **IPTA** and **(e)LISA**







Thanks for your attention

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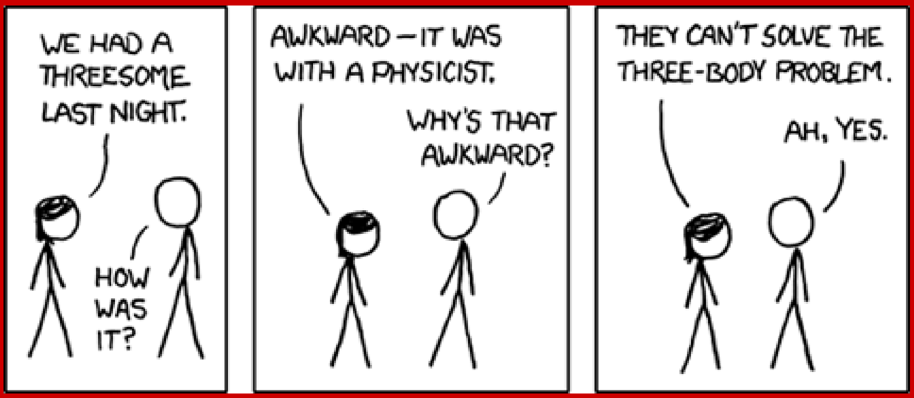
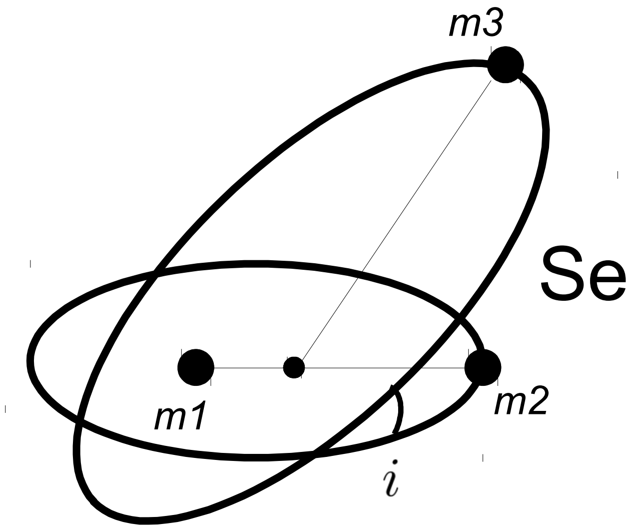


Kozai-Lidov mechanism

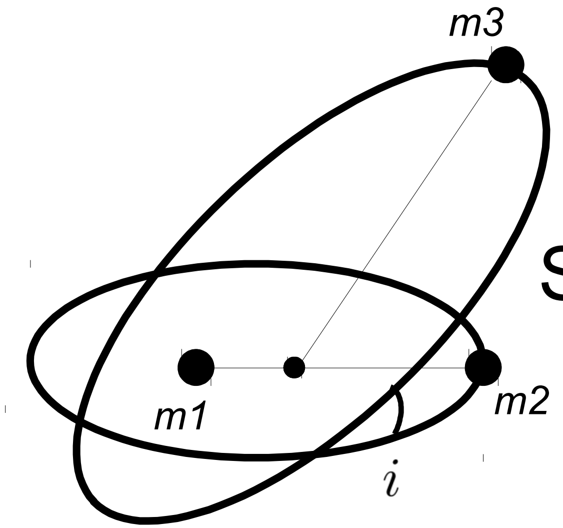


If relative inclination

$$i > 39.2^\circ \quad \text{and} \quad i < 140.8^\circ$$

Secular exchanges of angular momentum between inner and outer binary

Kozai-Lidov mechanism



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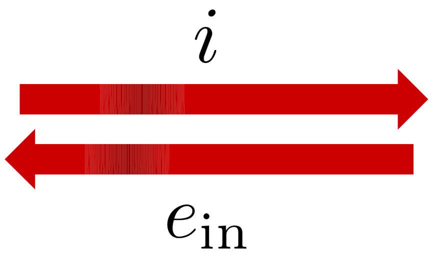


