

Primordial Gravitational Waves & eLISA Mission

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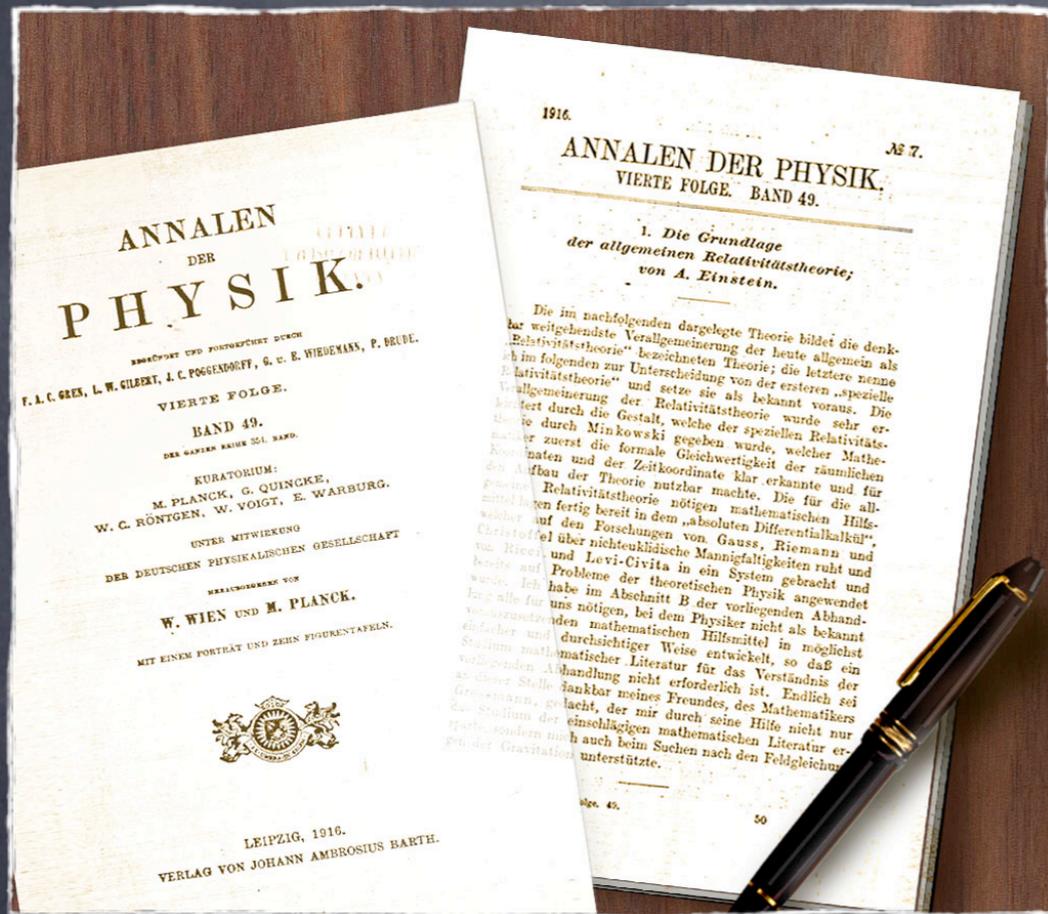
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XI Lisa Symposium- Zurich

OUTLINE

-  Introduction
-  Inflation and Primordial GW
-  Inflation and “Beyond” with eLISA
-  Conclusions

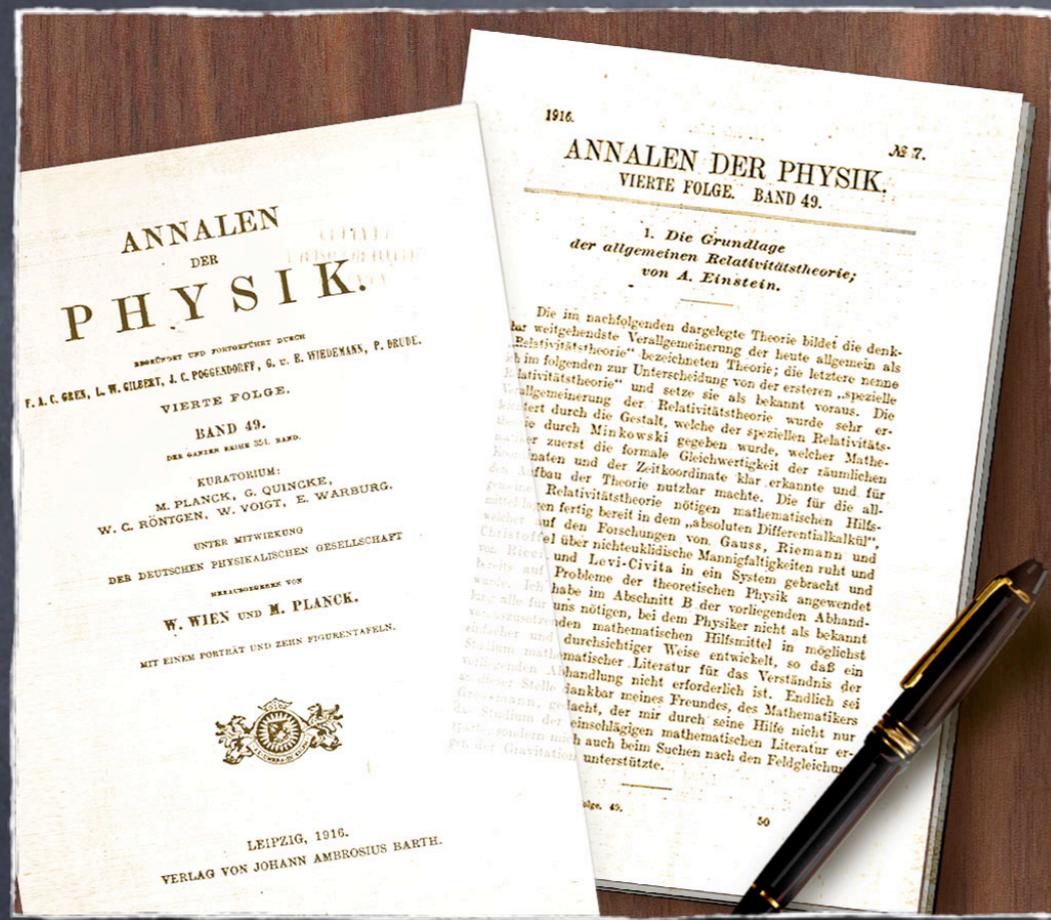
GR & Gravitational Waves



GR & Gravitational Waves

Predictions

- Geodesic Deviation
- Frame Dragging
- Gravitational Lensing
- Black Holes
- Gravitational Redshift
- Gravitational Waves



In 1916 Einstein predicted the existence of **gravitational waves**, since his linearized weak-field equations had wave solutions: **transverse waves of spatial strain** that travel at speed of light, generated by time variations of the mass quadrupole moment of the source

Why GW are important?



- Test better General Relativity

- Give information on the quantum nature of gravity

- GW Speed

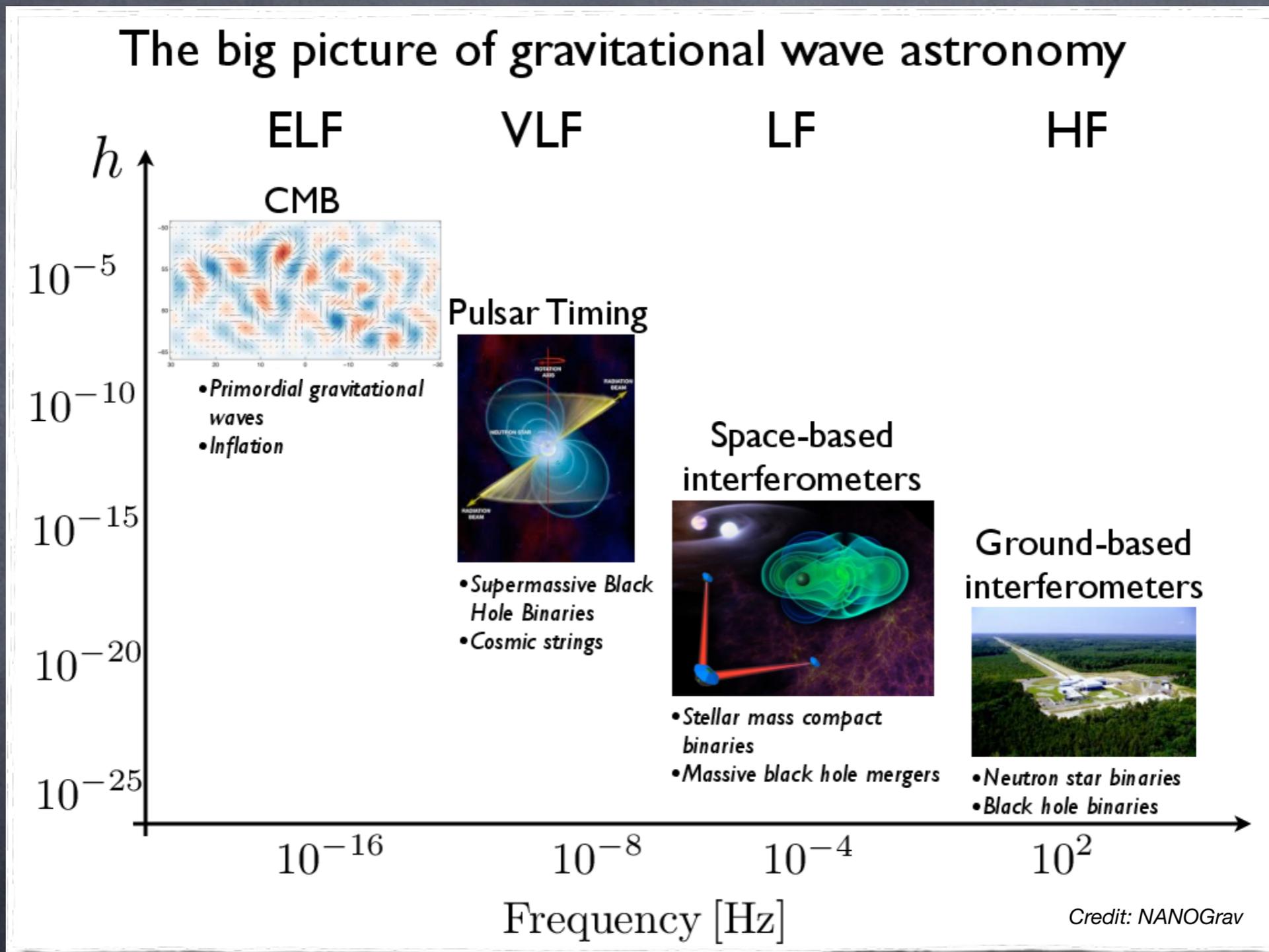
- Graviton mass

- Study compact objects and their properties

- Deep understanding of the physics of the early universe

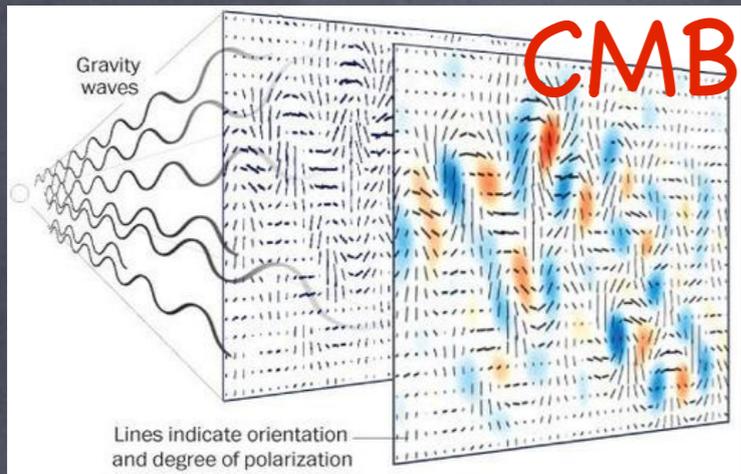
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The spectrum of GW



