



# Kern- und Teilchenphysik II

## Lecture 9: Deep Inelastic Scattering

(adapted from the Handout of Prof. Mark Thomson)

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<http://www.physik.uzh.ch/de/lehre/PHY213/FS2017.html>

# $e^- p$ Elastic Scattering at Very High $q^2$

★ At high  $q^2$  the Rosenbluth expression for elastic scattering becomes

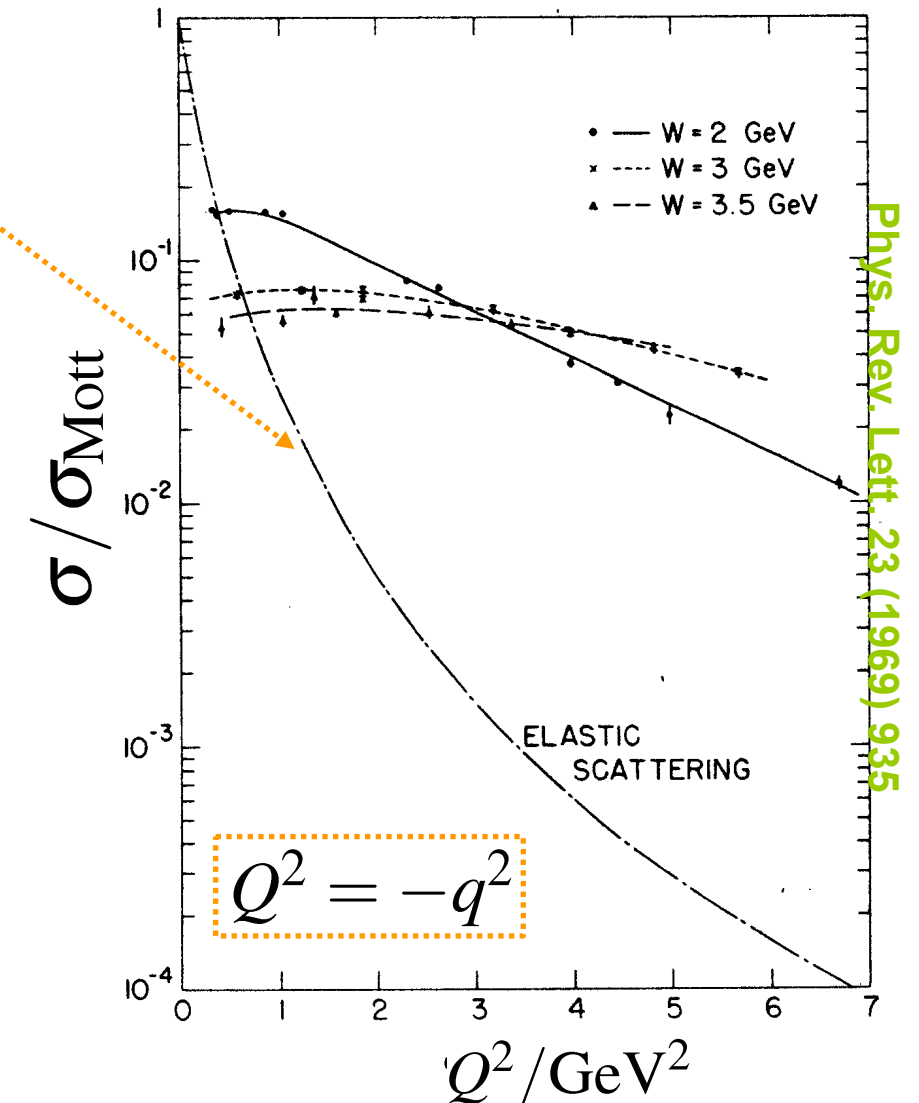
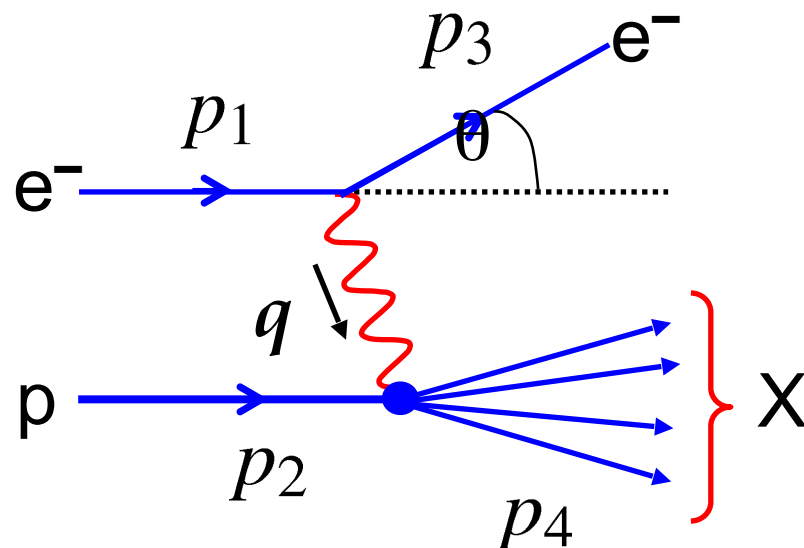
$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{q^2}{2M^2} G_M^2 \sin^2 \frac{\theta}{2}\right) \quad \tau = -\frac{q^2}{4M^2} \gg 1$$

• From  $e^- p$  elastic scattering, the proton magnetic form factor is

$$G_M(q^2) \approx \frac{1}{(1 + q^2/0.71 \text{ GeV}^2)^2} \quad \rightarrow \quad G_M(q^2) \propto q^{-4} \quad \text{at high } q^2$$

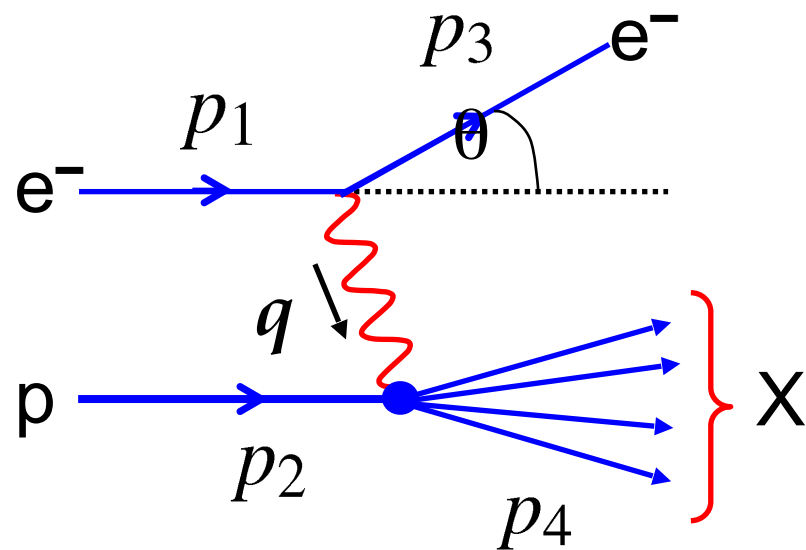
$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{elastic} \propto q^{-6}$$

• Due to the finite proton size, elastic scattering at high  $q^2$  is unlikely and inelastic reactions where the proton breaks up dominate.



M. Breidenbach et al.,

# Kinematics of Inelastic Scattering



- For inelastic scattering the mass of the final state hadronic system is no longer the proton mass,  $M$
- The final state hadronic system must contain at least one **baryon** which implies the final state invariant mass  $M_X > M$

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

★ For inelastic scattering introduce four new kinematic variables:  $x, y, \nu, Q^2$

★ Define:

$$x \equiv \frac{Q^2}{2p_2 \cdot q}$$

**Bjorken x**

(Lorentz Invariant)

where

$$Q^2 \equiv -q^2$$

$$Q^2 > 0$$

• Here  $M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2 \cdot q + M^2$

$\Rightarrow Q^2 = 2p_2 \cdot q + M^2 - M_X^2 \quad \Rightarrow Q^2 \leq 2p_2 \cdot q$

Note: in many text books  $W$  is often used in place of  $M_X$

hence

$$0 < x < 1 \text{ inelastic}$$

$$x = 1 \text{ elastic}$$

Proton intact  
 $M_X = M$

★ Define:  $y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$  (Lorentz Invariant)

• In the Lab. Frame:

$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$

$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$

$$\rightarrow y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

So  $y$  is the fractional energy loss of the incoming particle

$$0 < y < 1$$

• In the C.o.M. Frame (neglecting the electron and proton masses):

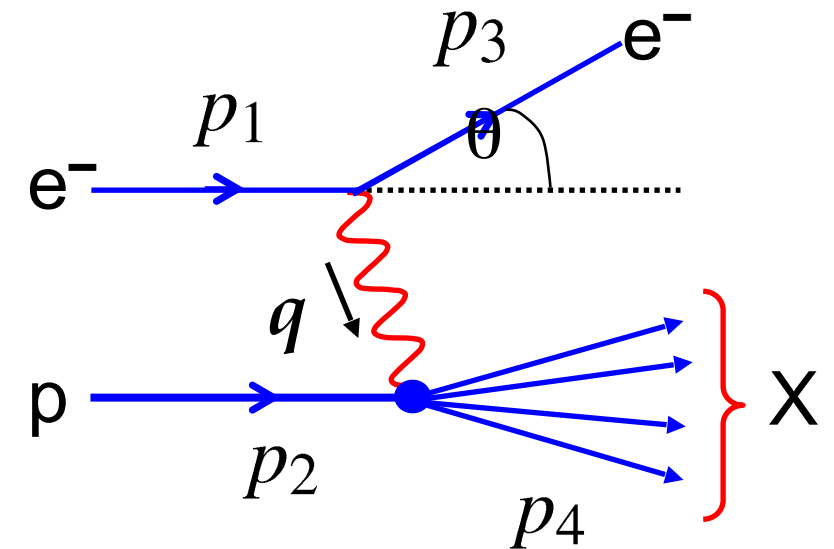
$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E \sin \theta^*, 0, E \cos \theta^*)$$

$$\rightarrow y = \frac{1}{2}(1 - \cos \theta^*) \quad \text{for } E \gg M$$

★ Finally Define:  $v \equiv \frac{p_2 \cdot q}{M}$  (Lorentz Invariant)

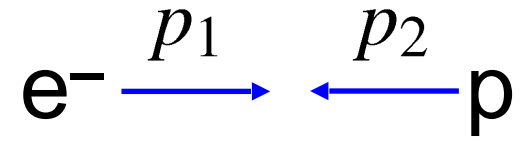
• In the Lab. Frame:  $v = E_1 - E_3$

$v$  is the energy lost by the incoming particle



# Relationships between Kinematic Variables

- Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy,  $s$ , for the electron-proton collision



$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + M^2 + \cancel{m_e^2}$$

Neglect mass of electron

$$2p_1 \cdot p_2 = s - M^2$$

- For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables

$$Q^2 \equiv -q^2 \quad x \equiv \frac{Q^2}{2p_2 \cdot q} \quad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad v \equiv \frac{p_2 \cdot q}{M}$$

are not independent.