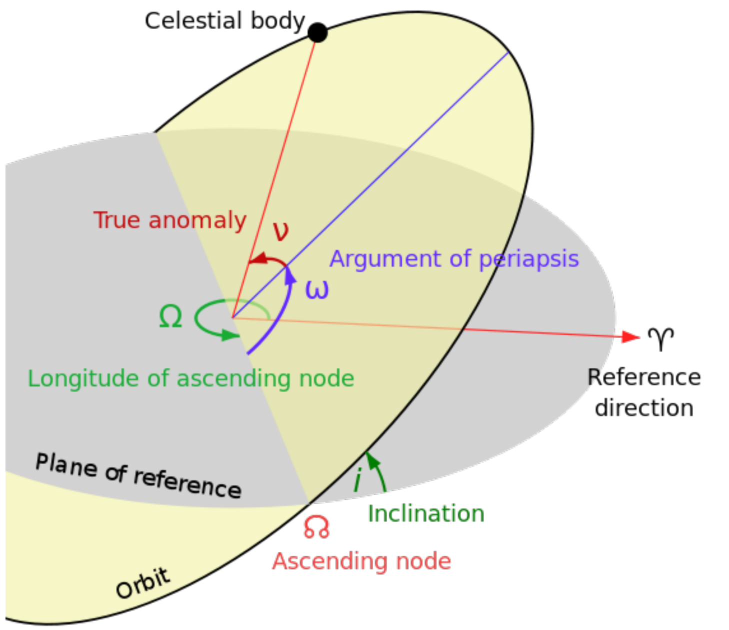
Large amplitude oscillations of the mutual inclination and of the inner binary eccentricity

$$\boxed{e_{\max} \simeq \sqrt{1 - \frac{5}{3}\cos^2 i}}$$

Initially very inclined orbits can reach high eccentricity!

Kozai-Lidov mechanism - Problems

KL implies libration
of the inner orbit argument of
pericenter ω



Some processes however force
circulation of ω

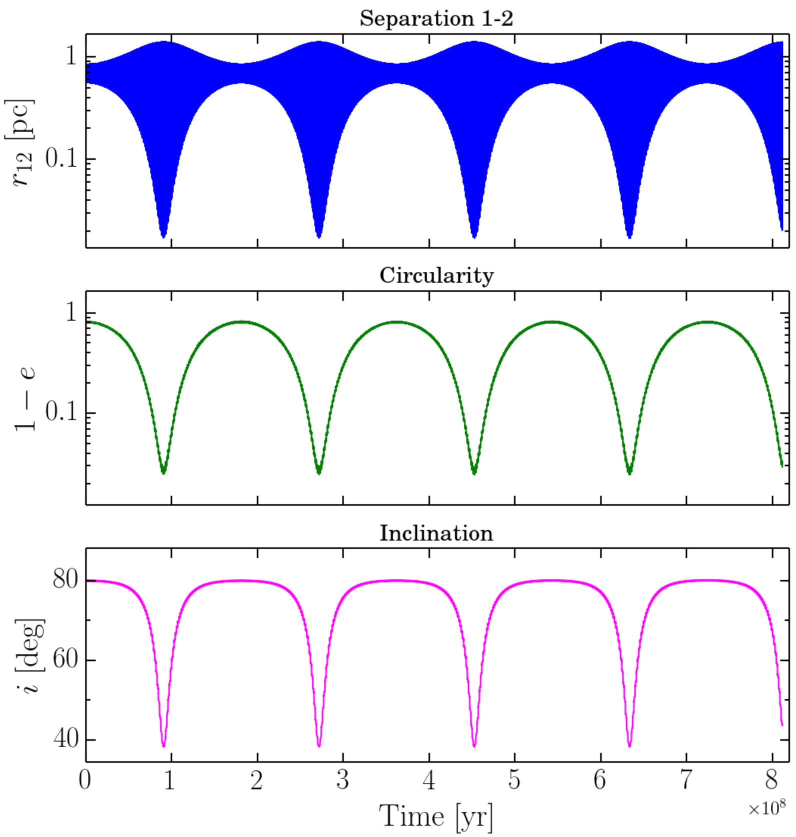
Relativistic
precession!

$$\frac{d\omega}{dt} = \frac{2\pi}{P} \frac{3GM}{c^2 a(1-e^2)^2}$$

Libration: oscillation about constant value
Circulation: monotonic increase from 0 to 2π

