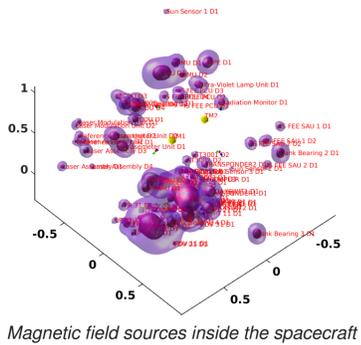


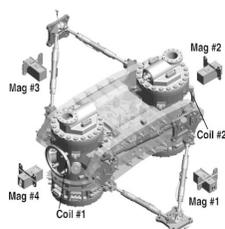
Abstract

- The magnetic field on-board LISA Pathfinder (LPF) plays an important role in the understanding of the LPF noise budget.
- Fluctuations of the field and the field gradient in the test mass position will directly translate into a force exerted on the test mass and therefore the magnetic field needs to be correctly characterised. This turns out to be a complex problem since there are more than 100 electronic devices inside the spacecraft contributing to the overall magnetic field and the only available information is the read-out from 4 tri-axial fluxgate magnetometers. Besides, in order to assess the force noise contribution to the differential force measurement, we are interested in the magnetic field in the position of the test masses (TMs). Since the magnetometers are relatively far from them an extrapolation is needed.
- Here, we show the approach we have followed to extrapolate the magnetic field using in-flight data. We will show how it seems we have obtained good results for the X & Y components of the field, while the results in Z do not seem concluding.



Magnetic diagnostics subsystem

- In order to obtain the contribution of the magnetic noise to the overall noise budget, LPF hosts a magnetic diagnostics subsystem, which consists in 4 tri-axial fluxgate magnetometers and 2 coils. With the magnetometers we are able to obtain measurements of the magnetic field \mathbf{B} in-flight, while the coils allow us to do controlled magnetic injections in order to study properties of the TMs, such as its magnetic moments \mathbf{M} & magnetic susceptibilities χ .



The magnetic diagnostics subsystem

Force on test masses

- The magnetic field \mathbf{B} & magnetic field gradient $\nabla\mathbf{B}$ present on the test mass (TM) position will couple with the magnetic moment \mathbf{M} & magnetic susceptibility χ of the TM, resulting in a force that will move the TM from its nominal free-fall position.

$$\mathbf{F} = \left\langle \left[(\mathbf{M} \cdot \nabla\mathbf{B}) + \frac{\chi}{\mu_0} (\mathbf{B} \cdot \nabla\mathbf{B}) \right] \right\rangle \mathbf{V}$$

- \mathbf{M} : magnetic moment of the TM [A.m²]
- χ : magnetic susceptibility of TM
- μ_0 : vacuum magnetic permeability $4\pi \times 10^{-7}$ [N.A⁻²]
- \mathbf{V} : TM volume [m³]

- With our characterization of the magnetic environment, we expect to obtain the values of \mathbf{B} & $\nabla\mathbf{B}$ in the TM's position using the values of \mathbf{B} measured at the magnetometers, so we will be able to compute this force afterwards.

Modellization

- Our modellization of the magnetic environment in order to be able to explain the magnetic field has been done only with data from the magnetometers. It will consist in several steps:
 - For each magnetic source inside the spacecraft, an equivalent dipole magnetic moment \mathbf{m} has been measured.
 - For all the known sources, since we know their magnetic moment \mathbf{m} and the distance \mathbf{r} to the TM's (or magnetometers), we can compute the magnetic field they are creating at the TM's (or magnetometers) position with the classical dipole formula:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\frac{3\mathbf{r}(\mathbf{m} \cdot \mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3} \right)$$

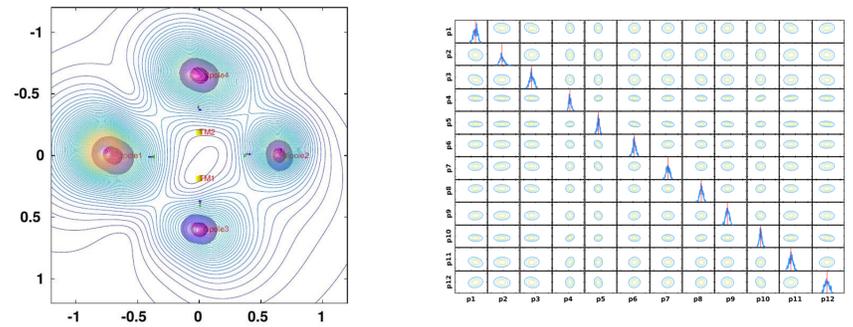
- For each source we compute \mathbf{B} in the position of the magnetometers, and sum over all the sources.
- Since surely there are sources that we are not considering creating magnetic field, and the magnetic moments \mathbf{m} assigned to our known sources can have errors, we will obtain a residual magnetic field, i.e. the sum over all the sources does not completely match the readout of the magnetometers.
- Through a Monte Carlo Markov Chain (MCMC) fit, we will place 4 "virtual" dipoles and assign them the magnetic moments \mathbf{m} that best explain this residual magnetic field.
- We only have 12 channels of data (4 magnetometers, 3 axis each), so we can only fit a maximum of 12 parameters if we want the fit to converge. Having this restriction, we will not fit the position of this "virtual" dipoles. Instead, we will assign them a fixed position and only fit their magnetic moments \mathbf{m} in order to only have 12 parameters to fit.

MCMC fit results

- The results of the fit for the magnetic moment \mathbf{m} of the "virtual" dipoles are presented in the following table. Below, it can also be seen the position of the dipoles with respect to the TM's, and the magnetic field lines they are causing. Next to it can also be seen the covariance matrix of the parameters from the fit. All parameters show a gaussian profile, which is a sign the fit has worked well.