- i.e. the scaling variables  $x$  and  $y$  can be expressed as

$$x = \frac{Q^2}{2Mv}$$

$$y = \frac{2M}{s - M^2} v$$

Note the simple relationship between  $y$  and  $v$

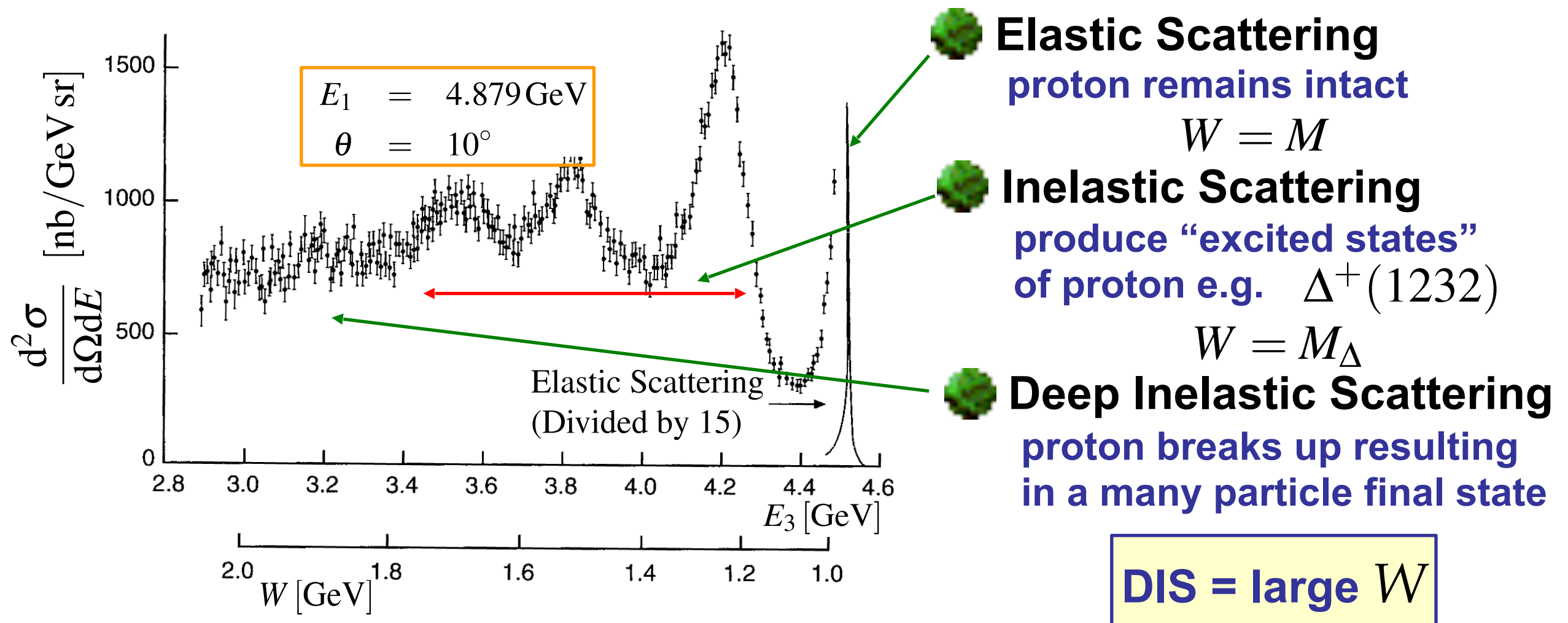
and  $xy = \frac{Q^2}{s - M^2} \Rightarrow Q^2 = (s - M^2)xy$

- For a fixed centre of mass energy, the interaction kinematics are completely defined by **any two** of the above kinematic variables (except  $y$  and  $v$ )
- For elastic scattering ( $x = 1$ ) there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.

# Inelastic Scattering

**Example:** Scattering of 4.879 GeV electrons from protons at rest

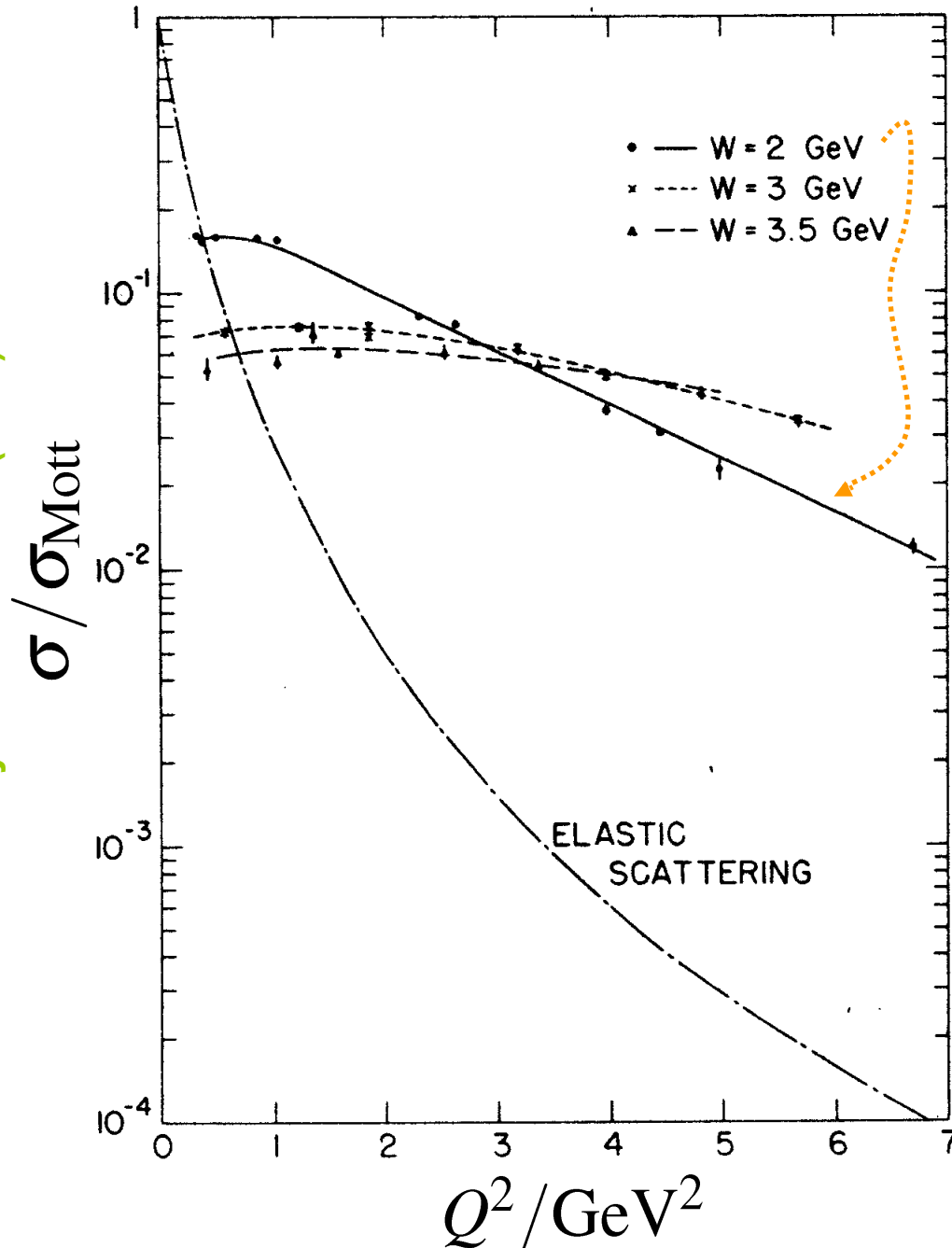
- Place detector at  $10^\circ$  to beam and measure the energies of scattered  $e^-$
- Kinematics fully determined from the electron energy **and** angle !
- e.g. for **this energy and angle** : the invariant mass of the final state hadronic system  $W^2 = M_X^2 = 10.06 - 2.03E_3$  (try and show this)



# Inelastic Cross Sections

- Repeat experiments at different angles/beam energies and determine  $q^2$  dependence of elastic and inelastic cross-sections

M. Breidenbach et al.,  
Phys. Rev. Lett. 23 (1969) 935



- Elastic scattering falls off rapidly with  $q^2$  due to the proton not being point-like (i.e. form factors)
- Inelastic scattering cross sections only weakly dependent on  $q^2$
- Deep Inelastic scattering cross sections almost independent of  $q^2$  !  
i.e. “Form factor”  $\rightarrow 1$



Scattering from point-like objects within the proton !

# Elastic → Inelastic Scattering

## ★ Recall: Elastic scattering

- **Only one independent variable.** In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left( \frac{G_E^2 + \tau G_M^2}{(1 + \tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \quad \tau = \frac{Q^2}{4M^2}$$

**Note:** here the energy of the scattered electron is determined by the angle.

- In terms of the Lorentz invariant kinematic variables can express this differential cross section in terms of  $Q^2$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ \frac{G_E^2 + \tau G_M^2}{(1 + \tau)} \left( 1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

which can be written as:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ f_2(Q^2) \left( 1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

## ★ Inelastic scattering

- For Deep Inelastic Scattering have **two independent variables**. Therefore need a double differential cross section



# Deep Inelastic Scattering

★ It can be shown that the most general Lorentz Invariant expression for  $e^-p \rightarrow e^-X$  inelastic scattering (via a single exchanged photon is):

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ \left(1 - y - \frac{M^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (1)$$

INELASTIC  
SCATTERING

c.f. 
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ \left(1 - y - \frac{M^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