Many sources => Many detectors

"Indirect" vs Direct GW detection



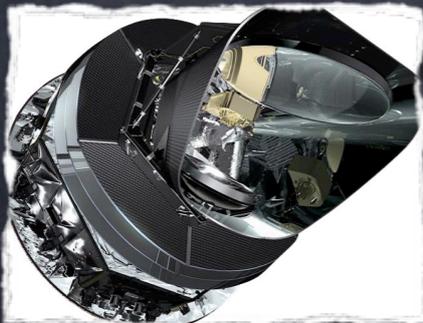
Tensor anisotropies
on last scattering surface

Polarization of CMB photons
through Thomson scattering
of electron and photon

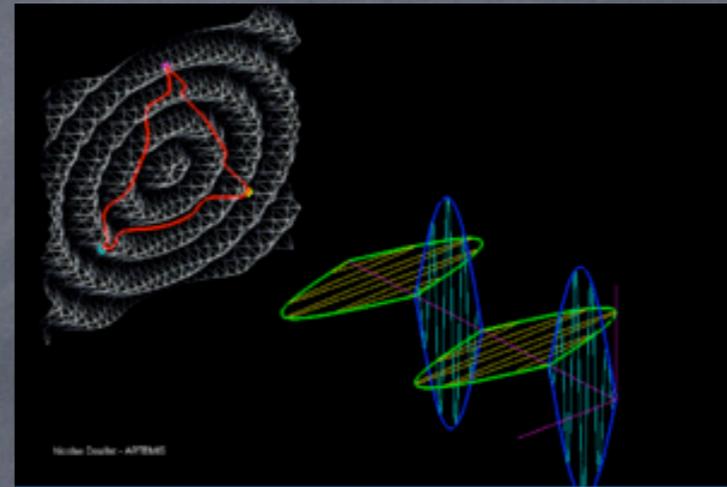
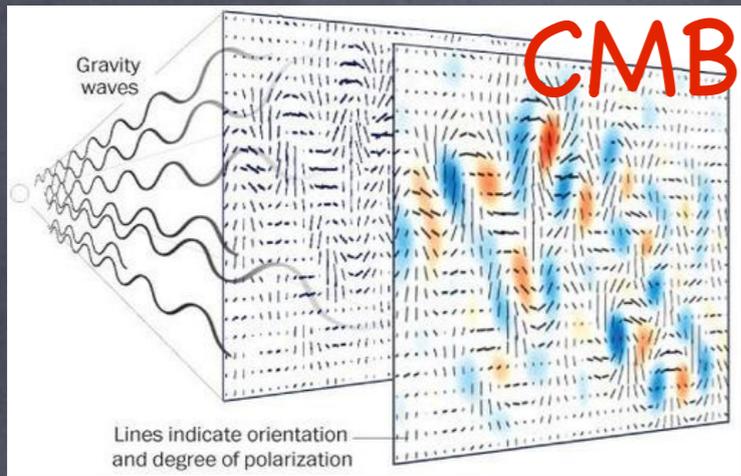
**Only Tensor perturbations
can source B-mode**

Poor and contaminated signal:

- foregrounds
- gravitational lensing (E→B at small scales)



"Indirect" vs Direct GW detection



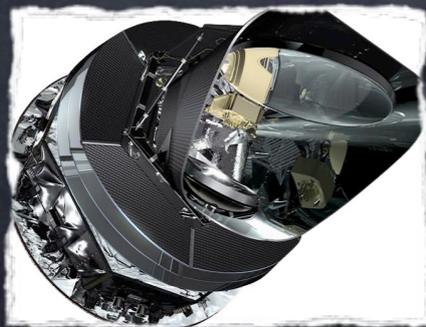
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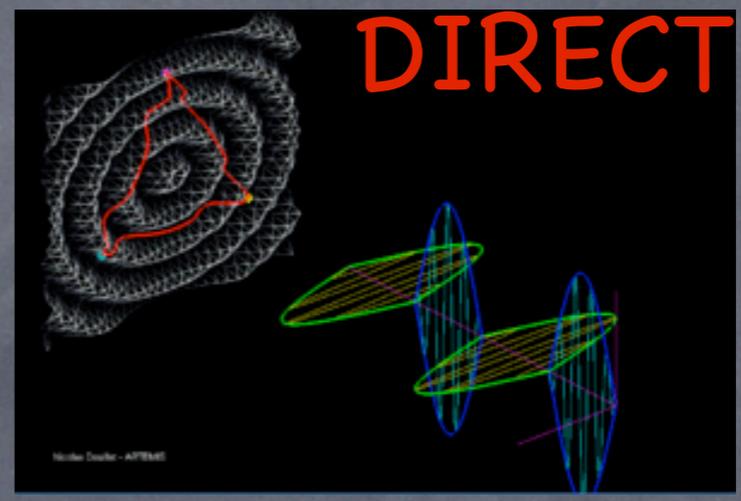
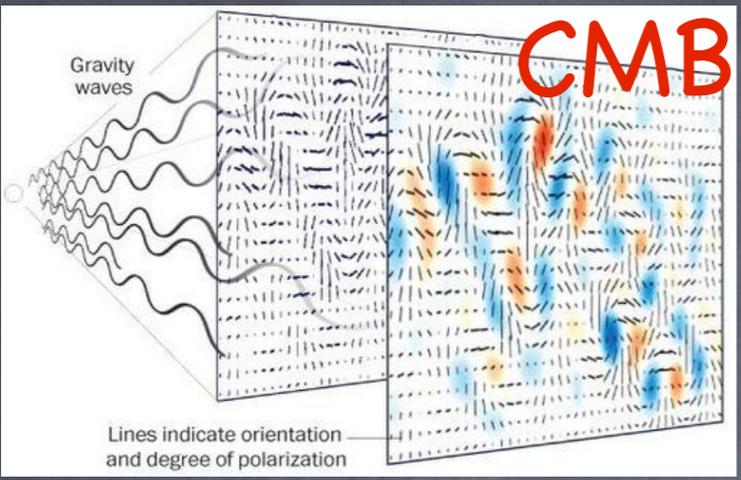
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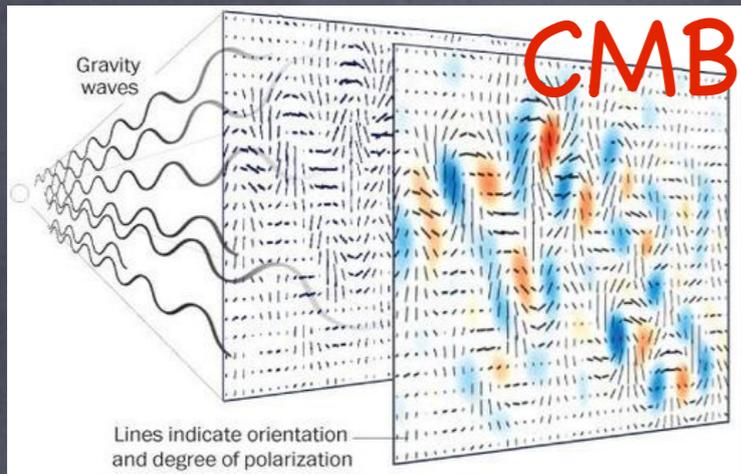
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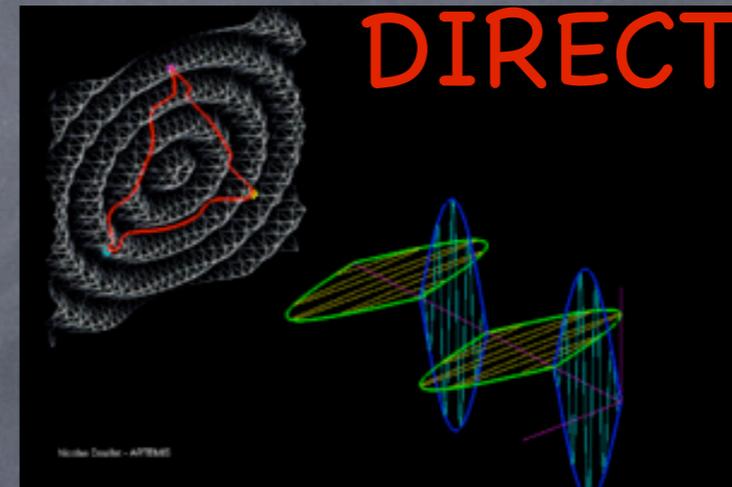
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GW travels freely until today

**Distortion of space as GW
passes detector arms**

- ground-based interferometers
- space-based interferometers
- pulsar timing arrays