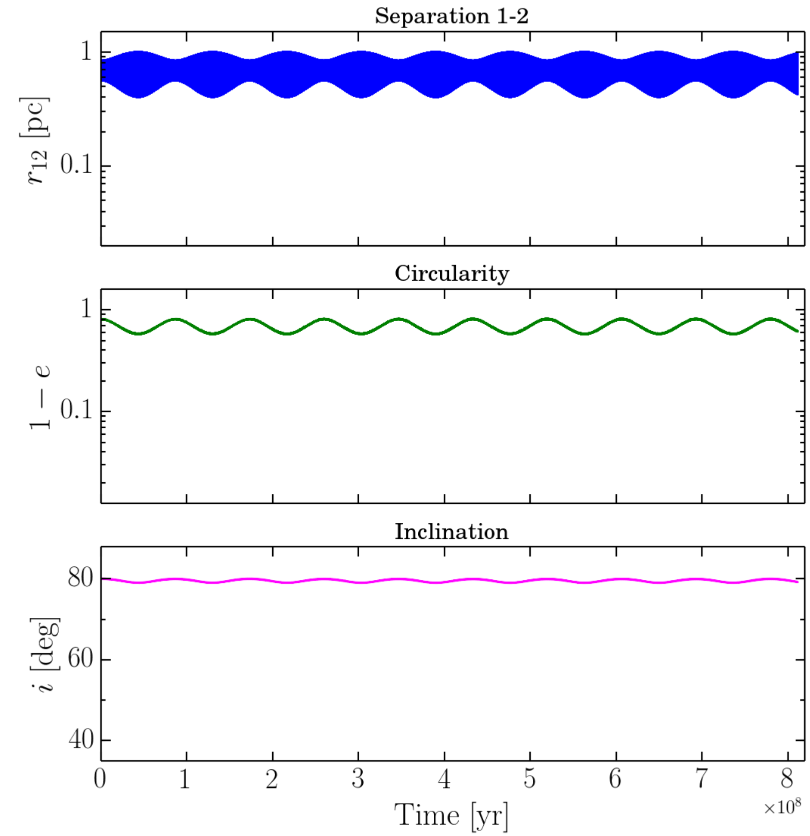
Kozai-Lidov mechanism

Newtonian dynamics



$$m_1 = 10^8$$
$$m_2 = 5 \times 10^7$$
$$m_3 = 2 \times 10^8$$
$$a_{\rm in} = 0.5 \text{ pc}$$
$$e_{\rm in} = 0.2$$
$$a_{\rm out} = 14.8 \text{ pc}$$
$$e_{\rm out} = 0.5$$
$$i = 80^\circ$$

Newtonian dynamics+GR



$$T_{\rm GR} < T_{\rm KL}$$

KL oscillations are strongly suppressed!

Triplets too hierarchical!

$$T_{\rm KL} \approx \frac{2\pi a_{\rm out}^3 (1-e_{\rm out}^2)^{3/2} \sqrt{m_1+m_2}}{G a_{\rm in}^{3/2} m_3}$$

Stellar Hardening

$$\frac{da}{dt} = -\frac{a^2 G\rho}{\sigma} H$$
$$\frac{de}{dt} = \frac{aG\rho}{\sigma} HK$$

Quinlan 1996; Sesana et al. 2006

Problem:
Orbit-averaged equations

Introduction of a fictitious force

Integrating out orbital motion

Stellar Hardening

$$\frac{da}{dt} = -\frac{a^2 G\rho}{\sigma} H$$
$$\frac{de}{dt} = \frac{aG\rho}{\sigma} HK$$