ELASTIC  
SCATTERING

We will soon see how this connects to the quark model of the proton

• **NOTE:** The form factors have been replaced by the **STRUCTURE FUNCTIONS**

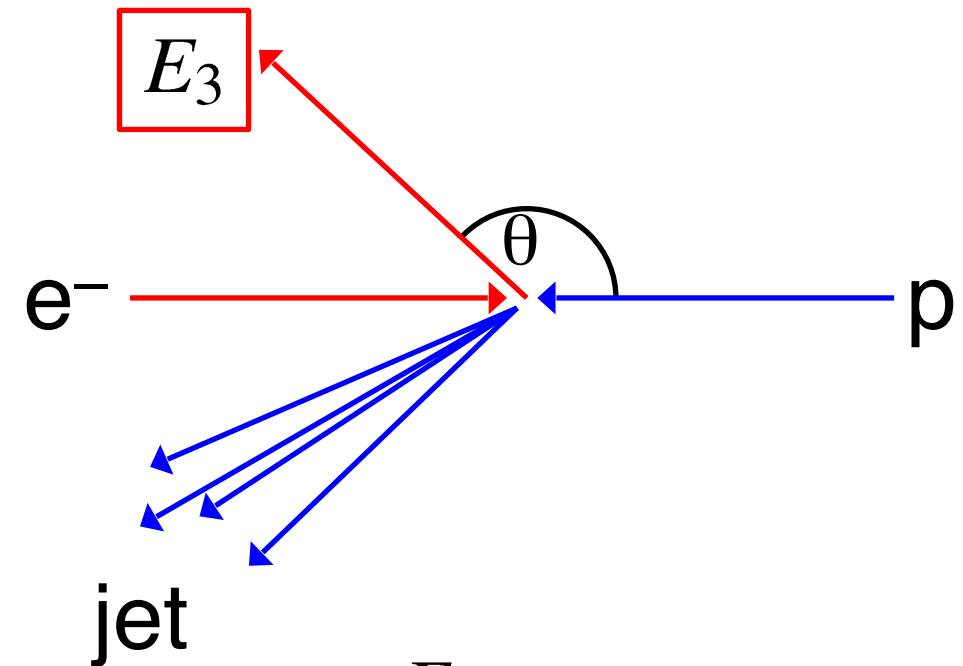
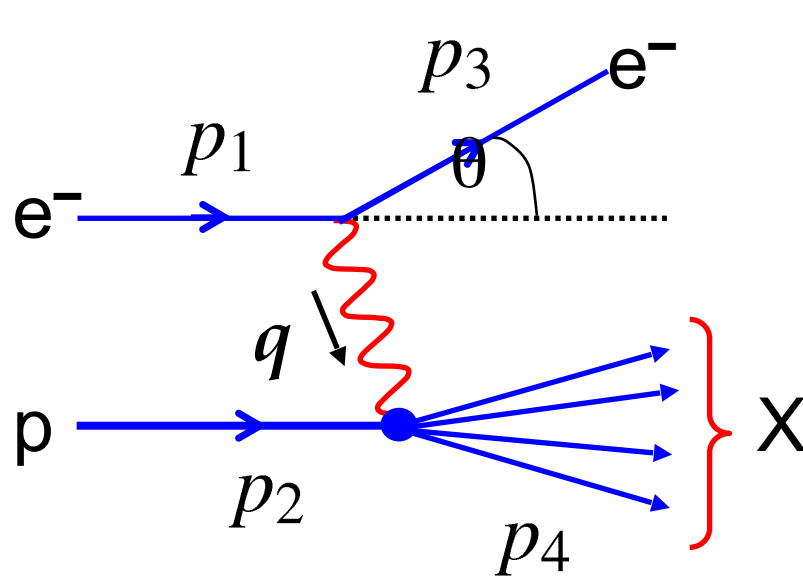
$$F_1(x, Q^2) \quad \text{and} \quad F_2(x, Q^2)$$

which are a function of  $x$  and  $Q^2$ : can not be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the **momentum distribution** of the quarks within the proton

★ In the limit of high energy (or more correctly  $Q^2 \gg M^2 y^2$ ) eqn. (1) becomes:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ (1 - y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (2)$$

- In the Lab. frame it is convenient to express the cross section in terms of the angle,  $\theta$ , and energy,  $E_3$ , of the scattered electron – experimentally well measured.



$$Q^2 = 4E_1 E_3 \sin^2 \theta / 2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad v = E_1 - E_3$$

- In the Lab. frame, Equation (2) becomes:

(see examples sheet Q13)

$$\frac{d^2 \sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta / 2} \left[ \frac{1}{v} F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right] \quad (3)$$

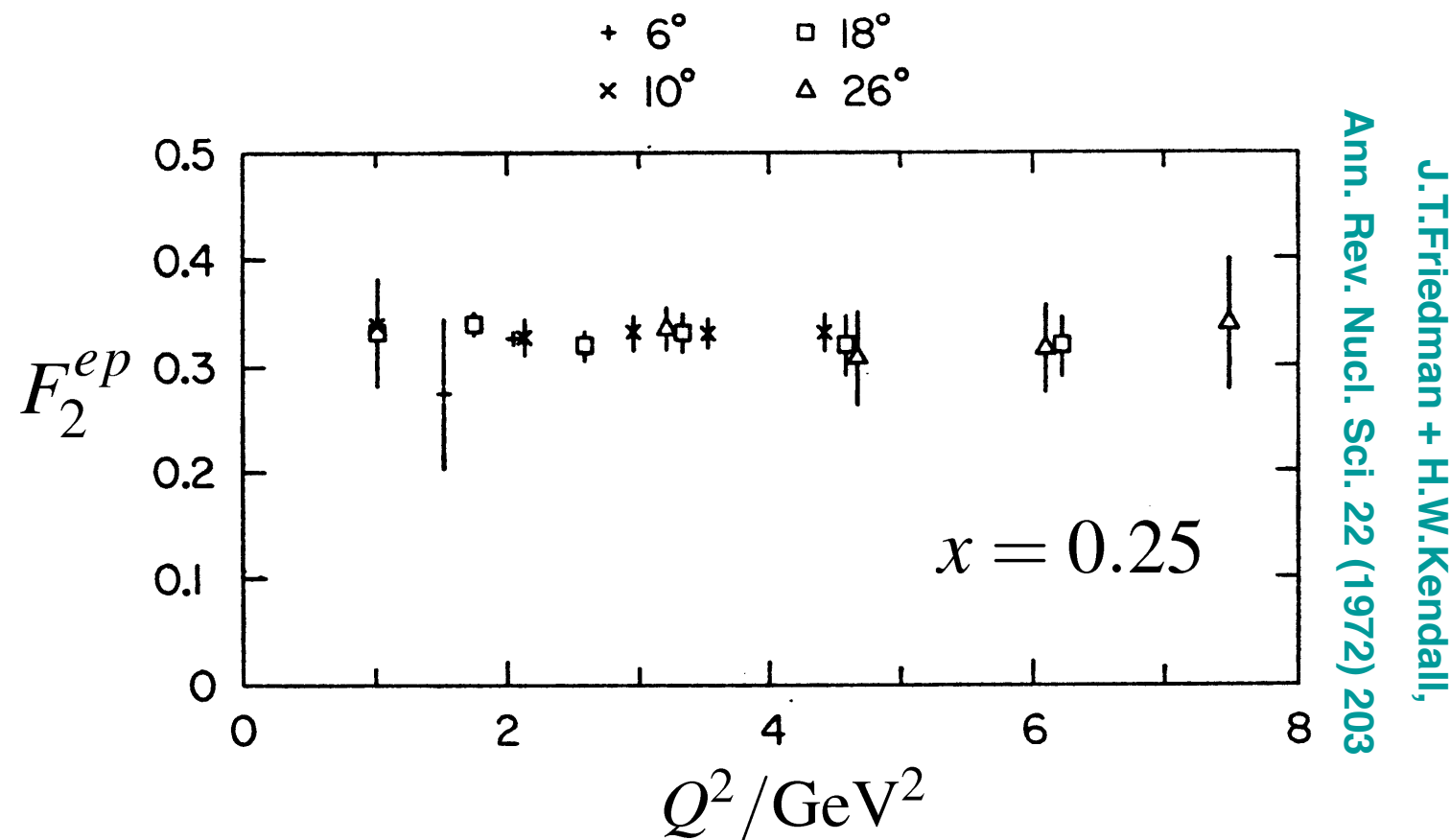
Electromagnetic Structure Function

Pure Magnetic Structure Function

# Measuring the Structure Functions

★ To determine  $F_1(x, Q^2)$  and  $F_2(x, Q^2)$  for a given  $x$  and  $Q^2$  need measurements of the differential cross section at several different scattering angles and incoming electron beam energies (see Q13 on examples sheet)

Example: electron-proton scattering  $F_2$  vs.  $Q^2$  at fixed  $x$



♦ Experimentally it is observed that both  $F_1$  and  $F_2$  are (almost) independent of  $Q^2$

# Bjorken Scaling and the Callan-Gross Relation

- ★ The near (see later) independence of the structure functions on  $Q^2$  is known as **Bjorken Scaling**, i.e.

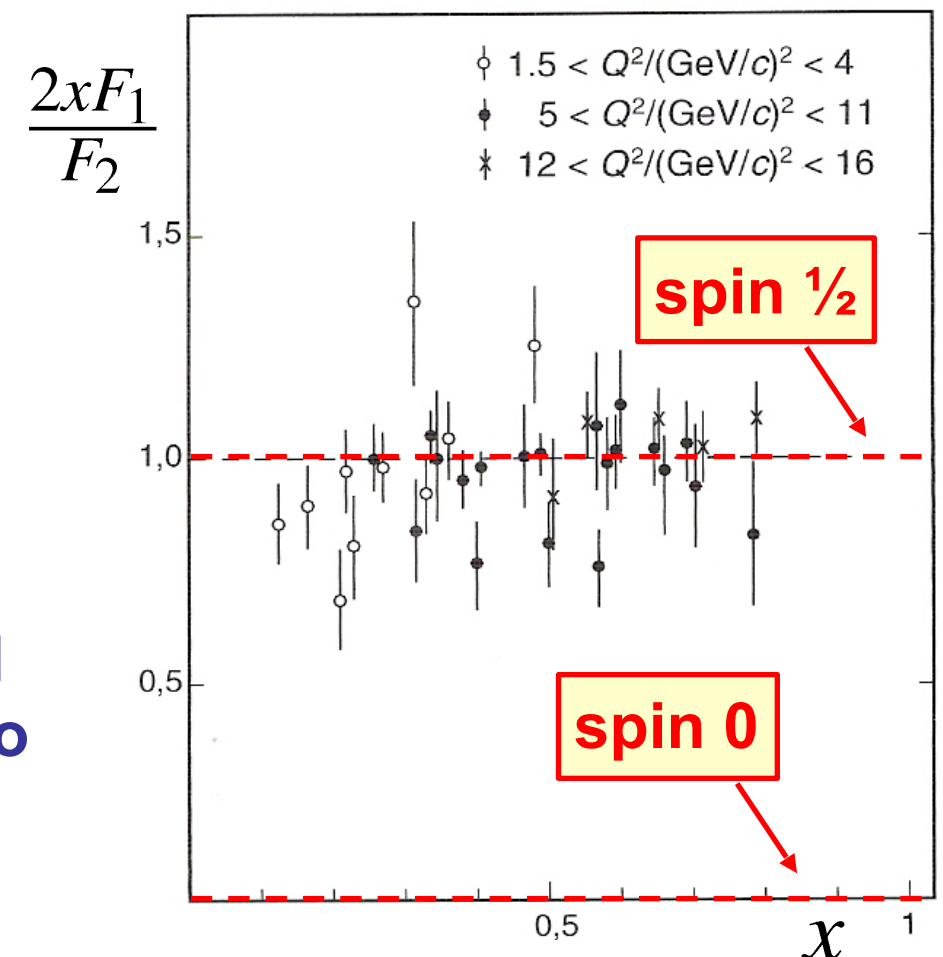
$$F_1(x, Q^2) \rightarrow F_1(x) \quad F_2(x, Q^2) \rightarrow F_2(x)$$

