



## Abstract

Thermal gradients on board the LISA Pathfinder mission can induce effects with a potential impact to perturb the main differential acceleration measurement between both free-falling test masses. Apart from thermal forces arising due to gradients around the test mass, thermo-elastic effects can also contribute to the instrument noise. There are two locations where such a distortion can be critical: the Optical Window (**OW**) and the struts. The **OW**, i.e. the interface between the optical bench and the test mass, is clamped in a Titanium ring and therefore is susceptible to mechanical stress or changes in the refractive index due to thermal gradients across the glass. Thermal fluctuations in these structure can induce changes on the interferometer readout. Both temperature sensors and heaters were located in these locations as part of the thermal diagnostics subsystems and experiments have been performed during operations. Here we report on the results of these experiments and first estimates on the contribution to the mission noise budget coming from thermo-elastic distortions.

## Introduction

On board LISA Pathfinder (**LPF**) there are two **OWs**, one for each Electrode Housing (**EH**).

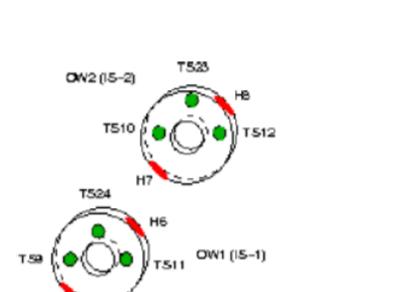


LTP, LISA Technology Package.

On the titanium ring of each **OW** there are three thermal sensors and two heaters. During the experiment, these heaters introduce thermal signals that will produce thermal fluctuations on the **OW**, which can be measured with the thermal sensors. These inputs will change the optical path length of the light beam traversing the glass. **LPF** will associate this change with movements of the **TM**, although no real force will be acting on it.



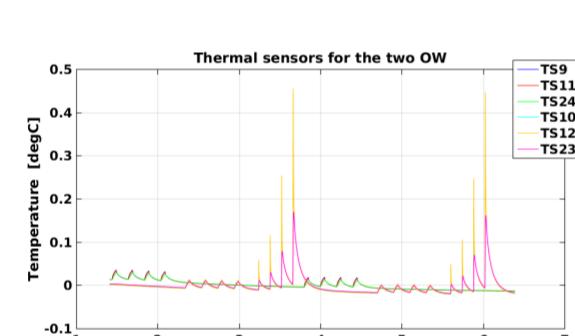
Sensors location on the OW.



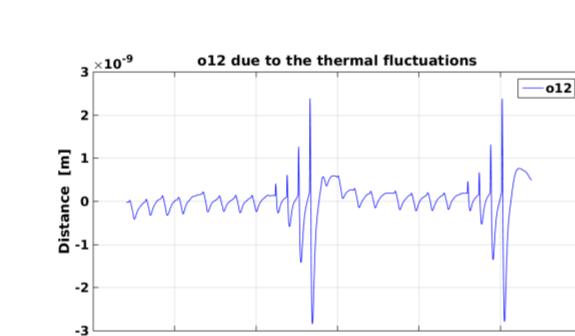
Sensors and heaters on the two OWs.

## Experiment

The experiment consisted on a series of long pulses (2 mW – 500 s) for the **OW1**, the same series for the **OW2** and then a sequence of fast pulses (200 mW – 5, 10, 20, 40 s) for the **OW2**. The three series were applied by both heaters. The first ones produce an increase of 20 mK and the later reach up to 450 mK.

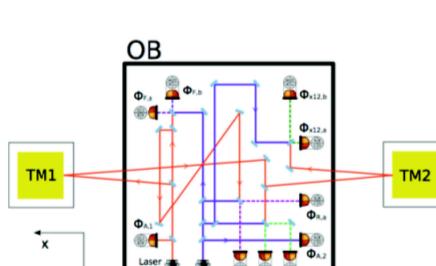


Thermal fluctuations produced.



Interferometric signal that appears due to the thermal fluctuations.

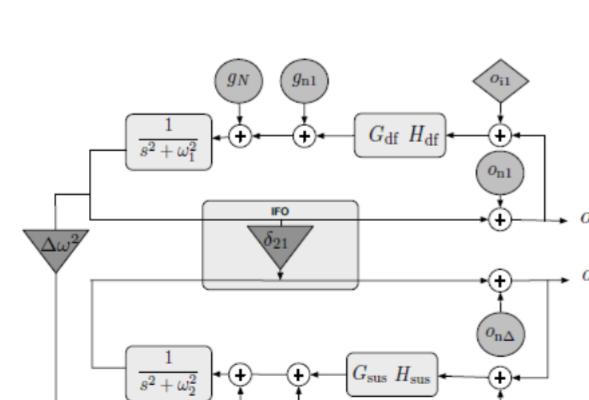
The same sequence was repeated twice. Before starting the second sequence, three photodiodes **PDs** close to the window were switched off to make sure that the heater activation was not affecting the photodiode read-out.



Optical lay-out. PDs masks during 2nd activation sequence are PD.F,A, PD.R,A, PD.x12,A.