GW sources and eLISA scientific goals

Astrophysics

- MBHBs
- EMRIs
- Compact WD

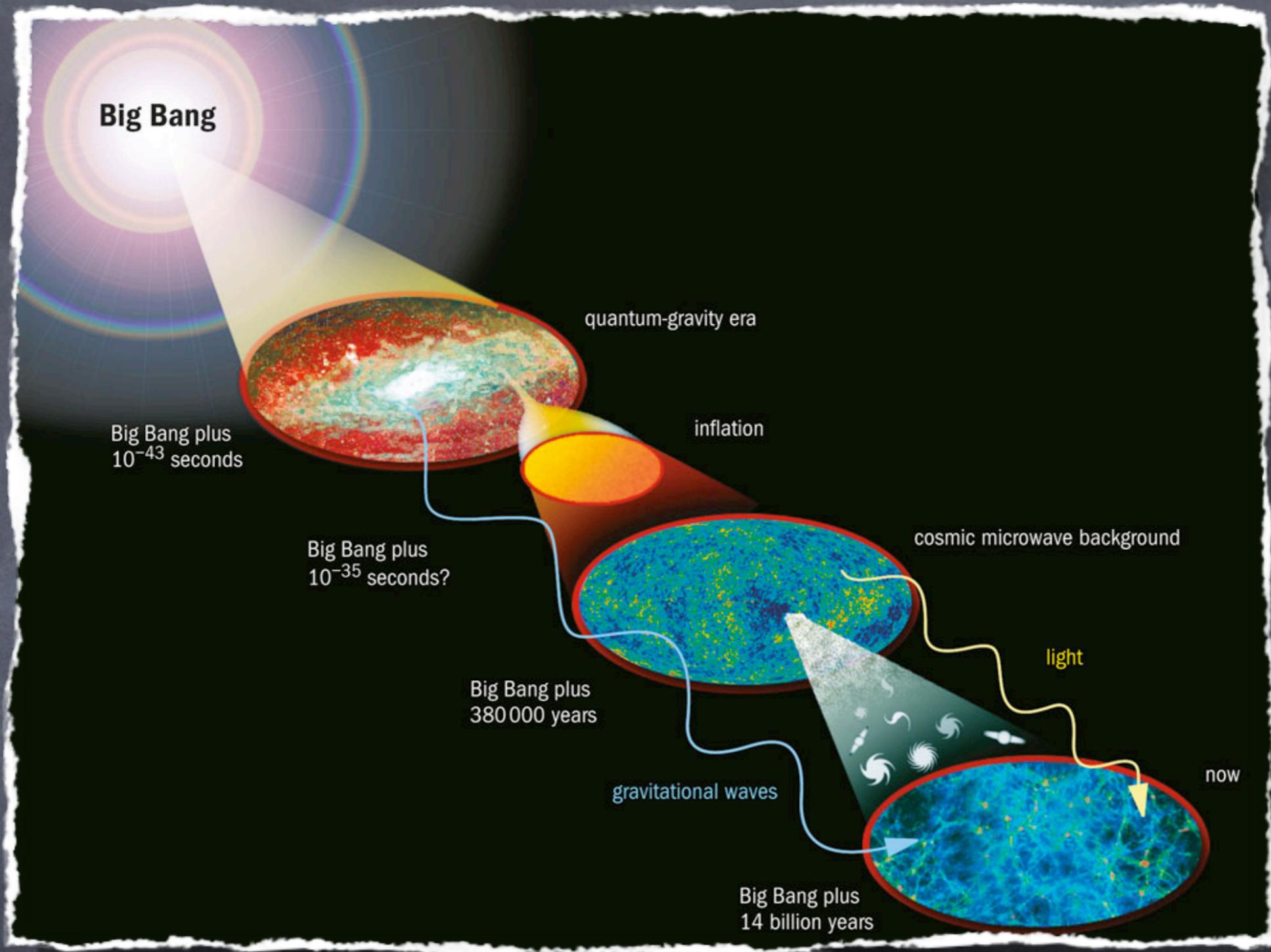
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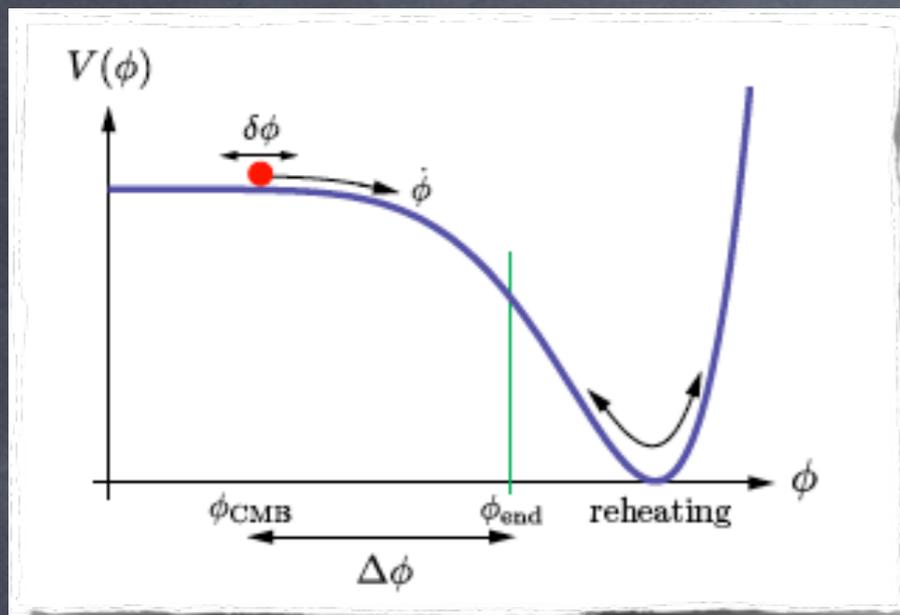
Cosmology

- EW Phase Transition BSM
- Other first-order PT
- Topological Defects
- Inflation and Beyond
- Standard sirens



Primordial GWs are out of equilibrium since the Planck scale (photons at 0.3 eV) so they carry information about the universe at really high energies

Inflation and Primordial GW



- Period of accelerated (exponential) expansion driven by a scalar field (inflaton) that rolls down on its flat potential

Solve Standard Big-Bang shortcomings

Generation of perturbations

Stretches the microphysics scales to super-horizon sizes

GW are represented by tensor perturbation h_{ij} of the FRW metric

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

Transverse and traceless $\partial_i h_{ij} = h_{ii} = 0 \rightarrow 2 \text{ D.O.F}$
(2 polarizations)

Dynamics governed by linearized Einstein eq:

$$\tilde{h}''_{ij}(\mathbf{k}, t) + \left(k^2 - \frac{a''}{a} \right) \tilde{h}_{ij}(\mathbf{k}, t) = 0$$

$\sim a^2 H^2$

Solutions

Sub-Horizon $\Rightarrow k \gg aH$: $h_{ij} \sim \cos(\omega\tau)/a$

Super-Horizon $\Rightarrow k \ll aH$: $h_{ij} \sim \text{const}$

Observational quantity on the CMB

$$\langle h(\mathbf{k}) h^*(\mathbf{k}') \rangle = \frac{2\pi^2}{k^3} \delta^{(3)}(\mathbf{k} + \mathbf{k}') P_h(\mathbf{k})$$

Single field slow-roll
Tensor Power Spectrum

$$P_T(k) = \frac{8}{M_{Pl}^2} \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{aH} \right)^{n_T}$$

$$n_T = -2\epsilon$$

$$\epsilon \equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2$$

Consistency relations

$$P_{S,T} = A_{S,T} \left(\frac{k}{k_*} \right)^{(n_s-1, n_t)+\dots}$$

$$A_S = \left(\frac{H^2}{2\pi\dot{\varphi}} \right)^2$$

$$A_T = \frac{8}{M_{Pl}^2} \left(\frac{H}{2\pi} \right)^2$$

$$r \equiv \frac{A_T(k_*)}{A_S(k_*)} = 16\epsilon = -8n_T$$

Importance of measuring the Tensor PS (at different scales)

Importance of measuring Violation of CR

$$r_{0.05} < 0.07 \quad (95\%CL) \quad \text{Bicep2/Keck +95GHz}$$

$$\frac{\Delta\varphi}{M_{Pl}} \gtrsim \mathcal{O}(1) \left(\frac{r}{0.01} \right)^{1/2}$$

Lyth Bound

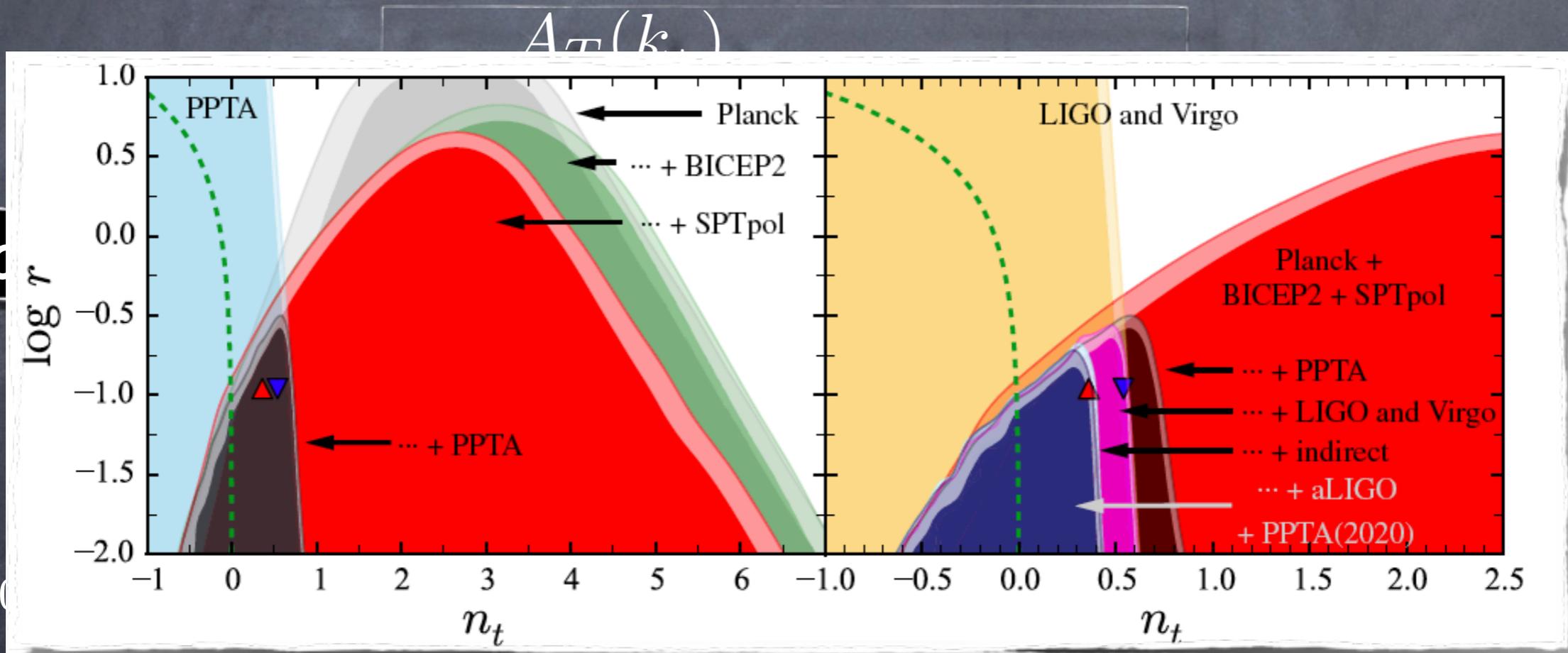
$$V = (1.88 \times 10^{16} \text{GeV})^4 \frac{r}{0.1}$$

Consistency relations

$$P_{S,T} = A_{S,T} \left(\frac{k}{k_*} \right)^{(n_s - 1, n_t) + \dots}$$

$$A_S = \left(\frac{H^2}{2\pi\dot{\varphi}} \right)^2$$

$$A_T = \frac{8}{M_{Pl}^2} \left(\frac{H}{2\pi} \right)^2$$



Impo

cales)

$$n_t = A \log_{10} \left(\frac{r}{0.11} \right) + B,$$

| Experiment | A | B |
|------------------------------|-------|------|
| CMB + PPTA | -0.13 | 0.68 |
| CMB + PPTA + LIGO | -0.06 | 0.54 |
| CMB + PPTA + LIGO + indirect | -0.04 | 0.36 |
| CMB + PPTA(2020) + aLIGO | -0.06 | 0.34 |