- It is strongly suggestive of scattering from **point-like** constituents within the proton

- ★ It is also observed that  $F_1(x)$  and  $F_2(x)$  are not independent but satisfy the **Callan-Gross relation**

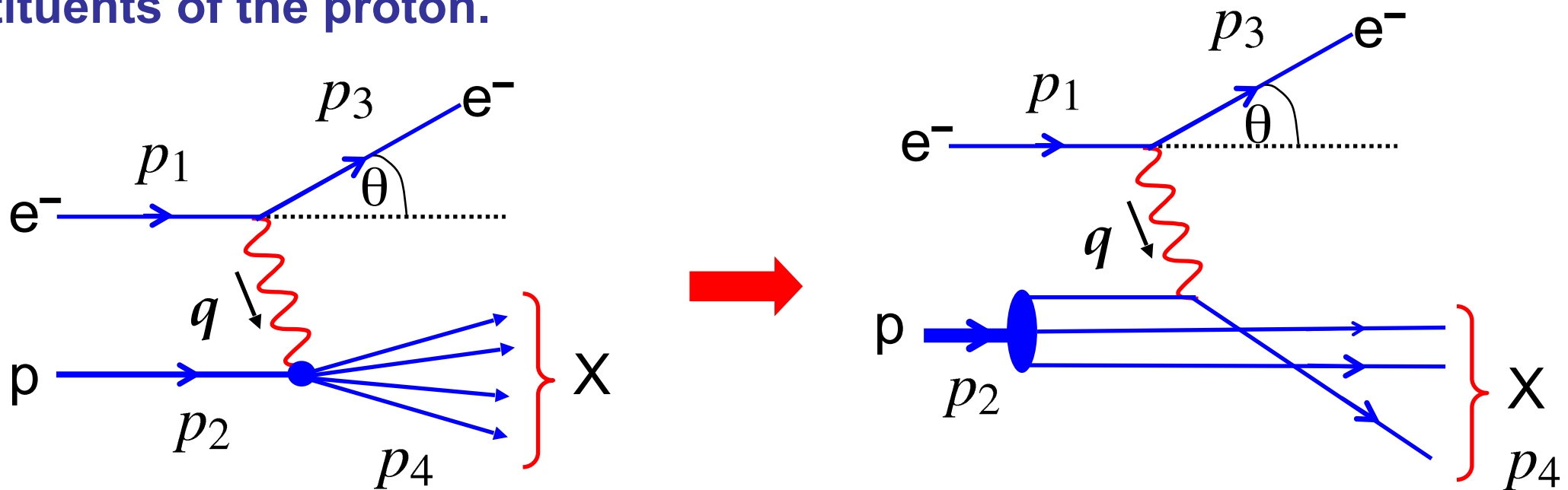
$$F_2(x) = 2xF_1(x)$$

- As we shall soon see this is exactly what is expected for scattering from **spin-half** quarks.
- **Note** if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e.  $F_1(x) = 0$



# The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents “**partons**”
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.

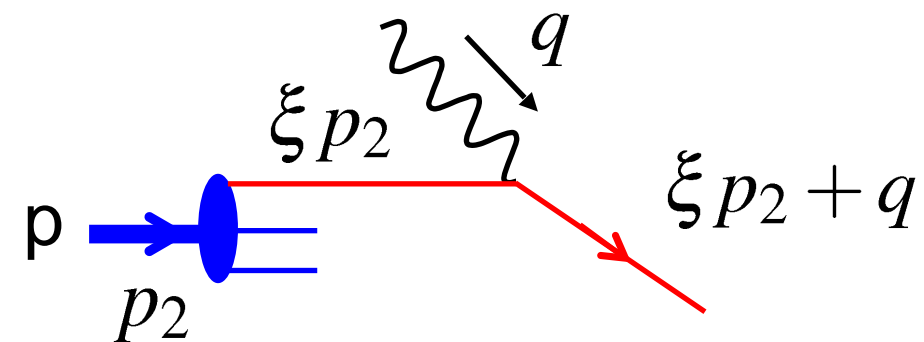
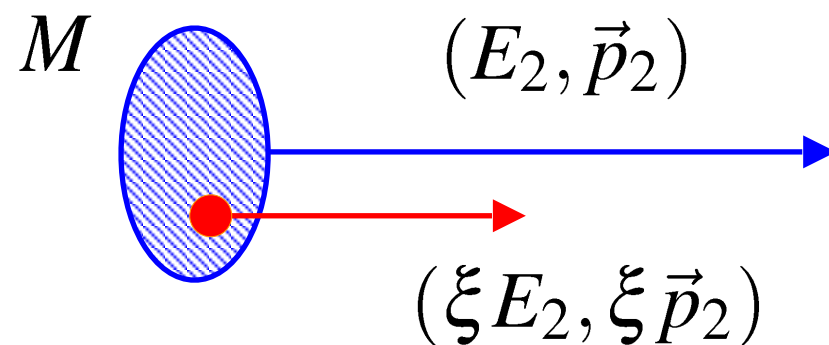


Scattering from a proton  
with structure functions

Scattering from a point-like  
quark within the proton

★ How do these two pictures of the interaction relate to each other?

- In the parton model the basic interaction is **ELASTIC** scattering from a “**quasi-free**” spin- $\frac{1}{2}$  quark in the proton, i.e. treat the quark as a free particle!
- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the “**infinite momentum frame**”, where we can neglect the proton mass and  $p_2 = (E_2, 0, 0, E_2)$
- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.
- Let the quark carry a fraction  $\xi$  of the proton's four-momentum.



- After the interaction the struck quark's four-momentum is  $\xi p_2 + q$

$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2 \cdot q = 0 \quad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\rightarrow \xi = \frac{Q^2}{2p_2 \cdot q} = x$$

**Bjorken  $x$  can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)**

- In terms of the proton momentum

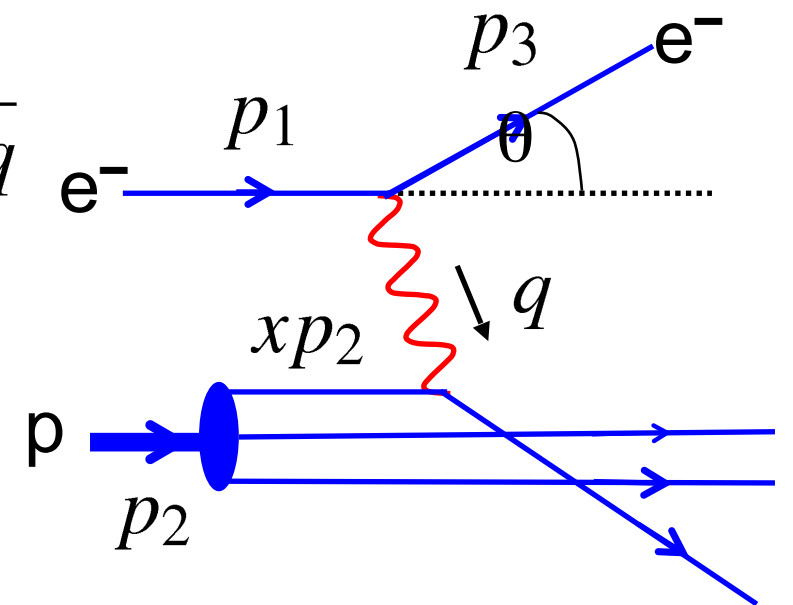
$$s = (p_1 + p_2)^2 \simeq 2p_1 \cdot p_2 \quad y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad x = \frac{Q^2}{2p_2 \cdot q}$$

- But for the underlying quark interaction

$$s^q = (p_1 + xp_2)^2 = 2xp_1 \cdot p_2 = xs$$

$$y_q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{xp_2 \cdot q}{xp_2 \cdot p_1} = y$$

$$x_q = 1 \quad (\text{elastic, i.e. assume quark does not break up})$$



- Previously derived the Lorentz Invariant cross section for  $e^- \mu^- \rightarrow e^- \mu^-$  elastic scattering in the ultra-relativistic limit (handout 4 + Q10 on examples sheet).

Now apply this to  $e^- q \rightarrow e^- q$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4} \left[ 1 + \left( 1 + \frac{q^2}{s_q} \right)^2 \right]$$

$e_q$  is quark charge, i.e.  
 $e_u = +2/3; \quad e_d = -1/3$

• Using  $-q^2 = Q^2 = (s_q - m^2)x_q y_q \quad \rightarrow \quad \frac{q^2}{s_q} = -y_q = -y$

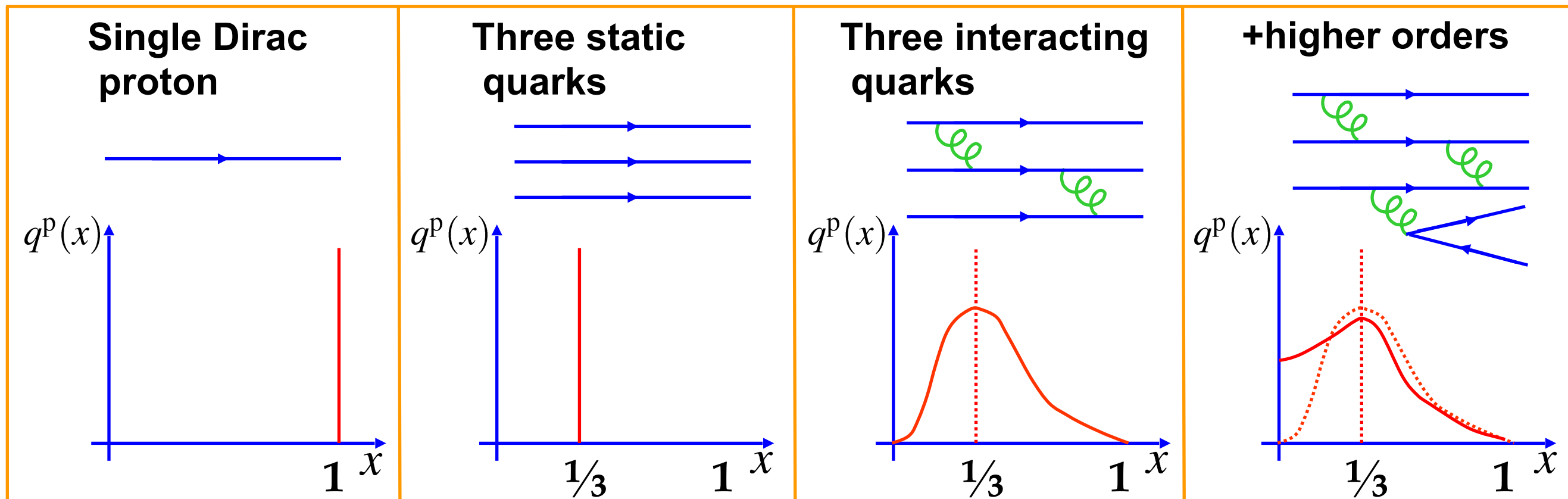
$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \left[ 1 + (1 - y)^2 \right]$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4} \left[ (1-y) + \frac{y^2}{2} \right] \quad (3)$$

- ★ This is the expression for the differential cross-section for **elastic**  $e^-q$  scattering from a quark carrying a fraction  $x$  of the proton momentum.
- Now need to account for distribution of quark momenta within proton

- ★ Introduce parton distribution functions such that  $q^p(x)dx$  is the number of quarks of type  $q$  within a proton with momenta between  $x \rightarrow x + dx$

Expected **form of the parton distribution function** ?