## Extracting the in-loop signal

The satellite reacts to a "fake" in-loop displacement. We need just to get the out-of-loop signal to the proceed with the fit.



Control loop: the boxes describe the interferometer (IFO), controllers and dynamics of the test masses. The circles represent noise contributions, diamonds are signal injection points and the triangles denote cross-couplings between the first ( $o_{x1}$ ) and second channel ( $o_{x\Delta}$ ).

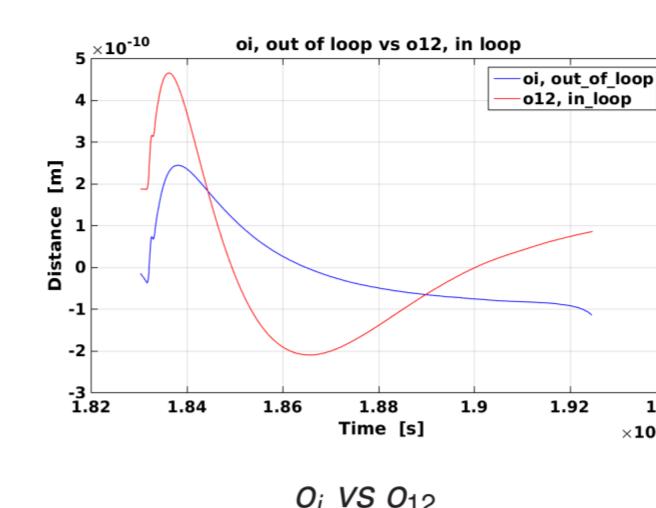
When the signals  $o_{i1}$  and  $o_{i\Delta}$  pass by the control loop are modified. Then we have:

$$\vec{o} = (\mathbf{D} \cdot \mathbf{S}^{-1} + \mathbf{C})^{-1} (\mathbf{C} \vec{o}_i + \vec{g}_n + \mathbf{D} \cdot \mathbf{S}^{-1} \vec{o}_n) \quad (1)$$

where  $\mathbf{D}$  is the dynamical matrix,  $\mathbf{C}$  is the controller, and  $\mathbf{S}$  stands for the sensing matrix (the interferometer in our case), i.e., the matrix translating the position of a test mass,  $\vec{q}$ , into the interferometer readout,  $\vec{o}$ . Subindex n stands for noise quantities, either sensing noise ( $\vec{o}_n$ ) or force noise ( $\vec{g}_n$ ) and subindex i stands for the injected signals ( $\vec{o}_i$ ).

A fit for our interferometric signal can be done if we subtract the modification of the control loop on this, i.e., we can use the above equation (1) to get the signal  $\vec{o}_i$  from  $\vec{o}$ .

## Modelling and data analysis



The following model can be used for the fit (for the **OW2**):

$$o_i = C_1 \cdot TS10 + C_2 \cdot \frac{d}{dt} TS10 + C_3 \cdot TS12 + C_4 \cdot \frac{d}{dt} TS12 + C_5 \cdot TS23 + C_6 \cdot \frac{d}{dt} TS23 \quad (2)$$

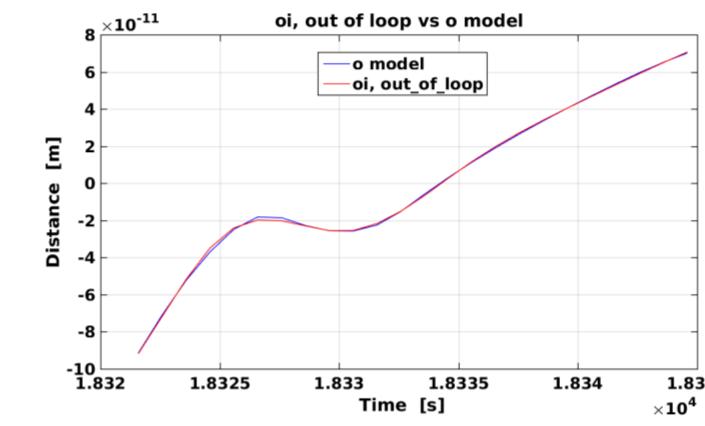
It is possible to divide each pulse in two parts and to do approximations in the equation of the model:

- 1) Peak that appears in the first part that goes up:

$$o_i = C_1 \cdot TS10 + C_2 \cdot \frac{d}{dt} TS10 + C_3 \cdot TS12 + C_4 \cdot \frac{d}{dt} TS12 \quad (3)$$

- 2) Rest of signal:

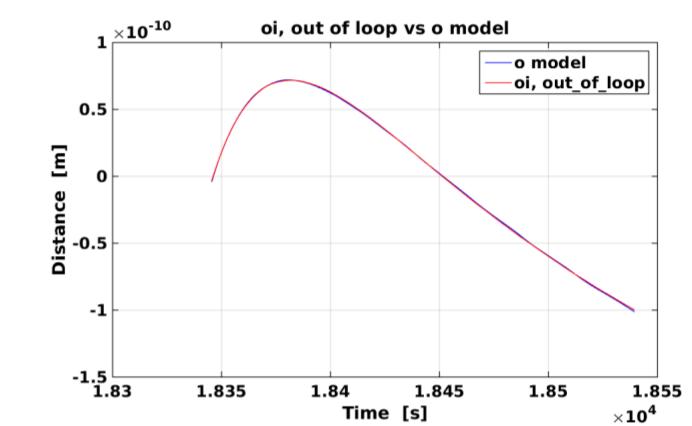
$$o_i = C_1 \cdot TS10 + C_3 \cdot TS12 + C_5 \cdot TS23 \quad (4)$$



Result of the fit with the approximation 1).