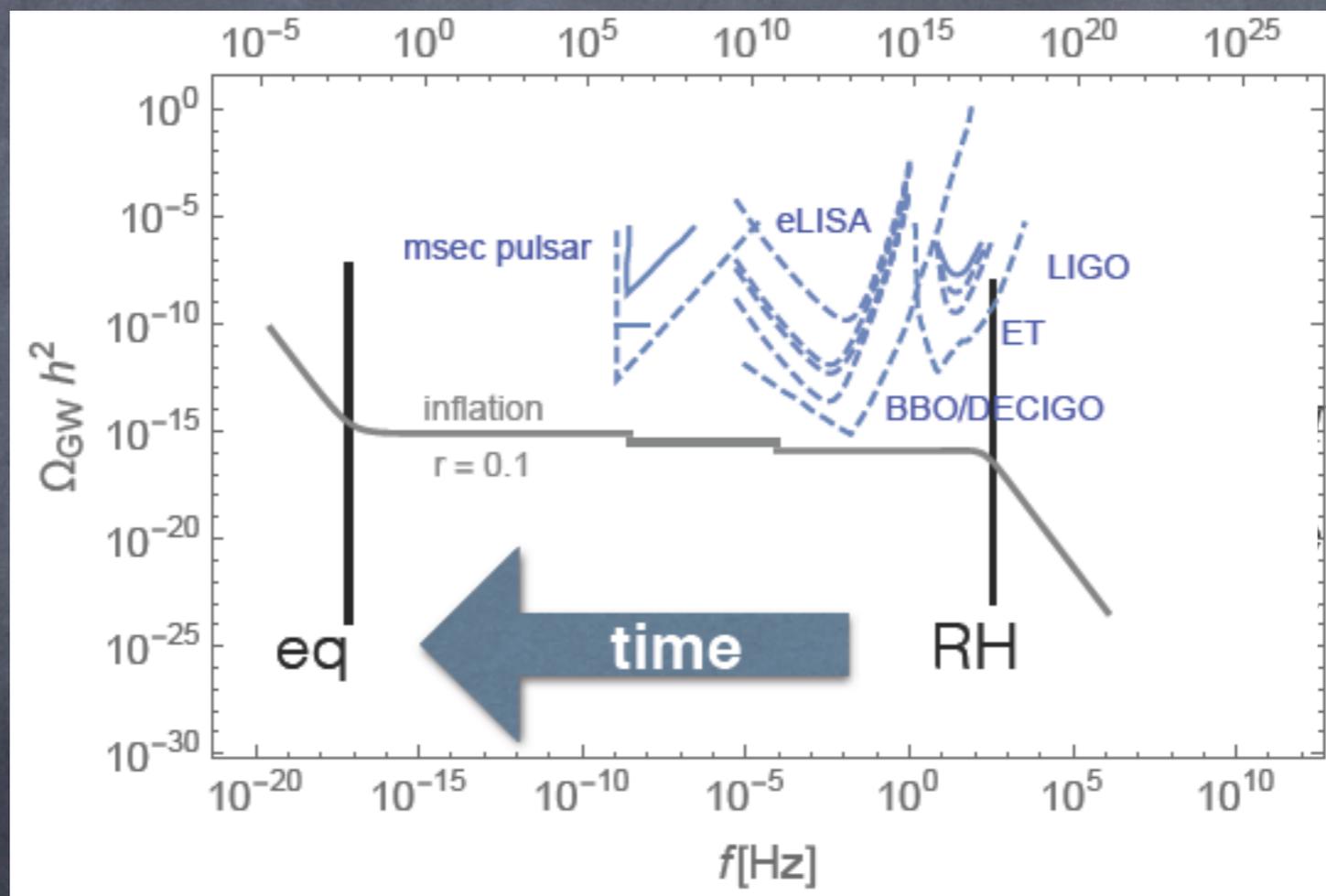
Inflationary GWs generated by the amplification of the vacuum fluctuations,
 have an amplitude **OUT** of eLISA range

$$h_0^2 \Omega_{\text{gw}}(f) \approx 5 \cdot 10^{-16} \left(\frac{H}{H_{\text{max}}} \right)^2$$

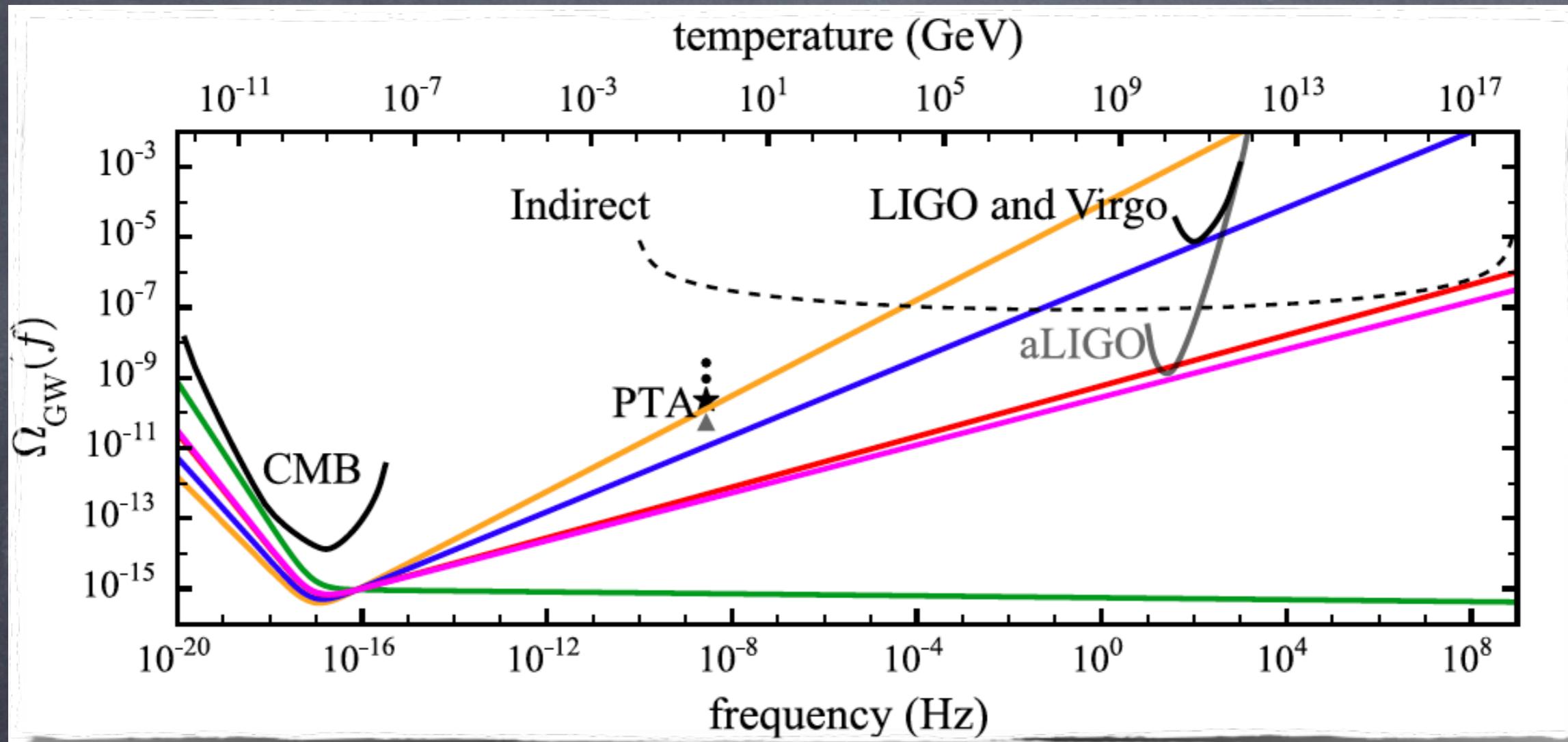
$$H_{\text{max}} \simeq 8.8 \times 10^{13} \text{ GeV}$$

current upper bound on energy scale of inflation

Planck 2015 results. XX



Current constraints on GW energy density



[P. D. Lasky et al., (1511.05994)]

Inflationary GWs and eLISA

$$\tilde{h}''_{ij}(\mathbf{k}, t) + \left(k^2 - \frac{a''}{a} \right) \tilde{h}_{ij}(\mathbf{k}, t) =$$

Inflationary GWs and eLISA

$$\tilde{h}''_{ij}(\mathbf{k}, t) + \left(k^2 - \frac{a''}{a} \right) \tilde{h}_{ij}(\mathbf{k}, t) = 16\pi G a \Pi_{ij}^{TT}(\mathbf{k}, t)$$

Π_{ij}^{TT} transverse-traceless part of the anisotropic stress Π_{ij} ($a^2 \Pi_{ij} = T_{ij} - p a^2 (\delta_{ij} + h_{ij})$)

The processes that give rise to a **non-zero** tensor **anisotropic stress**
in the Early Universe can directly **source GW**
potentially detectable by eLISA

Inflationary GWs and eLISA

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The processes that give rise to a **non-zero** tensor **anisotropic stress** in the Early Universe can directly **source GW** potentially detectable by eLISA

GW Energy Density $\rho_{gw} = \frac{\langle \dot{h}_{ij} \dot{h}_{ij} \rangle}{32\pi G} = \int \frac{df}{f} \frac{d\rho_{gw}}{d \log f}$

Present-day GW frequency $f = \frac{k}{2\pi} \frac{a}{a_0}$

Energy density per log frequency interval

$$h^2 \Omega_{GW}(f) = \frac{h^2}{\rho_c} \frac{d\rho_{gw}}{d \log f}$$

Inflationary setup

Second order GWs

Particle production during inflation

(see M. Pieroni's talk)

Spectator fields

EFT of broken space diff

Inflationary PT

Post-Inflationary setup

GW from (p)reheating

Thermal background

Kination-domination

Merging of primordial BHs

(see J. G. Bellido's talk)

Alternatives to Inflation

String Cosmology

Pre-Big-Bang models

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Particle production during inflation

[J. Cook, L. Sorbo (arXiv:1109.0022)]

[N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

Axion-inflation model

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{4}F^2 - \frac{\varphi}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\xi \equiv \frac{\dot{\varphi}}{2fH}$$

φ inflaton=pseudo-scalar axion

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}_{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

The rolling inflaton excites, through the coupling, quanta of EM field



A_+ is exponentially amplified as ξ becomes large (>0), while A_- has no amplification

by the rolling field φ

$$(A_+ \propto e^{\pi\xi})$$



The production of gauge quanta prolongs inflation because it sources inflaton perturbations through the inverse decay $\delta A + \delta A \rightarrow \varphi$



EM field sources also tensor fluctuations(GW) $\delta A + \delta A \rightarrow \delta g$

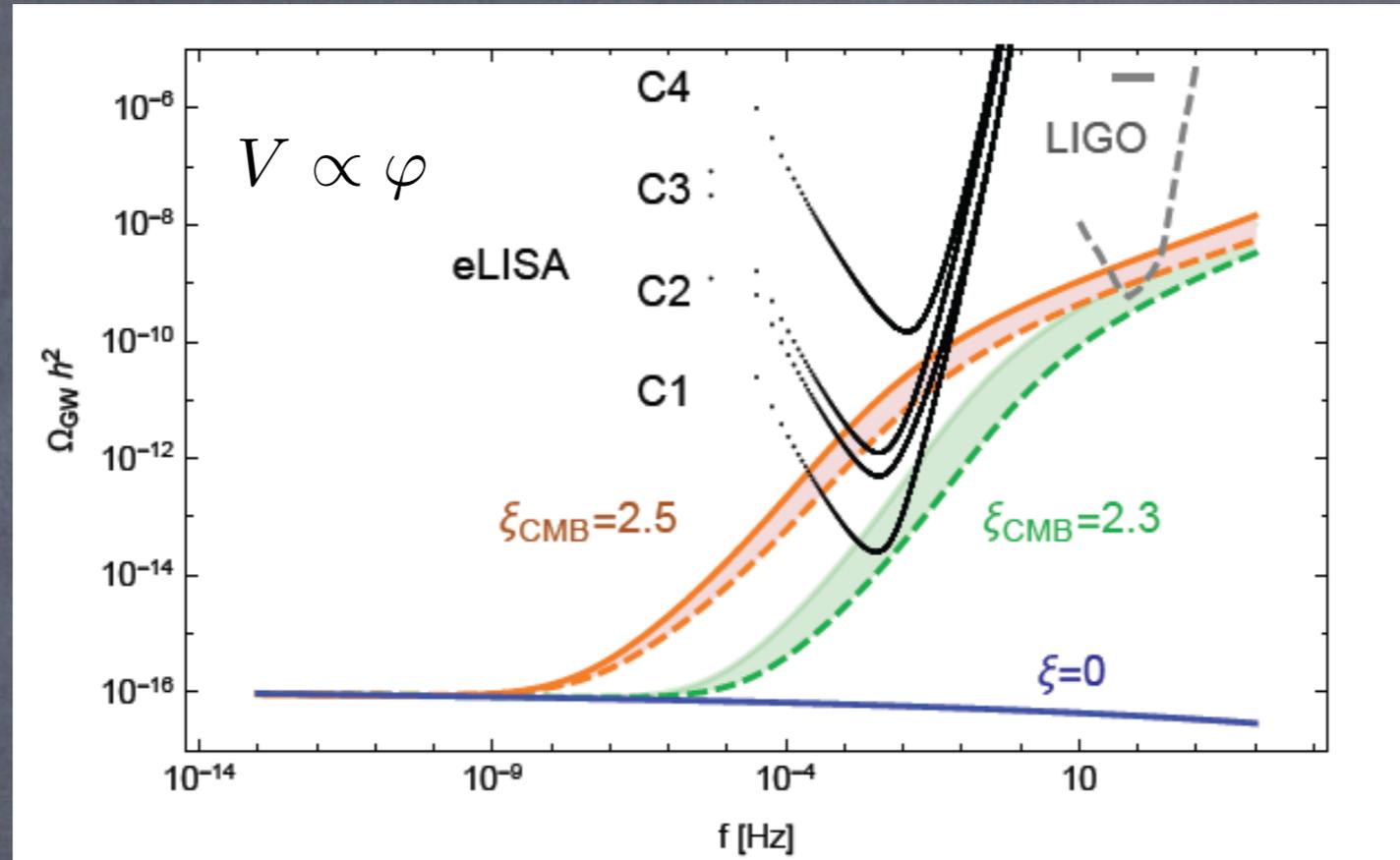
$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = -\frac{2a^2}{M_P^2}(E_i E_j + B_i B_j)^{TT}$$