- The cross section for scattering from a particular quark type within the proton which in the range  $x \rightarrow x + dx$  is

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ (1-y) + \frac{y^2}{2} \right] \times e_q^2 q^p(x) dx$$

- Summing over all types of quark within the proton gives the expression for the **electron-proton** scattering cross section

$$\frac{d^2\sigma^{\text{ep}}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ (1-y) + \frac{y^2}{2} \right] \sum_q e_q^2 q^p(x) \quad (5)$$

- Compare with the **electron-proton** scattering cross section in terms of structure functions (equation (2) ):

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[ (1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (6)$$

- ★ By comparing (5) and (6) obtain the parton model prediction for the structure functions in the general L.I. form for the differential cross section

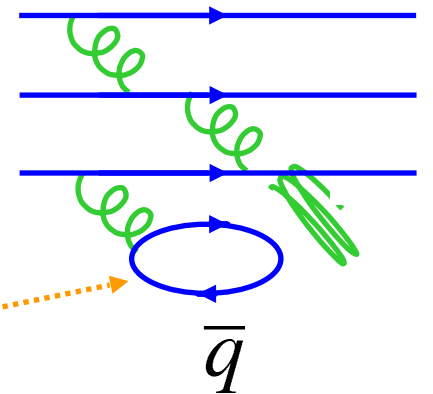
$$F_2^p(x, Q^2) = 2xF_1^p(x, Q^2) = x \sum_q e_q^2 q^p(x)$$



Can relate measured structure functions to the underlying quark distributions

## The parton model predicts:

- **Bjorken Scaling**  $F_1(x, Q^2) \rightarrow F_1(x)$      $F_2(x, Q^2) \rightarrow F_2(x)$ 
  - Due to scattering from **point-like particles** within the proton
- **Callan-Gross Relation**  $F_2(x) = 2xF_1(x)$ 
  - Due to scattering from **spin half Dirac particles** where the magnetic moment is directly related to the charge; hence the “electro-magnetic” and “pure magnetic” terms are fixed with respect to each other.
- At present parton distributions cannot be calculated from QCD
  - Can't use perturbation theory due to large coupling constant
- Measurements of the structure functions enable us to determine the parton distribution functions !
- For electron-proton scattering we have:
$$F_2^p(x) = x \sum_q e_q^2 q^p(x)$$
  - Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks (will neglect the small contributions from heavier quarks)



● **For electron-proton scattering have:**

$$F_2^{\text{ep}}(x) = x \sum_q e_q^2 q^{\text{p}}(x) = x \left( \frac{4}{9} u^{\text{p}}(x) + \frac{1}{9} d^{\text{p}}(x) + \frac{4}{9} \bar{u}^{\text{p}}(x) + \frac{1}{9} \bar{d}^{\text{p}}(x) \right)$$

● **For electron-neutron scattering have:**

$$F_2^{\text{en}}(x) = x \sum_q e_q^2 q^{\text{n}}(x) = x \left( \frac{4}{9} u^{\text{n}}(x) + \frac{1}{9} d^{\text{n}}(x) + \frac{4}{9} \bar{u}^{\text{n}}(x) + \frac{1}{9} \bar{d}^{\text{n}}(x) \right)$$

● **Now assume “isospin symmetry”, i.e. that the neutron (ddu) is the same as a proton (uud) with up and down quarks interchanged, i.e.**

$$d^{\text{n}}(x) = u^{\text{p}}(x); \quad u^{\text{n}}(x) = d^{\text{p}}(x)$$

**and define the neutron distributions functions in terms of those of the proton**

$$\begin{aligned} u(x) &\equiv u^{\text{p}}(x) = d^{\text{n}}(x); & d(x) &\equiv d^{\text{p}}(x) = u^{\text{n}}(x) \\ \bar{u}(x) &\equiv \bar{u}^{\text{p}}(x) = \bar{d}^{\text{n}}(x); & \bar{d}(x) &\equiv \bar{d}^{\text{p}}(x) = \bar{u}^{\text{n}}(x) \end{aligned}$$

**giving:**

$$F_2^{\text{ep}}(x) = 2xF_1^{\text{ep}}(x) = x \left( \frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{4}{9} \bar{u}(x) + \frac{1}{9} \bar{d}(x) \right) \quad (7)$$

$$F_2^{\text{en}}(x) = 2xF_1^{\text{en}}(x) = x \left( \frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{4}{9} \bar{d}(x) + \frac{1}{9} \bar{u}(x) \right) \quad (8)$$

● **Integrating (7) and (8) :**

$$\int_0^1 F_2^{\text{ep}}(x) dx = \int_0^1 x \left( \frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x)] \right) dx = \frac{4}{9} f_u + \frac{1}{9} f_d$$

$$\int_0^1 F_2^{\text{en}}(x) dx = \int_0^1 x \left( \frac{4}{9} [d(x) + \bar{d}(x)] + \frac{1}{9} [u(x) + \bar{u}(x)] \right) dx = \frac{4}{9} f_d + \frac{1}{9} f_u$$

●  $f_u = \int_0^1 [xu(x) + x\bar{u}(x)] dx$  **is the fraction of the proton momentum carried by the up and anti-up quarks**

**Experimentally**

$$\int F_2^{\text{ep}}(x) dx \approx 0.18$$

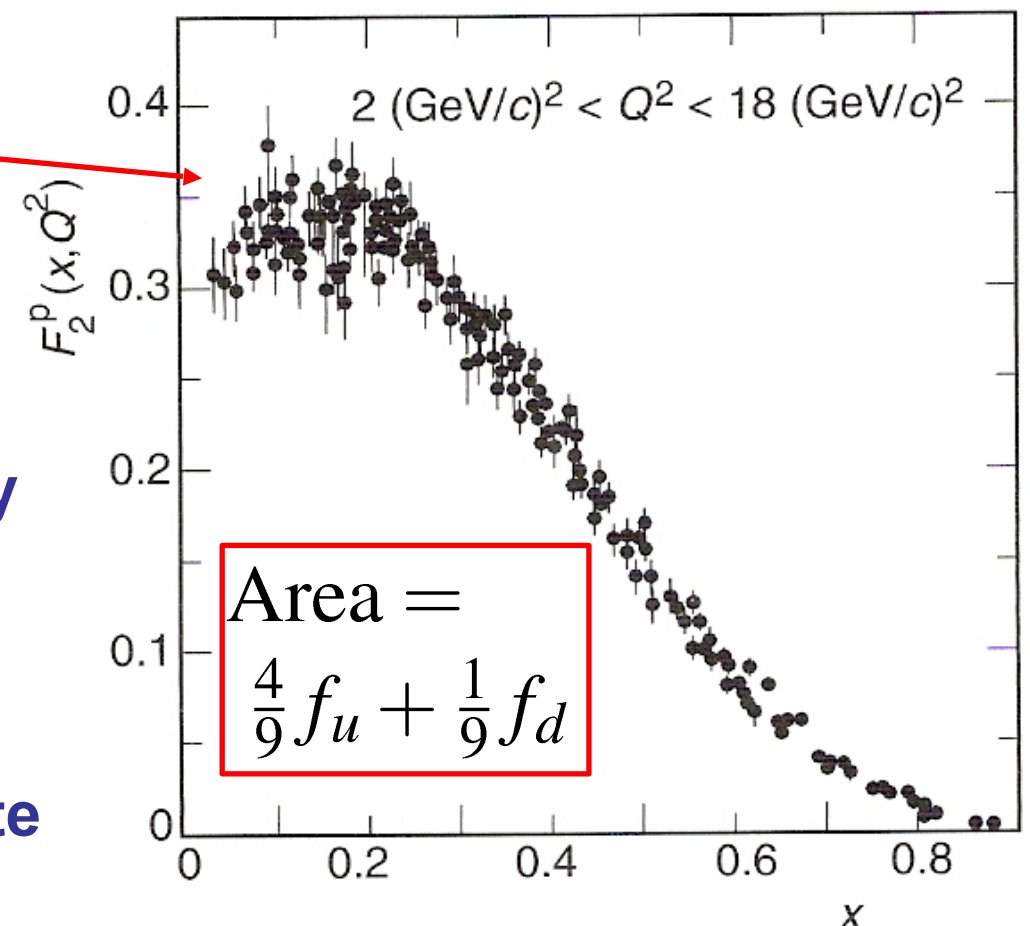
$$\int F_2^{\text{en}}(x) dx \approx 0.12$$



$$f_u \approx 0.36 \quad f_d \approx 0.18$$

● **In the proton, as expected, the up quarks carry twice the momentum of the down quarks**

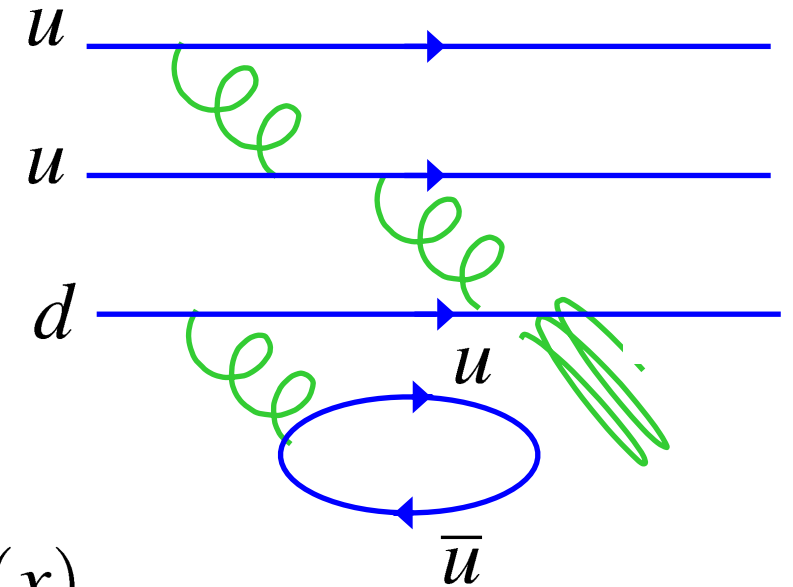
● **The quarks carry just over 50 % of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to **electron-nucleon** scattering).**



# Valence and Sea Quarks

- As we are beginning to see the proton is complex...