Coefficient	Value
$C_1$	$(4.8 \pm 0.2) \cdot 10^{-10} [\text{m degC}^{-1}]$
$C_2$	$(2.07 \pm 0.03) \cdot 10^{-09} [\text{m degC}^{-1}]$
$C_3$	$(-3.9 \pm 0.1) \cdot 10^{-08} [\text{m degC}^{-1}]$
$C_4$	$(-1.40 \pm 0.03) \cdot 10^{-07} [\text{m degC}^{-1}]$

Parameters obtained for the first fit.



Result of the rest of signal 2).

Coefficient	Value
$C_1$	$(7.2 \pm 0.4) \cdot 10^{-10} [\text{m degC}^{-1}]$
$C_3$	$(-1.5 \pm 0.4) \cdot 10^{-09} [\text{m degC}^{-1}]$
$C_5$	$(2.291 \pm 0.008) \cdot 10^{-08} [\text{m degC}^{-1}]$

Parameters obtained for the second fit.

## Physical model of the thermoelastic effect

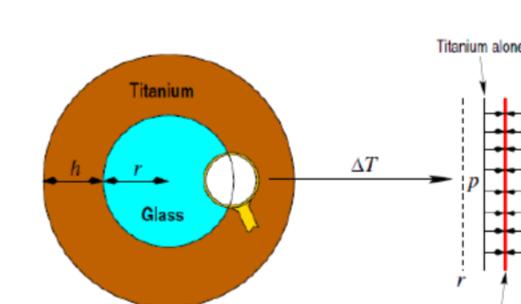
Two different kind of thermal effects are sources of changes in the optical path length:

- Temperature-dependent changes of the refractive index.

$$\frac{d\phi}{dT_{\text{free}}} = 2\pi \frac{L}{\lambda} \left[ \frac{dn}{dT} + (n-1)\alpha_E \right] \quad (5)$$

where  $\phi$  would be our  $o_i$ ,  $L$  the thickness of the glass,  $n$  the index of refraction,  $\lambda$  is the wavelength of the light, and  $\alpha_E$  is the thermal expansion factor of the glass.

The result of the substitution of nominal values in  $d\phi/dT_{\text{free}}$  is  $\sim 21 \text{ mrad} \cdot K^{-1}$ , while if we consider the sensor TS23 as more representative of the glass temperature and we look the model with the approximation 2 (second part of the fit), this value is given by  $C_5$ , that is  $\sim 33 \text{ mrad} \cdot K^{-1}$ .



Schematics of the dilation of the **OW** glass and the clamping titanium flange that produce mechanical stress.

- Mechanical stress induced changes of the refractive index.

- Low frequency:

$$\frac{d\phi}{dT_{\text{Stress}}} = \beta 2\pi \frac{L}{\lambda} \frac{\alpha_{Ti} - \alpha_{Glass}}{E_{Ti}^{-1} + (h/r)E_{Glass}^{-1}}, \quad (6)$$

- High frequency:

$$\frac{d\phi}{dT_{\text{Stress}}} = \beta 2\pi \frac{L}{\lambda} \frac{\alpha_{Ti}}{E_{Ti}^{-1} + (h/r)E_{Glass}^{-1}}, \quad (7)$$

where  $E$  is the Young's modulus and  $\alpha$  is the thermal expansion coefficient.

The result of the substitution of nominal values for the part of high frequency is  $d\phi/dT_{\text{Stress}} \sim 15 \text{ mrad} \cdot K^{-1}$ , while if we consider the first part of the fit with the approximation 1), this value is given by  $C_1-C_3$ , that is  $\sim 15 \text{ mrad} \cdot K^{-1}$ . For the part of low frequency, the theory predicts  $d\phi/dT_{\text{Stress}} \sim 2.5 \text{ mrad} \cdot K^{-1}$ , and with our model this value is given by  $C_1-C_3$  of 2), that is  $\sim 6 \text{ mrad} \cdot K^{-1}$ . Both are in agreement with the theoretical result.

## Conclusion

The total thermal effect consists in the sum of two effects:

- Optical path length changes induced by the thermal expansion: the values obtained with the above model are in agreement with the theoretical values.
- Optical path length changes induced by mechanical stress: the value obtained with the above model is similar to the expected theoretical value.

## References

- M Nofrarias, A F Garcia Marín, A Lobo, G Heinzel, J Ramos-Castro, J Sanjuán and K Danzmann, **Thermal diagnostics of the optical window on board LISA Pathfinder**, October 2007, Classical and Quantum Gravity.
- M Nofrarias et al, **Bayesian parameter estimation in the second LISA Pathfinder mock data challenge**, December 2010, Physical Review.