"Magnetic field"

$$\mathbf{B} = \frac{1}{a^2} \nabla \times \mathbf{A}$$

"Electric field"

$$\mathbf{E} = -\frac{\mathbf{A}'}{a^2}$$



| Name | C1 | C2 | C3 | C4 |
|------------------|----------|----------|----------|---------------------|
| Full name | N2A5M5L6 | N2A1M5L6 | N2A2M5L4 | N1A1M2L4 |
| # links | 6 | 6 | 4 | 4 |
| Arm length [km] | 5M | 1M | 2M | 1M |
| Duration [years] | 5 | 5 | 5 | 2 |
| Noise level | N2 | N2 | N2 | N1 |

after LISA PathFinder

N2= LPF expected
N1= 10 times LPF expected

Chiral Gravitational Waves signal

Useful to distinguish origin of the signal

High scalar non-Gaussian contribution

Limits from the CMB $\xi < 2.5$

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F^2 - \alpha_2 \frac{\chi}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

φ inflaton

χ pseudo-scalar field

$$\xi \equiv \frac{\alpha_2 \dot{\chi}}{2fH}$$

Since there is no direct coupling between the inflaton and the axion the inflaton perturbations, also sourced by the gauge field, are negligible wrt the gravity wave production.

Tensor contribution amplified by ξ

$$\delta A + \delta A \rightarrow \delta\chi$$

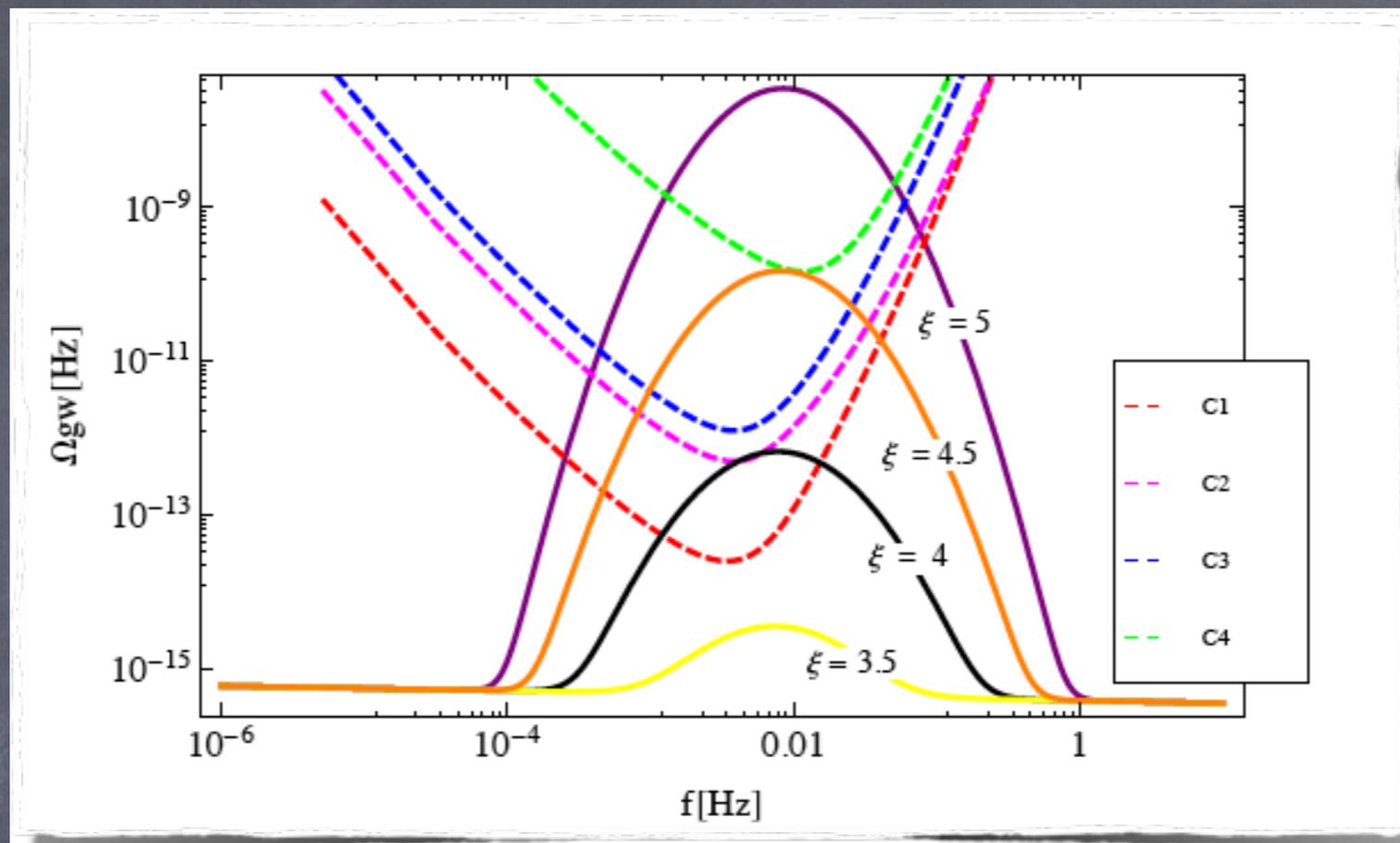
~~$$\delta A + \delta A \rightarrow \delta\varphi$$~~ and
$$\delta A + \delta A \rightarrow \delta h$$

Since there is no direct coupling between χ and φ

$$\delta A + \delta A \rightarrow \delta\varphi \sim \text{negligible}$$

The model produces:

- Large gravitational wave signal (observable B modes)
- Sufficiently small scalar perturbations
(in agreement with CMB constraints)



Very model dependent

The bump depends on the dynamics of χ

Possibility to test the inflaton field at the latest stage of inflation, for which we have very poor information

Possibility to test the inflaton coupling(s)

Peculiar Features

Parity violating signal

$$\Delta\chi = \frac{P_T^+ - P_T^-}{P_T^+ + P_T^-} \propto \xi$$

High tensor CMB non-Gaussian signal

consistency
relations

What about measuring
PARITY VIOLATION and NON-GAUSSIANITY
with eLISA?

[S. G. Crowder et al. (arXiv:1212.4165)]

[N. Barnaby et al. (arXiv:1206.6117)]

[N. Bartolo et al. (arXiv:1505.02193)]

Spectator Field

[M. Biagetti et al., (1305.7241)]

[M. Biagetti et al., (1411.3029)]

[T. Fujita et al., (1411.3658)]

$$\mathcal{L} = \frac{1}{2} M_{Pl}^2 R - \frac{1}{2} (\partial\varphi)^2 - V(\varphi) + P(X, \sigma)$$

$$X = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma$$

spectator responsible only for perturbations

$$c_s \equiv \frac{P_X}{(P_X + P_{XX} \dot{\sigma}_0^2)} \neq 0$$

$$s \equiv \frac{H \dot{c}_s}{c_s} \neq 0$$

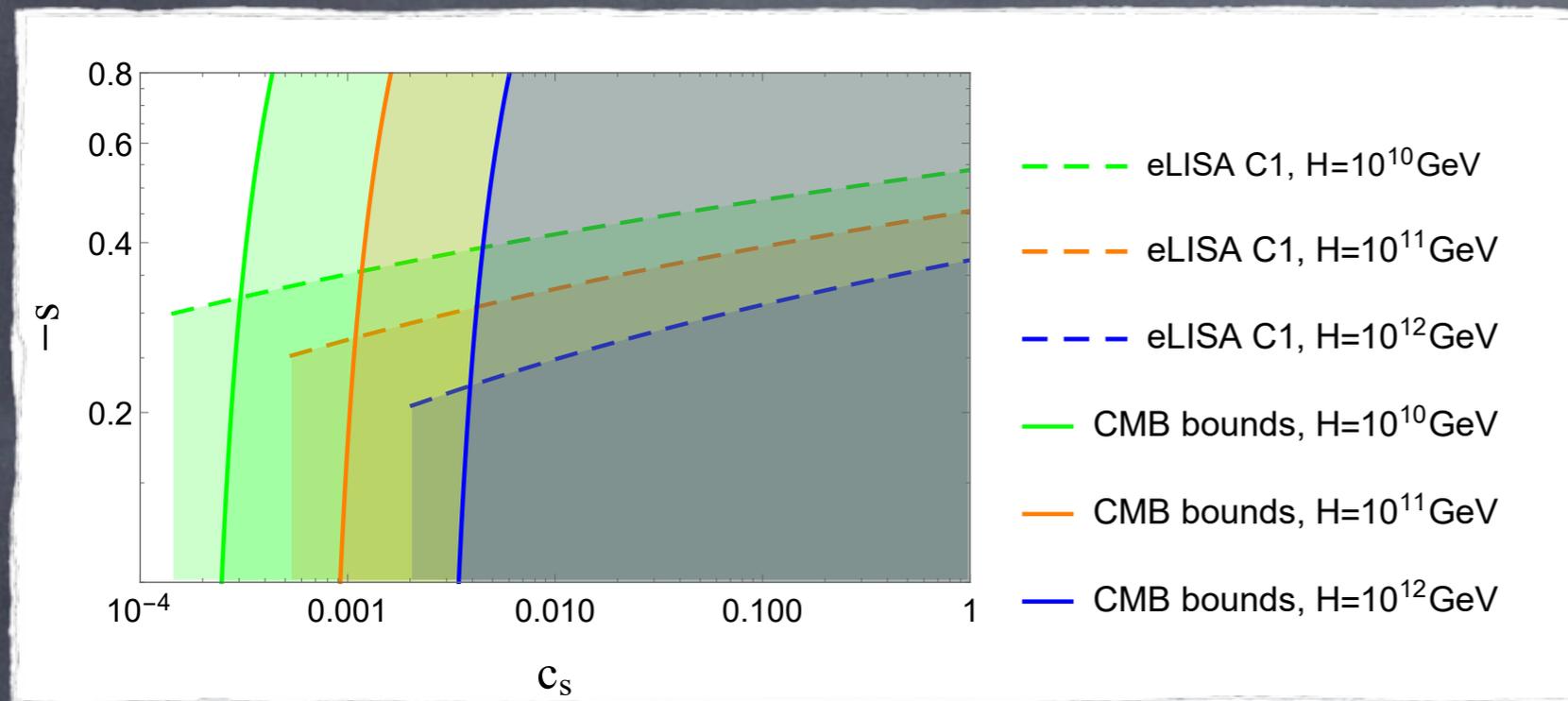
$$P_R = P_R^{(v)} + P_R^{(\sigma)} \simeq \frac{H^2}{4\epsilon M_{pl}^2} + \frac{1}{32\pi c_s^7} \frac{H^4}{M_{pl}^4}$$

$$P_t = P_t^{(v)} + P_t^{(\sigma)} \simeq \frac{2H^2}{M_{pl}^2} + \frac{8}{15\pi c_s^3} \frac{H^4}{M_{pl}^4}$$

$$n_T^{(\sigma)} \simeq -4\epsilon - 3s$$

$$n_S^{(\sigma)} \simeq -4\epsilon - 7s$$

From a **NON DETECTION** of primordial GW by eLISA we can put a limit on the spectral index of the sourced GW, for a given value of the sourced GW amplitude on CMB scales