- The parton distribution function  $u^p(x) = u(x)$  includes contributions from the “**valence**” quarks and the virtual quarks produced by gluons: the “**sea**”



- Resolving into valence and sea contributions:

$$u(x) = u_V(x) + u_S(x) \quad d(x) = d_V(x) + d_S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) \quad \bar{d}(x) = \bar{d}_S(x)$$

- The proton contains two valence up quarks and one valence down quark and would expect:

$$\int_0^1 u_V(x) dx = 2 \quad \int_0^1 d_V(x) dx = 1$$

- But no **a priori** expectation for the total number of sea quarks !
- But sea quarks arise from gluon quark/anti-quark pair production and with  $m_u = m_d$  it is reasonable to expect

$$u_S(x) = d_S(x) = \bar{u}_S(x) = \bar{d}_S(x) = S(x)$$

- With these relations (7) and (8) become

$$F_2^{\text{ep}}(x) = x \left( \frac{4}{9} u_V(x) + \frac{1}{9} d_V(x) + \frac{10}{9} S(x) \right) \quad F_2^{\text{en}}(x) = x \left( \frac{4}{9} d_V(x) + \frac{1}{9} u_V(x) + \frac{10}{9} S(x) \right)$$

Giving the ratio

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

- The sea component arises from processes such as  $g \rightarrow \bar{u}u$ . Due to the  $1/q^2$  dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of **low energy**  $q/\bar{q}$

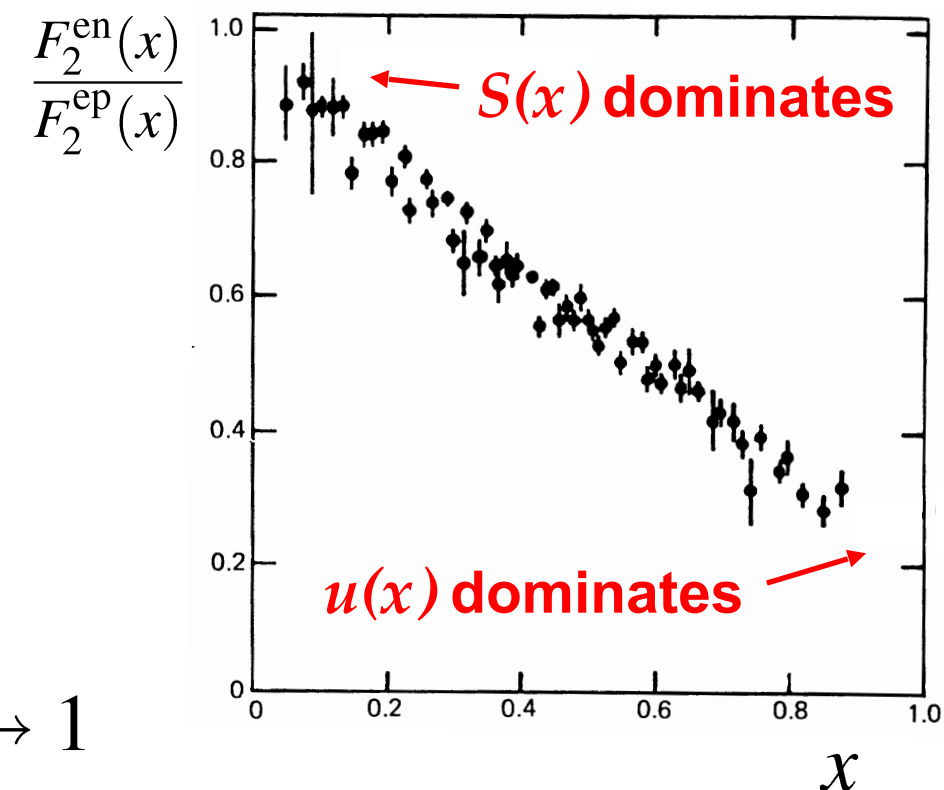
- Therefore at low  $x$  expect the sea to dominate:

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \rightarrow 1 \quad \text{as } x \rightarrow 0$$

**Observed experimentally**

- At high  $x$  expect the sea contribution to be small

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \rightarrow \frac{4d_V(x) + u_V(x)}{4u_V(x) + d_V(x)} \quad \text{as } x \rightarrow 1$$



**Note:**  $u_V = 2d_V$  would give ratio **2/3** as  $x \rightarrow 1$

**Experimentally**  $F_2^{\text{en}}(x)/F_2^{\text{ep}}(x) \rightarrow 1/4$  as  $x \rightarrow 1$

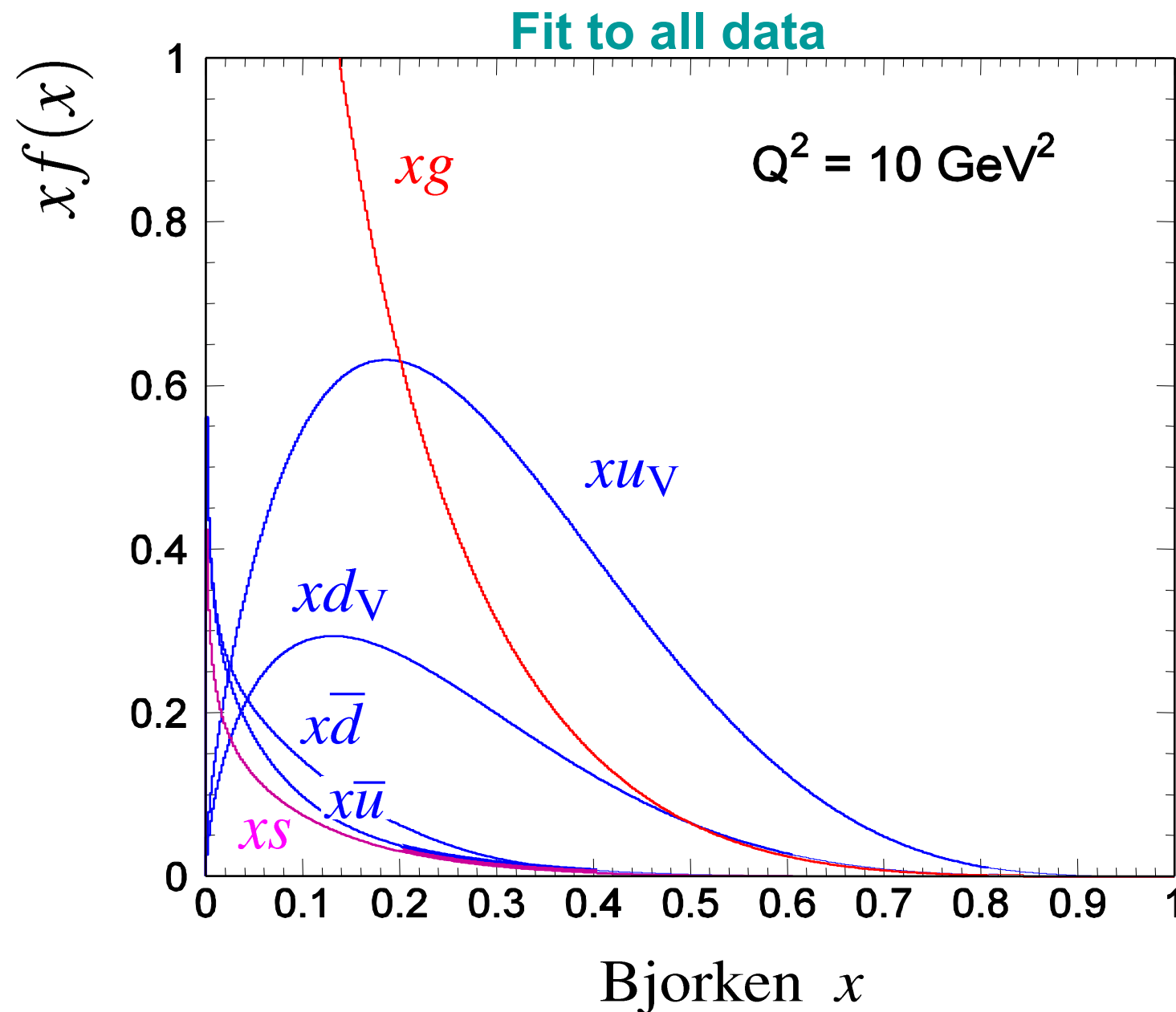
$$\rightarrow d(x)/u(x) \rightarrow 0 \quad \text{as } x \rightarrow 1$$

**This behaviour is not understood.**



# Parton Distribution Functions

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering
- Hadron-hadron collisions give information on gluon pdf  $g(x)$



## Note:

- Apart from at large  $x$   
 $u_v(x) \approx 2d_v(x)$
- For  $x < 0.2$   
gluons dominate
- In fits to data assume  
 $u_s(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$   
not understood –  
exclusion principle?
- Small strange quark  
component  $s(x)$