Quinlan 1996; Sesana et al. 2006

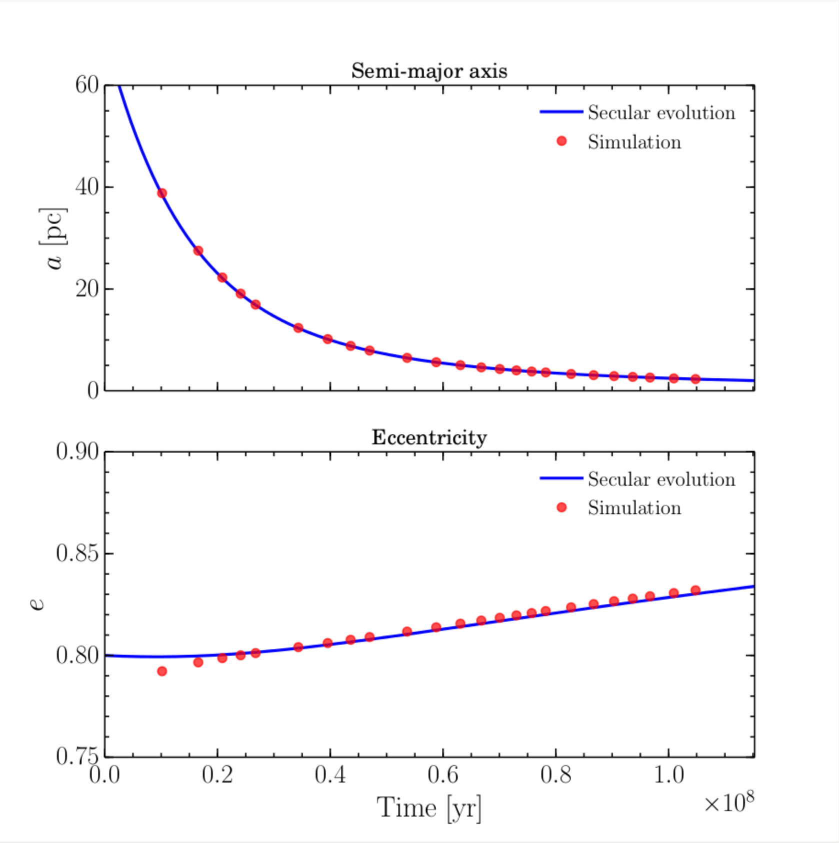
Problem:
Orbit-averaged equations

Introduction of a fictitious force

$$\vec{f} = \underbrace{A \frac{\vec{v}\cdot\vec{r}}{r}\left[\frac{\vec{r}-(\vec{r}\cdot\vec{v})\vec{v}/v^2}{\sqrt{r^2-(\vec{v}\cdot\vec{r})^2/v^2}}\right]}_{\text{Changes eccentricity}} \underbrace{- B rv\,\vec{v}}_{\text{Changes semi-major axis}}$$

Changes eccentricity

Changes semi-major axis



A, B

tuned to macth evolution
predicted by orbit-averaged equations

Hamiltonians

Newtonian order

$$H_0 = \frac{1}{2}\sum_\alpha \frac{|\vec{p}_\alpha|^2}{m_\alpha} - \frac{G}{2}\sum_\alpha \sum_{\beta\neq\alpha} \frac{m_\alpha m_\beta}{r_{\alpha\beta}}$$

1PN order

$$\begin{aligned} H_1 = & -\frac{1}{8}\sum_\alpha m_\alpha \left(\frac{|\vec{p}_\alpha|^2}{m_\alpha^2}\right)^2 \\ & - \frac{G}{4}\sum_\alpha \sum_{\beta\neq\alpha} \frac{1}{r_{\alpha\beta}}\left[6\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2 - 7\vec{p}_\alpha\cdot\vec{p}_\beta - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)\right] \\ & + \frac{G^2}{2}\sum_\alpha \sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha} \frac{m_\alpha m_\beta m_\gamma}{r_{\alpha\beta} r_{\alpha\gamma}} \end{aligned}$$

2.5PN order

$$H_{2.5} = \frac{G}{45}\dot{\chi}_{(4)ij}(\vec{x}_{\alpha'}, \vec{p}_{\alpha'}; t)\chi_{(4)ij}(\vec{x}_\alpha, \vec{p}_\alpha)$$