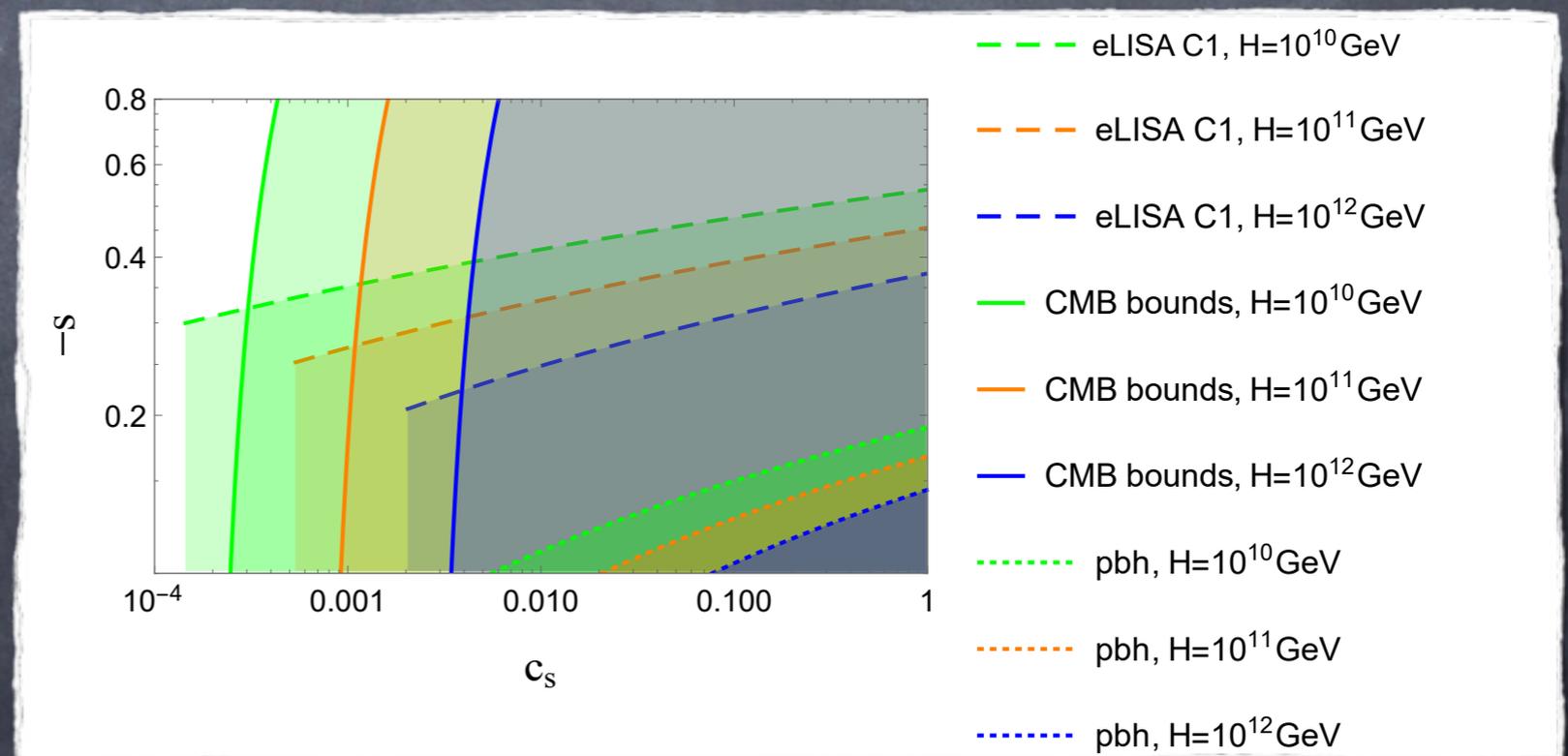


**PRELIMINARY
ANALYSIS**

$$A_{0.05}^{(S)} = 2.21 \times 10^{-9} \quad (65\% CL) \quad [\text{Planck 2013 XVI}]$$

$$\epsilon = 0.0068 \quad (95\% CL) \quad (\text{PlanckTT} + \text{lowP})$$

$$r_{0.05} < 0.09 \quad (95\% CL) \quad [\text{BICEP2} / \text{Keck Array VI}]$$



EFT of broken space diffeo

[N. Bartolo, D. Cannone, A. R., G. Tasinato (arXiv:1511.07414)]

[D. Cannone, G. Tasinato, D. Wands (arXiv:1409.6568)]

General Relativity \Rightarrow invariance under $x^\mu \rightarrow x^\mu + \xi^\mu(x^\nu)$

During inflation $t \rightarrow t + \xi(x^\mu)$ is broken

What happens if $x^i \rightarrow x'^i(t, x^j)$ is broken? ($\phi = \phi(x^i)$)

If space-diffeo are broken the graviton can acquire an **effective mass** and an effective sound speed during inflation

$$S_h = \frac{M_{Pl}^2}{4} \int d\eta d^3x a^2(\eta) \left\{ (h'_{ij})^2 - c_T^2 (\partial_l h_{ij})^2 - m^2 h_{ij}^2 \right\}.$$

Tensor Power Spectrum

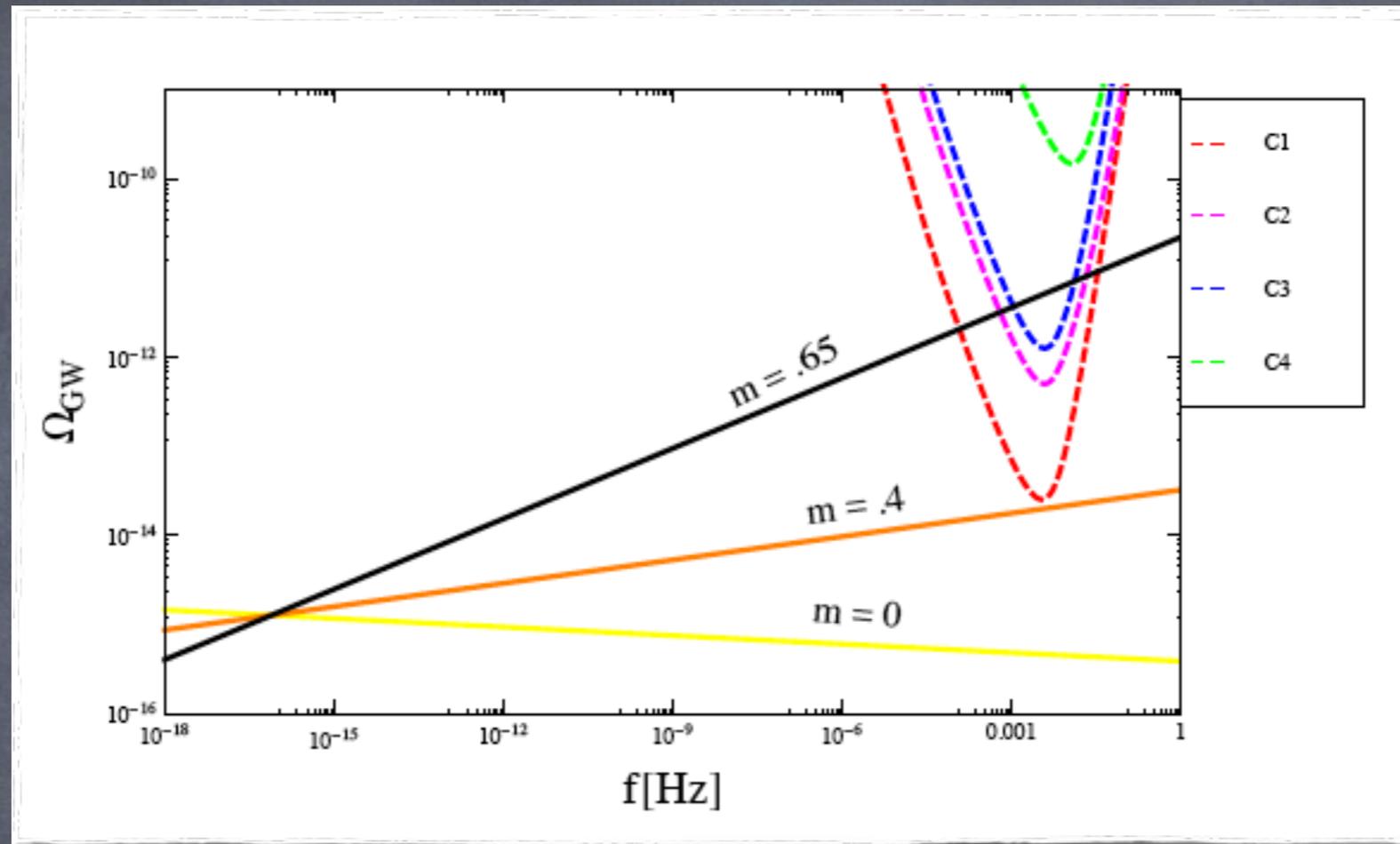
$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M_{Pl}^2 c_T} \left(\frac{k}{k_*} \right)^{n_T}$$

Spectral index

$$n_T = -2\epsilon + \frac{2}{3} \frac{m^2}{H^2} \left(1 + \frac{4}{3} \epsilon \right)$$

We can generate a **blue tensor spectrum** w/o violating Null Energy Condition

A "sufficiently" blue tensor spectrum can be detectable by eLISA



Limits on
tensor spectral
index



Limits on
(effective)
graviton mass

$$m_g \leq 1.2 \times 10^{-22} \text{ eV}/c^2 \quad (90\% \text{ CL})$$

GW from post-inflationary processes

(p)reheating through parametric effects

Resonance parameter

$$q \sim \frac{g^2 \Phi_*^2}{\omega^2}$$

g (coupling constant)

Φ_* (initial amplitude of the inflaton)

$\omega^2 \equiv V''$ (frequency of the oscillations)

$$g^2 \phi^2 \chi^2$$

(scalar)

$$g^2 \phi^2 A_\mu A^\mu$$

(vector)

$$g \phi \bar{\psi} \psi$$

(fermion)

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(fermion)