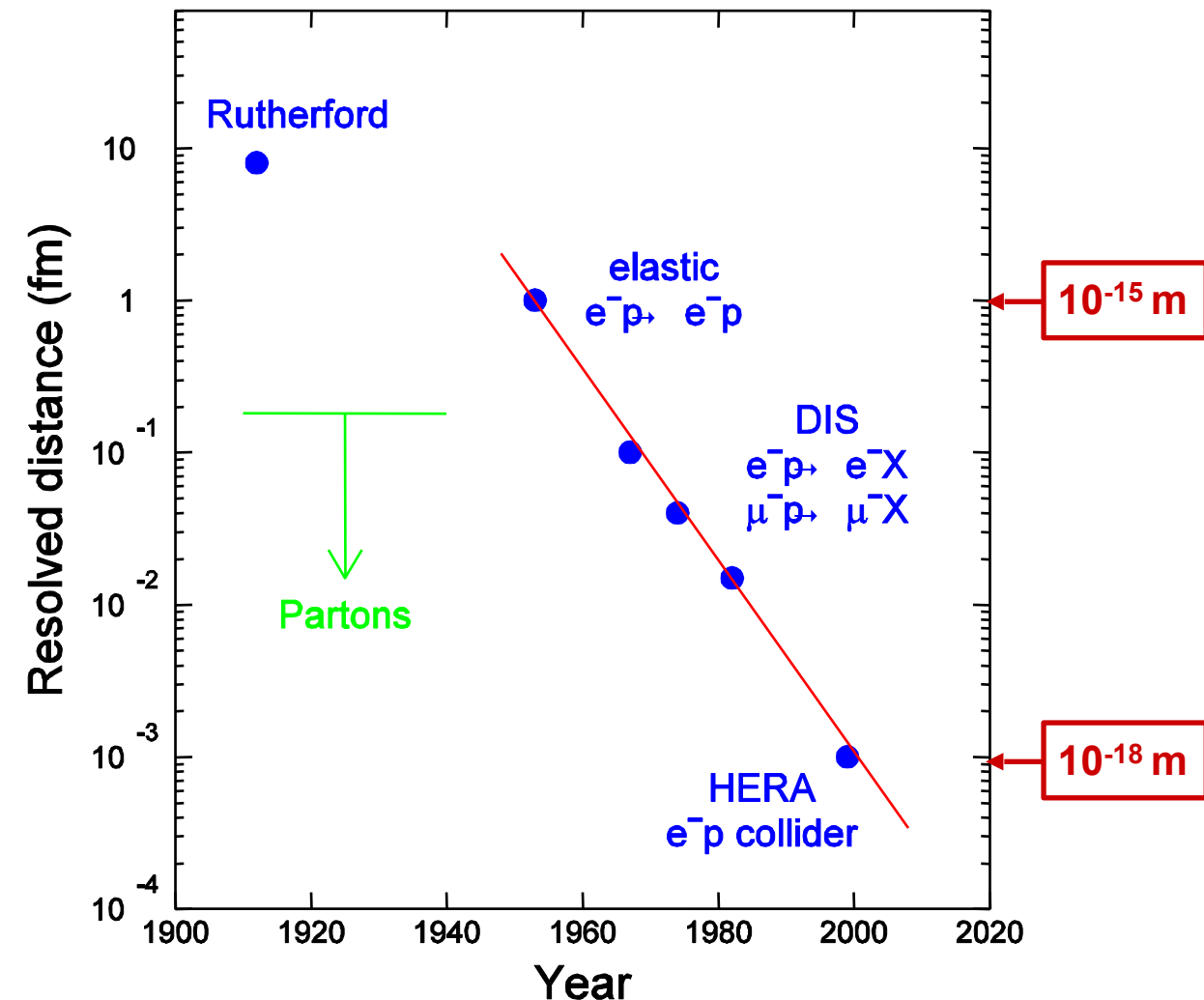
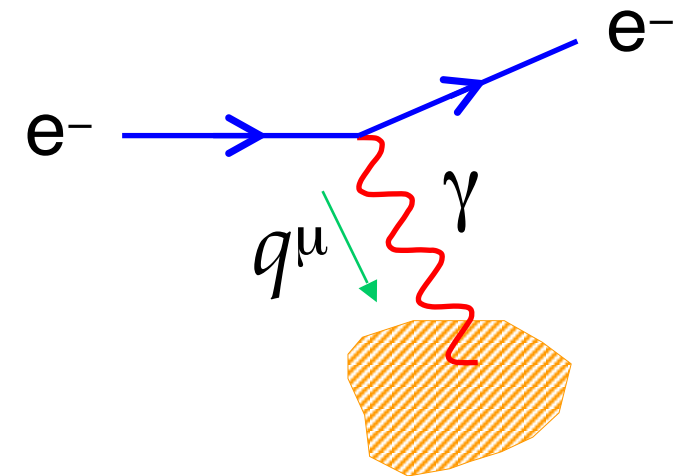
# Scaling Violations

- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy
- Non-point like nature of the scattering becomes apparent when  $\lambda_\gamma \sim$  size of scattering centre

$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1 \text{ GeV fm}}{|\vec{q}| (\text{GeV})}$$

- Scattering from point-like quarks gives rise to **Bjorken scaling**: no  $q^2$  cross section dependence
- IF quarks were not point-like, at high  $q^2$  (when the wavelength of the virtual photon  $\sim$  size of quark) would observe rapid decrease in cross section with increasing  $q^2$ .
- To search for quark sub-structure want to go to highest  $q^2$

**HERA**

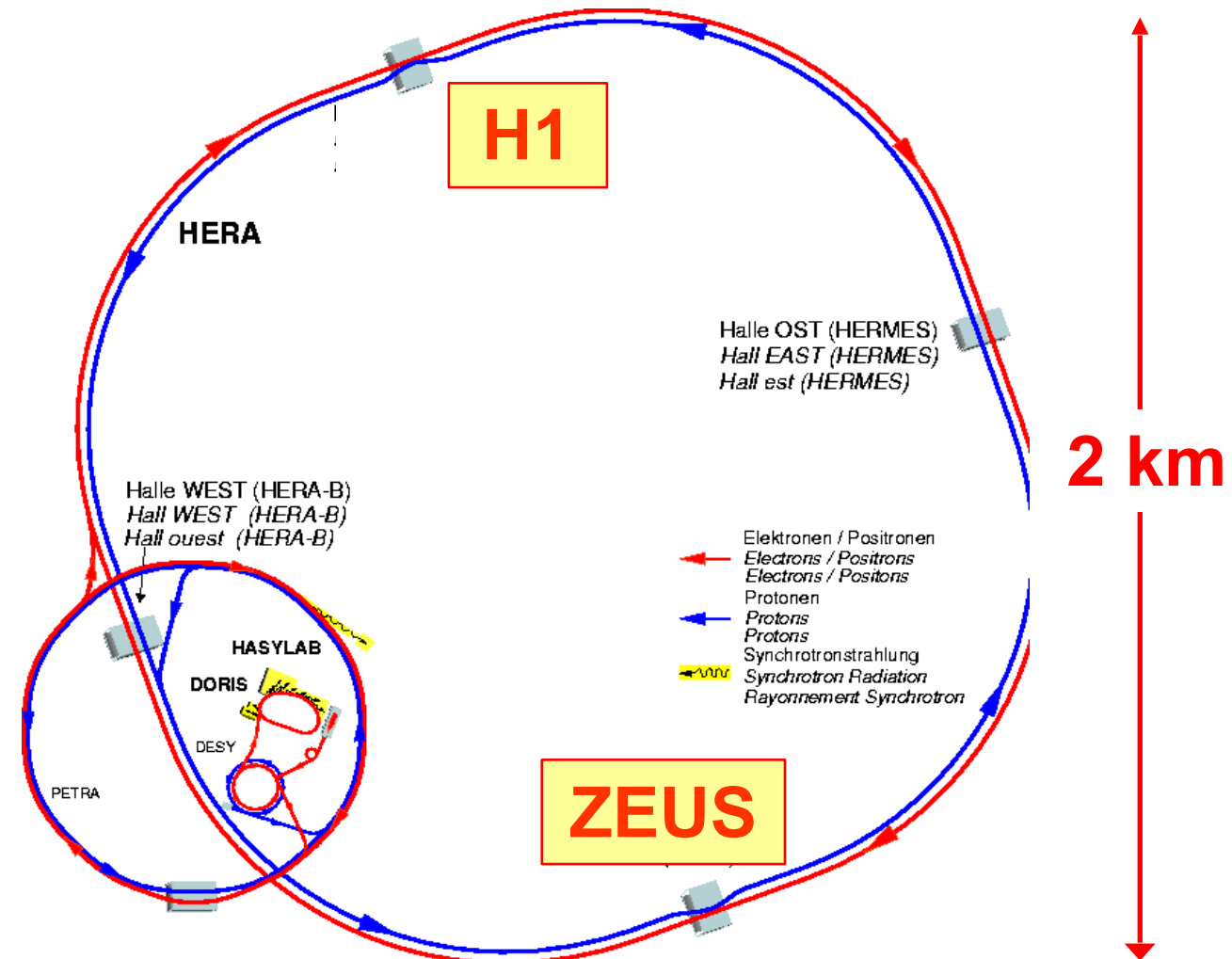




# HERA $e^\pm p$ Collider : 1991-2007

★ DESY (Deutsches Elektronen-Synchrotron) Laboratory, Hamburg, Germany

$e^\pm$   $\xrightarrow{27.5 \text{ GeV}}$   $\xleftarrow{820 \text{ GeV}} p$   $\sqrt{s} = 300 \text{ GeV}$