	X [m]	Y [m]	Z [m]	m_x [mA m ²]	m_y [mA m ²]	m_z [mA m ²]
Dipole 1	0	-0.7	0.53	120 ± 10	239 ± 3	-60 ± 10
Dipole 2	0	0.65	0.54	80 ± 6	17 ± 2	-86 ± 6
Dipole 3	0.6	0	0.52	29 ± 1	154 ± 2	17 ± 4
Dipole 4	-0.65	0	0.53	81 ± 2	198 ± 4	-10 ± 7

Assigned positions to the virtual dipoles and magnetic moments estimated using a MCMC fit.



Magnetic field lines and position of the virtual dipoles wrt the TMs.

Covariance matrix of the estimated parameters.

- Now, if we sum again the contribution of all the known sources plus the ones from the "virtual" dipoles, the estimated magnetic field in the magnetometers match pretty well the magnetic field the magnetometers are reading, as can be seen in the following table.

Axis	Magnetometer PX		Magnetometer MX	
	B_{measured} [nT]	$B_{\text{estimated}}$ [nT]	B_{measured} [nT]	$B_{\text{estimated}}$ [nT]
x	864.57 ± 0.07	845 ± 3	820.33 ± 0.07	815 ± 2
y	-897.01 ± 0.06	-900 ± 1	-453.55 ± 0.04	-445 ± 1
z	82.08 ± 0.07	50 ± 3	91.19 ± 0.06	82 ± 2

Axis	Magnetometer PY		Magnetometer MY	
	B_{measured} [nT]	$B_{\text{estimated}}$ [nT]	B_{measured} [nT]	$B_{\text{estimated}}$ [nT]
x	-112.65 ± 0.06	-123 ± 1	-84.61 ± 0.07	-85 ± 2
y	595.51 ± 0.04	599 ± 1	1037.57 ± 0.06	1051 ± 1
z	384.63 ± 0.06	356 ± 3	527.39 ± 0.07	504 ± 3

Comparison of estimated and measured magnetic field in the magnetometers.

Results comparison

- In the following tables can be seen a comparison between \mathbf{B} & $\nabla\mathbf{B}$ estimated at the TMs position and the values predicted by an independent study [1].

Axis	TM 1		TM 2		TMs volume	$\partial B_x / \partial x$ [nT/m]	$\partial B_y / \partial x$ [nT/m]	$\partial B_z / \partial x$ [nT/m]
	Airbus $B_{\text{predicted}}$ [nT]	$B_{\text{estimated}}$ [nT]	Airbus $B_{\text{predicted}}$ [nT]	$B_{\text{estimated}}$ [nT]				
x	267	130 ± 20	454	110 ± 10	TM1 estimated	2400 ± 200	-1400 ± 100	-400 ± 300
y	-515	-140 ± 30	-39	90 ± 30	TM1 Airbus	[15025]	[2905]	[14847]
z	171	650 ± 60	64	220 ± 50	TM2 estimated	-1300 ± 100	400 ± 100	1100 ± 300
					TM2 Airbus	[11407]	[1473]	[15041]
					Between TMs	$\partial B_x / \partial x$ [nT/m]	$\partial B_y / \partial x$ [nT/m]	$\partial B_z / \partial x$ [nT/m]
					TM1-TM2 estimated	49 ± 4	-618 ± 3	1151 ± 8
					Between Magnet.	$\partial B_x / \partial x$ [nT/m]	$\partial B_y / \partial x$ [nT/m]	$\partial B_z / \partial x$ [nT/m]
					Px - Mx (data)	54.86 ± 0.07	-601.30 ± 0.09	-11.83 ± 0.09

Comparison between estimated values for \mathbf{B} & $\nabla\mathbf{B}$ with Airbus predictions and magnetometers data.

- B results:** on one hand, results in X & Y are in agreement with Airbus predictions. On the other hand, the results in Z do not agree.
- $\nabla\mathbf{B}$ results between TMs:** the estimated gradient along TMs is similar to the measured gradient across the magnetometers behind each TM. Again, the Z component estimation is not as good as X & Y estimations.
- $\nabla\mathbf{B}$ results in TMs volume:** values for $\nabla\mathbf{B}$ are difficult to estimate only with data from the magnetometers, since they are far away and we can not know how the magnetic field changes along the TMs faces. Here, we obtain big values of $\nabla\mathbf{B}$ because we are placing a big virtual dipole near each TM, and since \mathbf{B} decays very fast, the gradient is strong. Since this dipole is not real (it does not exist in the spacecraft) this result may not be very realistic.
- Requirements:** results are under the mission requirements for \mathbf{B} & $\nabla\mathbf{B}$ in the TMs position, which are $|\mathbf{B}| < 10000$ nT & $|\nabla\mathbf{B}| < 5\sqrt{3} \times 10^3 \sim 8660$ nT/m

Conclusions

- Specifying the location of virtual dipoles we obtain a good fit, but it would be better to make a fit with real dipoles, not virtual.
- We have obtained good values for some of the parameters, specially in the X & Y components.
- On the other hand, results in Z does not seem as good, probably because we lack information in Z: all magnetometers are in the same Z plane, so we do not have information of how the magnetic field changes in Z.
- Even when these results could not seem solid on their own, they can be useful to use along with the results coming from the coils experiments, specially because there, it is difficult to disentangle the components coming from \mathbf{B} & $\nabla\mathbf{B}$. Using this results, for example, we can assign the value obtained here for \mathbf{B} and then obtain $\nabla\mathbf{B}$ using the experiments with the coils.

References

- [1] TRENKEL C. *et al.*, Prediction of spacecraft magnetic field. S2.ASU.TN.2523, (2015).