Amplitude

$$\Omega_{\text{GW}} \propto q^p \propto g^{2p}$$

$$p \sim \mathcal{O}(1)$$

Peak frequency

$$f \propto g^{1/2}$$

These scenarios predict a bg of GW with very large amplitude peaked at very high frequencies

$$h^2 \Omega_{\text{GW}}^{(\text{peak})} \lesssim 10^{-11} \quad f \gtrsim 10^8 \quad (\text{OUT OF eLISA RANGE})$$

(p)reheating through spinodal instabilities

Peak frequency and Amplitude are decoupled

The vacuum energy of the waterfall field v_* controls the amplitude

Amplitude

$$\Omega_{GW} \propto v_*$$

Peak frequency

$$f \propto \lambda^{1/2}$$

λ self-coupling of the waterfall field

(p)reheating through spinodal instabilities

Peak frequency and Amplitude are decoupled

The vacuum energy of the waterfall field v_* controls the amplitude

Amplitude

$$\Omega_{GW} \propto v_*$$

Peak frequency

$$f \propto \lambda^{1/2}$$

λ self-coupling of the waterfall field

In order to be in the eLISA sensitivity range of frequency and amplitude the coupling constant must be

$$\lambda \lesssim \mathcal{O}(10^{-28})$$

very unnatural

Kination-Domination

[B. Spokoiny, Phys. Lett. B **315**, 40 (1993) [gr-qc/9306008]]

“Stiffness” period after inflation

$$w = (K - V)/(K + V) \simeq +1$$

If a kination-domination period lasts sufficiently long, from the end of inflation until somewhere just before BBN, it is in principle possible that the, otherwise slightly red-tilted inflationary spectrum of GW, becomes highly blue-tilted, becoming a target for the eLISA mission

It does not affect the modes that affect the CMB

Summary of the inflationary scenarios

| Model \ Effect | Blue Tilted | Single Peak | Other Effects | eLISA Detect. |
|---------------------------------------|-------------|-------------|----------------------|-----------------------------|
| Vacuum Ampl. | × | × | × | × |
| Second order GW | ✓ | × | ? | × |
| Particle production (gauge fields) | ✓ | ✓ | Parity Violation | ✓ |
| Spectator Field | ✓ | × | × | ✓ |
| EFT of Broken Diff. | ✓ | ✓ | × | ✓ |
| Inflation Ph. Tr. | × | ✓ | Voids generation (?) | ✓ |
| (p)Reheating | × | ✓ | Anisotropies | ✓ (very fine-tuned only) |
| Thermal Backg. | × | ✓ | × | × |
| Kination-domination (stiff phase) | ✓ | × | × | ? |
| PBH after Inflation | × | ✓ | DM candidates | ✓ |
| String Cosmology | ✓ | × | × | × |
| Pre-Big-Bang models | ✓ | × | × | ? |

Conclusions

The aLIGO detection officially open the decade of GWs

GWs allow to test energy scale not accessible at collider

Primordial GW gives information on the early Universe

The complementarity of CMB and direct GW measurements (eLISA) provides a powerful probe of the physics of cosmic inflation (Tensor Spectral tilt)

Consistency relations

“Inflationary” physics with eLISA:

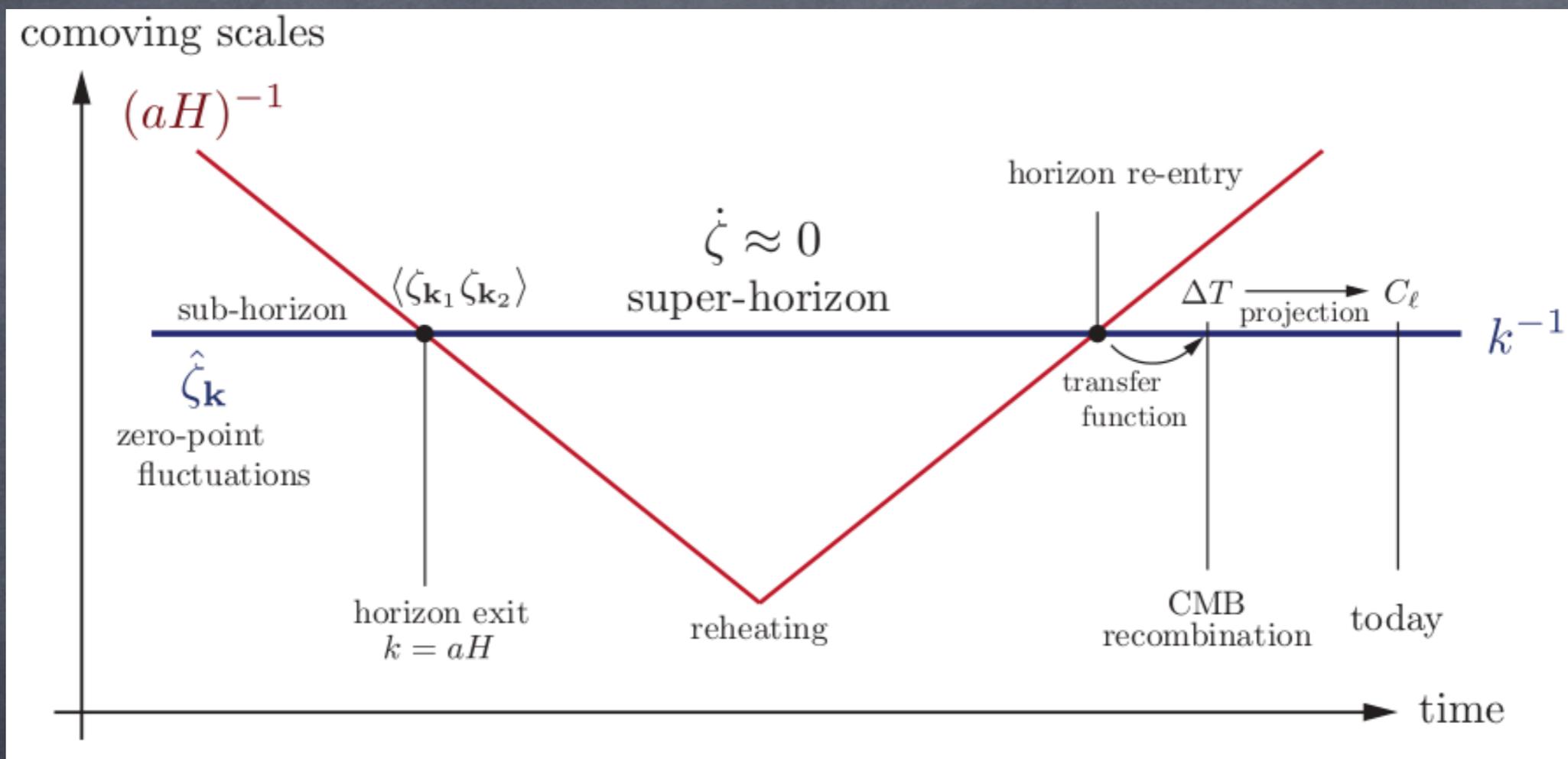
Possibility to test latest stage of inflation and possible couplings

The NON Detection of GW constrains cosmological parameters

Possibility to test new pattern of symmetries

Possibility to give informations on the post-inflationary period

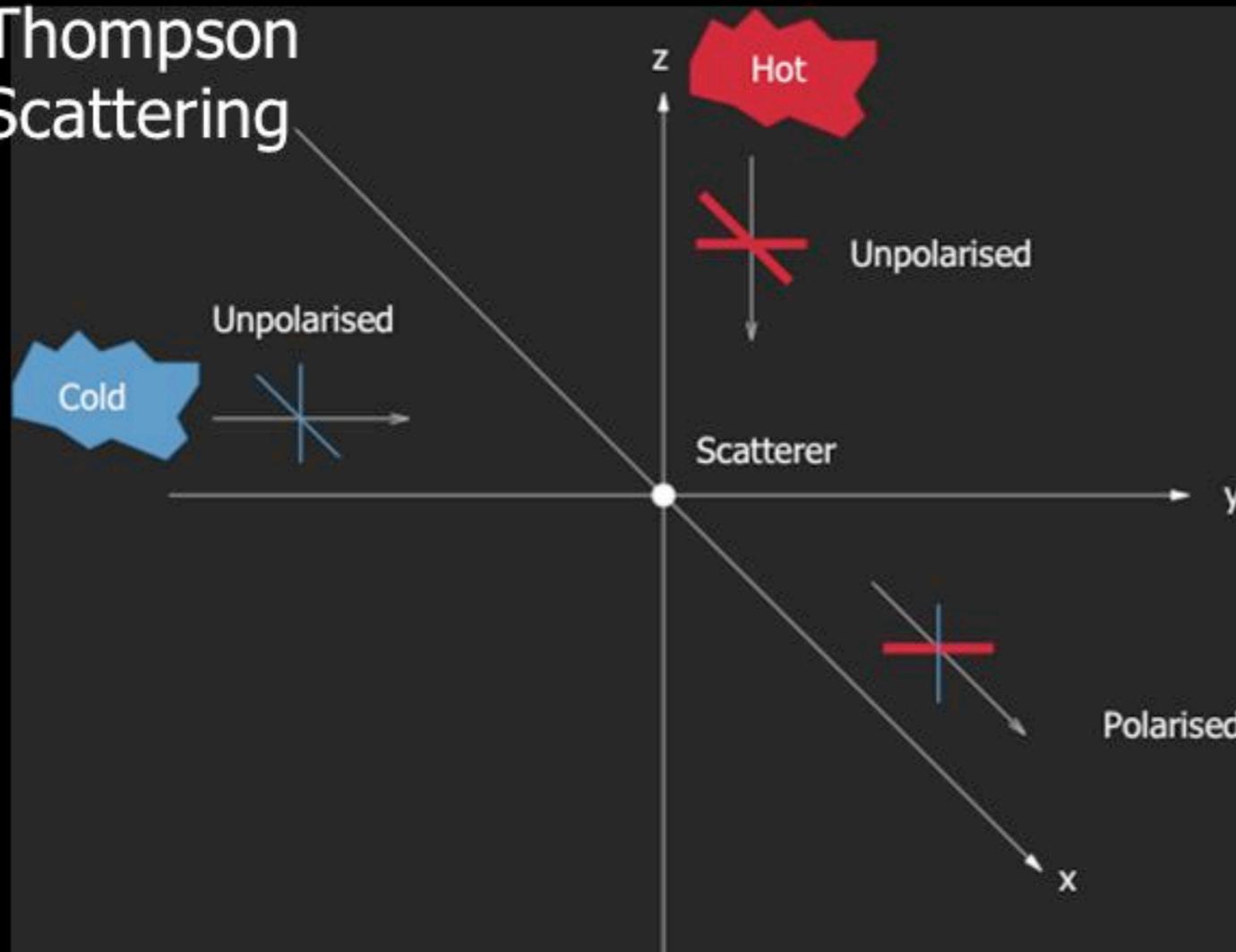
Inflation and stretching of CPs



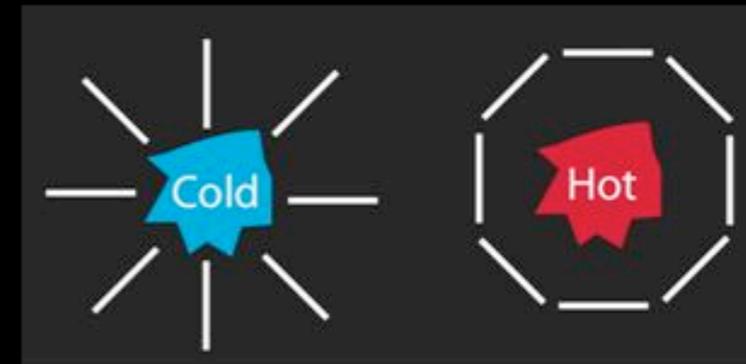
Polarisation of the CMB

8

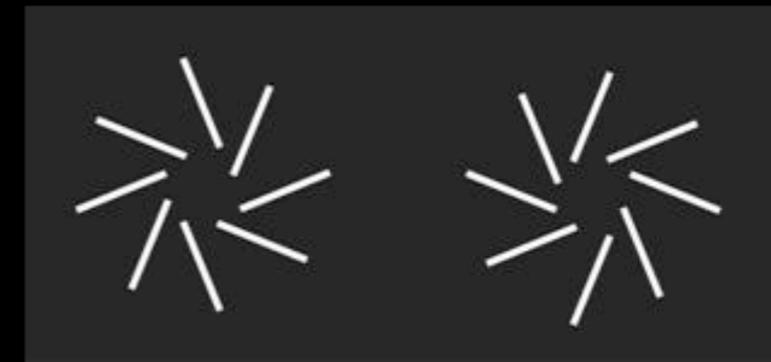
Thompson Scattering



E mode (Grad)