$$\begin{aligned} H_2 = & \frac{1}{16}\sum_\alpha m_\alpha \left(\frac{|\vec{p}_\alpha|^2}{m_\alpha^2}\right)^3 + \frac{G}{16}\sum_\alpha\sum_{\beta\neq\alpha}\frac{(m_\alpha m_\beta)^{-1}}{r_{\alpha\beta}}\Bigg[10\left(\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2\right)^2 - 11|\vec{p}_\alpha|^2|\vec{p}_\beta|^2 - 2(\vec{p}_\alpha\cdot\vec{p}_\beta)^2 \\ & + 10|\vec{p}_\alpha|^2(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2 - 12(\vec{p}_\alpha\cdot\vec{p}_\beta)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta) - 3(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)^2(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2\Bigg] \\ & + \frac{G^2}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha}\frac{1}{r_{\alpha\beta}r_{\alpha\gamma}}\Bigg[18\frac{m_\beta m_\gamma}{m_\alpha}|\vec{p}_\alpha|^2 + 14\frac{m_\alpha m_\gamma}{m_\beta}|\vec{p}_\beta|^2 - 2\frac{m_\alpha m_\gamma}{m_\beta}(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)^2 \\ & - 50m_\gamma(\vec{p}_\alpha\cdot\vec{p}_\beta) + 17m_\alpha(\vec{p}_\beta\cdot\vec{p}_\gamma) - 14m_\gamma(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta) \\ & + 14m_\alpha(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma) + m_\alpha(\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)\Bigg] \\ & + \frac{G^2}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha}\frac{1}{r_{\alpha\beta}^2}\Bigg[2m_\beta(\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma) + 2m_\beta(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma) \\ & + \frac{m_\alpha m_\beta}{m_\gamma}\big(5(\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})|\vec{p}_\gamma|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{n}_{\alpha\gamma})(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)^2 - 14(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)(\vec{n}_{\alpha\gamma}\cdot\vec{p}_\gamma)\big)\Bigg] \\ & + \frac{G^2}{4}\sum_\alpha\sum_{\beta\neq\alpha}\frac{m_\alpha}{r_{\alpha\beta}^2}\left[\frac{m_\beta}{m_\alpha}|\vec{p}_\alpha|^2 + \frac{m_\alpha}{m_\beta}|\vec{p}_\beta|^2 - 2(\vec{p}_\alpha\cdot\vec{p}_\beta)\right] \\ & + \frac{G^2}{2}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{(n^i_{\alpha\beta}+n^i_{\alpha\gamma})(n^j_{\alpha\beta}+n^j_{\gamma\beta})}{(r_{\alpha\beta}+r_{\beta\gamma}+r_{\gamma\alpha})^2}\Bigg[8m_\beta(p_{\alpha i}p_{\gamma j}) - 16m_\beta(p_{\alpha j}p_{\gamma i}) \\ & + 3m_\gamma(p_{\alpha i}p_{\beta j}) + 4\frac{m_\alpha m_\beta}{m_\gamma}(p_{\gamma i}p_{\gamma j}) + \frac{m_\beta m_\gamma}{m_\alpha}(p_{\alpha i}p_{\alpha j})\Bigg] \\ & + \frac{G^2}{2}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha m_\beta m_\gamma}{(r_{\alpha\beta}+r_{\beta\gamma}+r_{\gamma\alpha})r_{\alpha\beta}}\Bigg[8\frac{\vec{p}_\alpha\cdot\vec{p}_\gamma - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)}{m_\alpha m_\gamma} \\ & - 3\frac{\vec{p}_\alpha\cdot\vec{p}_\beta - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)(\vec{n}_{\alpha\beta}\cdot\vec{p}_\beta)}{m_\alpha m_\beta} - 4\frac{|\vec{p}_\gamma|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\gamma)^2}{m_\gamma^2} - \frac{|\vec{p}_\alpha|^2 - (\vec{n}_{\alpha\beta}\cdot\vec{p}_\alpha)^2}{m_\alpha^2}\Bigg] \\ & - \frac{G^3}{2}\sum_\alpha\sum_{\beta\neq\alpha}\left(\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}} + \frac{1}{2}\sum_{\gamma\neq\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}}\right) \\ & - \frac{3G^3}{8}\sum_\alpha\sum_{\beta\neq\alpha}\left(\sum_{\gamma\neq\alpha}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\alpha\gamma}} + \sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}^2 r_{\beta\gamma}}\right) \\ & - \frac{3G^3}{8}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}r_{\alpha\gamma}r_{\beta\gamma}} \\ & - \frac{G^3}{64}\sum_\alpha\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}\frac{m_\alpha^2 m_\beta m_\gamma}{r_{\alpha\beta}r_{\alpha\gamma}^3 r_{\beta\gamma}}\Bigg[18r_{\alpha\gamma}^2 - 60r_{\beta\gamma}^2 - 24r_{\alpha\gamma}(r_{\alpha\beta}+r_{\beta\gamma}) \\ & + 60\frac{r_{\alpha\gamma}r_{\beta\gamma}^2}{r_{\alpha\beta}} + 56r_{\alpha\beta}r_{\beta\gamma} - 72\frac{r_{\beta\gamma}^3}{r_{\alpha\beta}} + 35\frac{r_{\beta\gamma}^4}{r_{\alpha\beta}^2} + 6r_{\alpha\beta}^2\Bigg] - \frac{G^3}{4}\sum_\alpha\sum_{\beta\neq\alpha}\frac{m_\alpha^2 m_\beta^2}{r_{\alpha\beta}^3}. \end{aligned}$$

2PN order