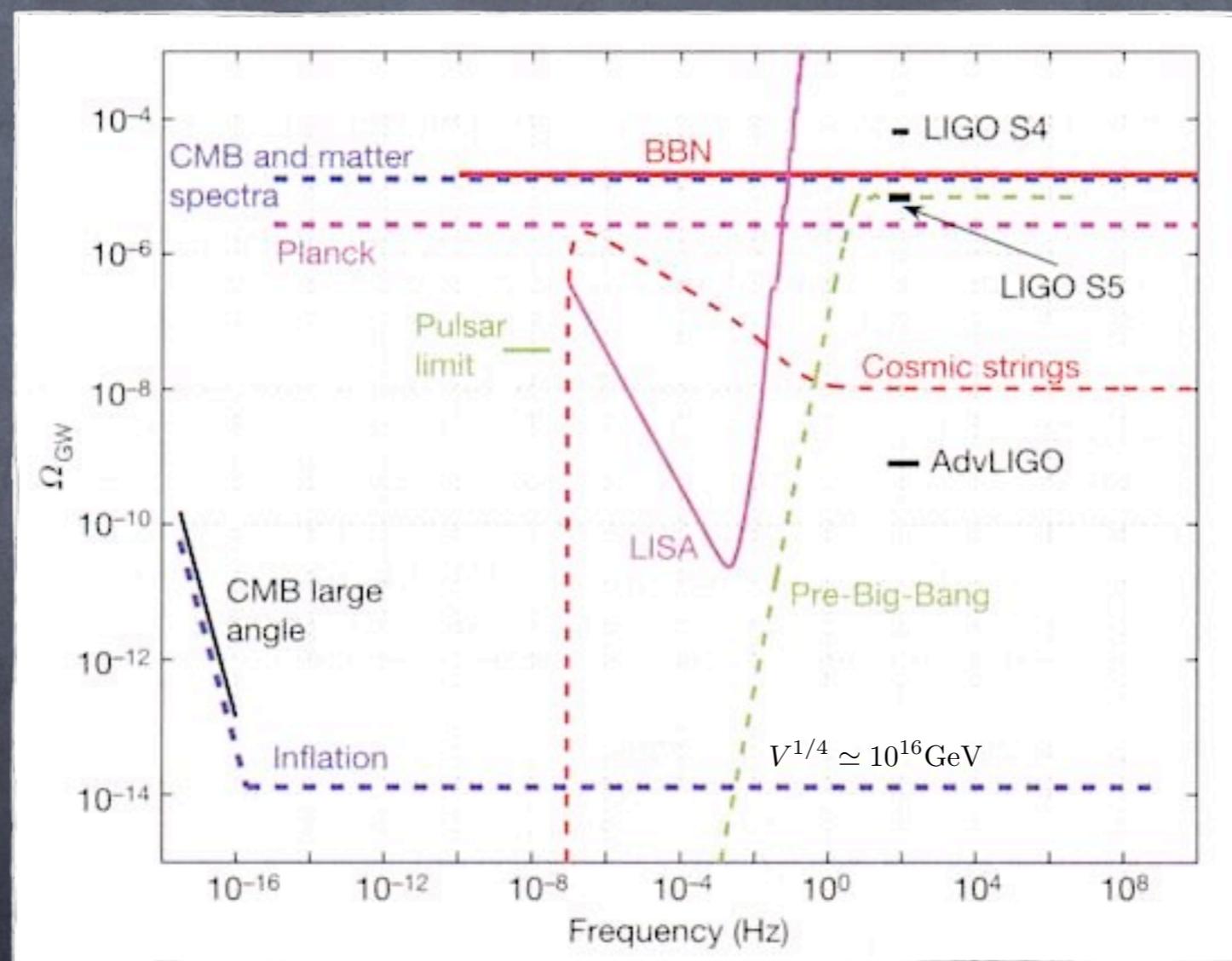
B mode (Curl)



- Tensor quadrupole doesn't show axial symmetry \rightarrow B mode polarisation

Kamionkowski, Kosowsky & Stebbins 1997

Zaldarriaga & Seljak 1997



Current observational upper bounds on the amplitude of GW spectrum

Why eLISA?

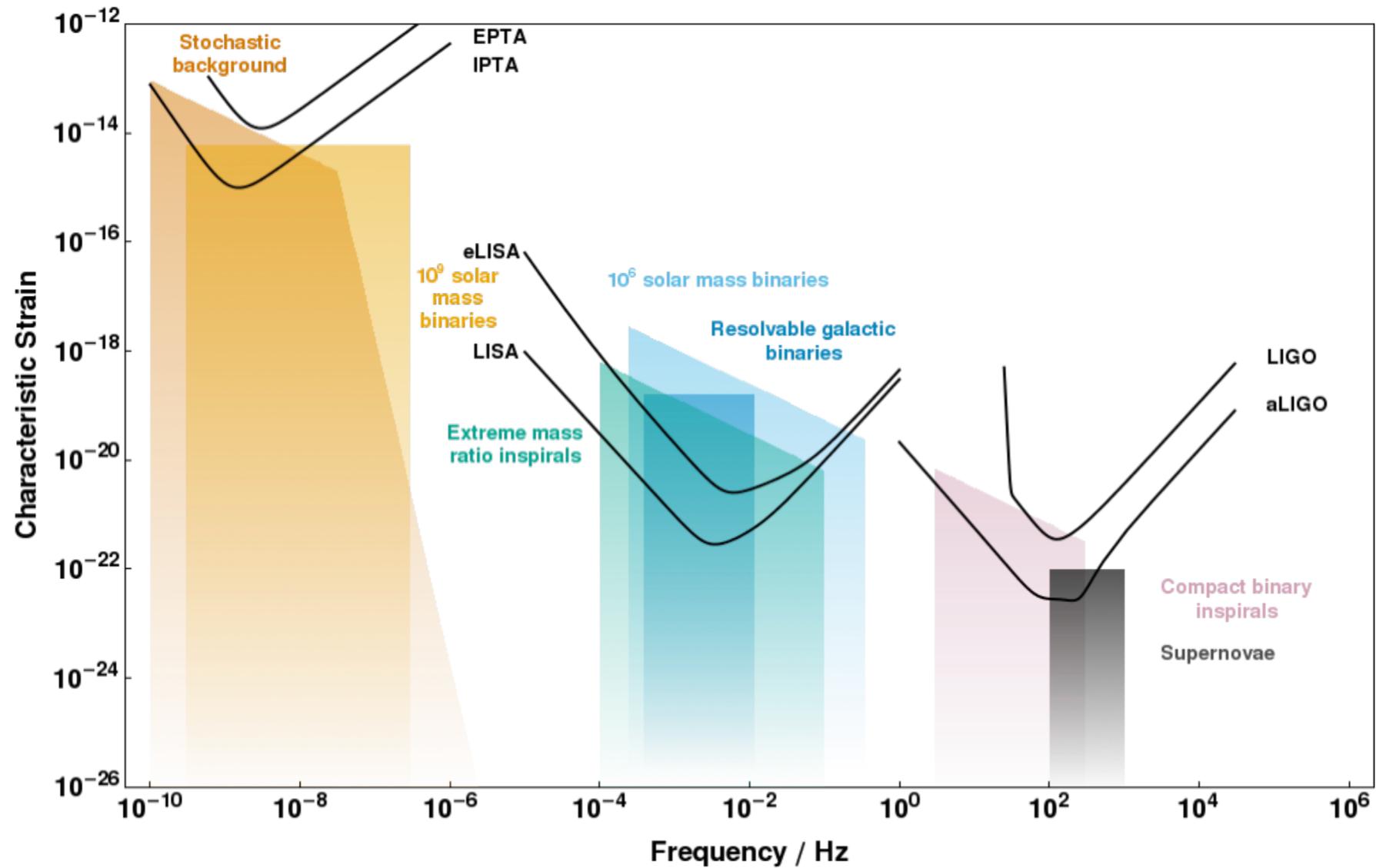


Image credit: [Wikimedia Commons/](#)
C. Cole, R. Berry, C. Berry

$$f_c \simeq 2.6 \times 10^{-5} \text{ Hz } \epsilon_*^{-1} \left(\frac{T_*}{1 \text{ TeV}} \right) \left(\frac{g_*}{100} \right)^{1/6}$$

Only for GW emitted by causal sources

$$\lambda_* = \epsilon_* H_*^{-1}$$

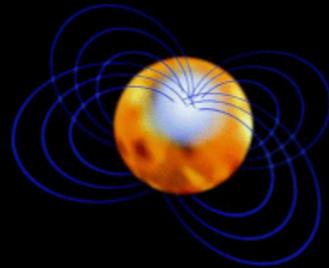
Wavelength Horizon length

$$\left(\text{or } f_* = \frac{H_*}{\epsilon_*} \right)$$

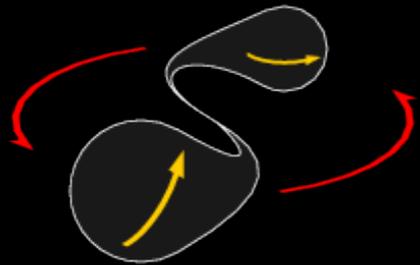
$\epsilon_* \leq 1$ param. depend. on the dynamics of the GW source

Sources of Gravitational Waves

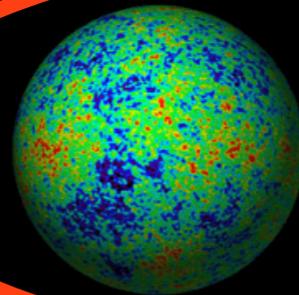
- 
- **Supernova:** Explosion caused by the collapse of an old, burnt-out star
 - Produces a burst of gravitational radiation, *if it is non-symmetric!*



- **Neutron star:** A city-sized atomic nucleus!
- Can spin at up to 600 cycles per second
- Emits continuous gravitational radiation (again, if it is non-symmetric)



- **Merging compact binary:** Collision of two stellar remnants (neutron stars or *black holes*)
- Produce a sweeping “chirp” as they spiral together



- **Primordial background:** Leftover radiation from the beginning of the Universe
- Tells us about the state of the Universe at *or before* the Big Bang!
- Sounds like “noise” with a characteristic spectrum

? Other sources

Strain Spectrum

$$S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{\text{GW}}(f)}{f^3}$$

Strain

$$h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{\text{GW}}(f)} \left(\frac{100\text{Hz}}{f} \right)^{3/2} \text{Hz}^{-1/2}$$