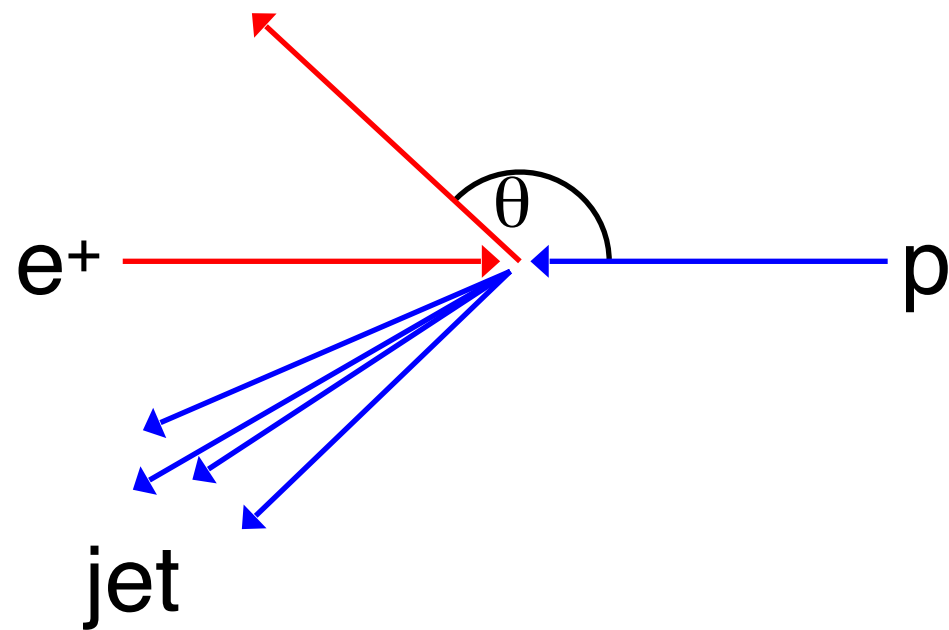


★ Two large experiments : H1 and ZEUS

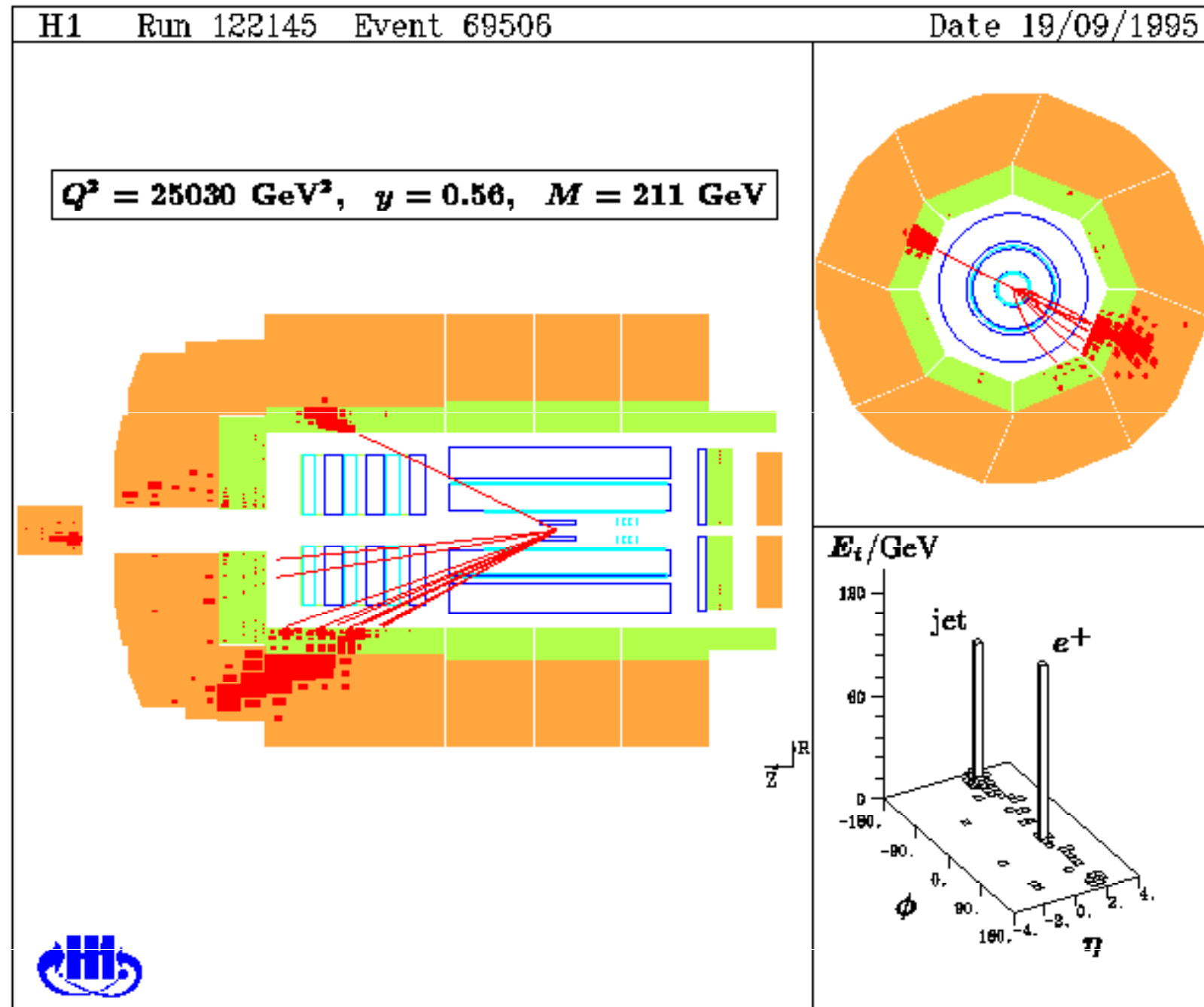
★ Probe proton at very high  $Q^2$  and very low  $x$

# Example of a High $Q^2$ Event in H1

★ Event kinematics determined from electron angle and energy



★ Also measure hadronic system (although not as precisely) – gives some redundancy



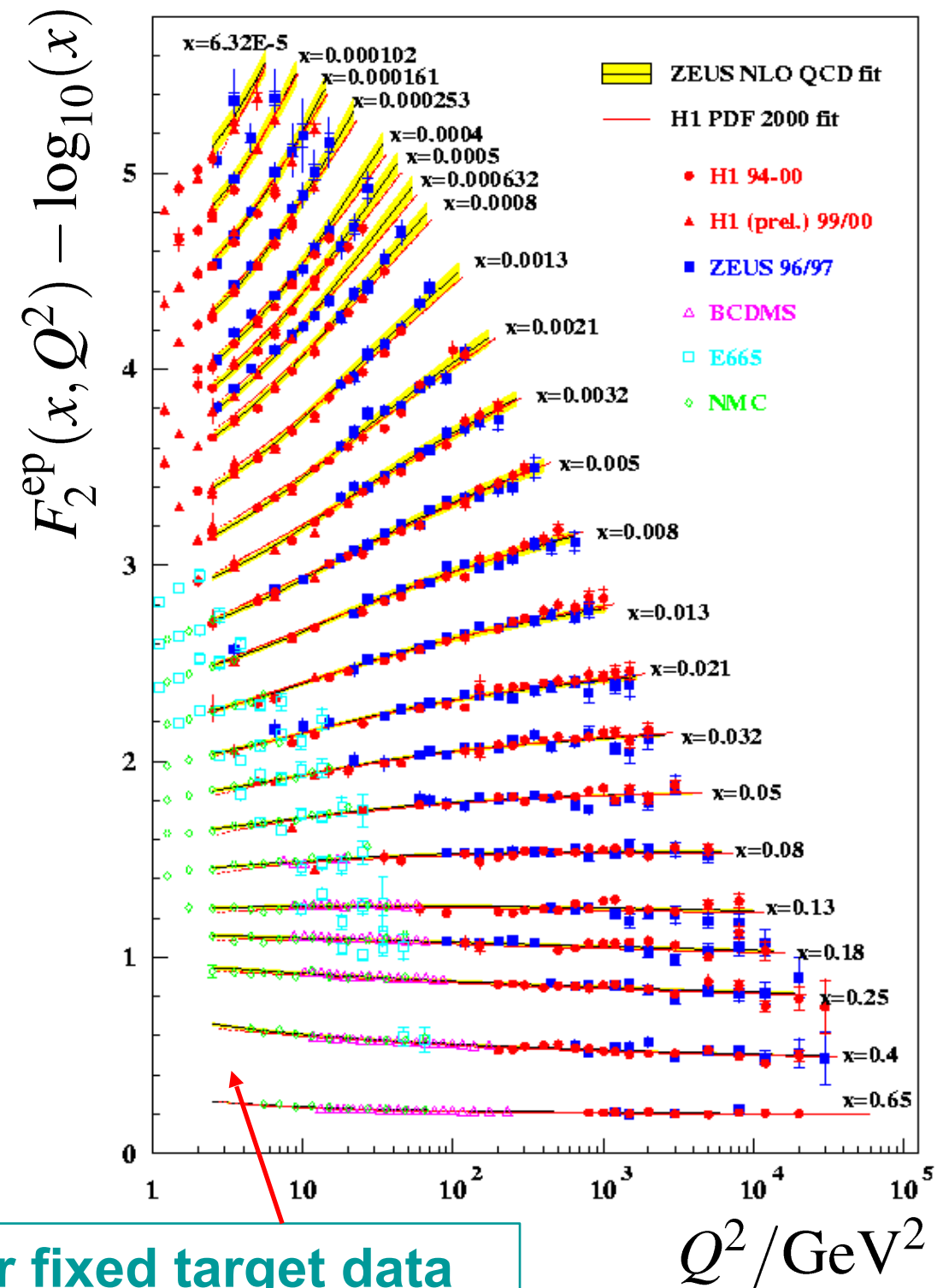
# $F_2(x, Q^2)$ Results

- ★ No evidence of rapid decrease of cross section at highest  $Q^2$

→  $R_{\text{quark}} < 10^{-18} \text{ m}$

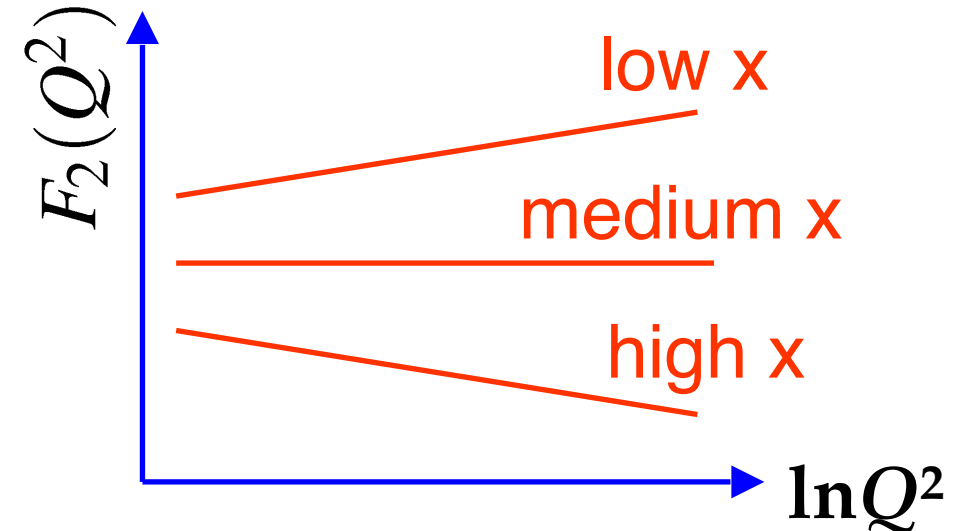
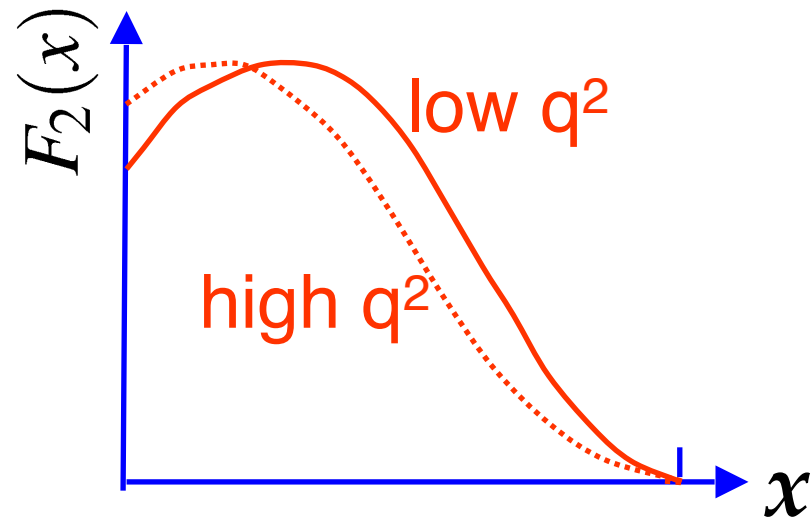
- ★ For  $x > 0.05$ , only weak dependence of  $F_2$  on  $Q^2$  : consistent with the expectation from the quark-parton model
- ★ But observe clear scaling violations, particularly at low  $x$

$$F_2(x, Q^2) \neq F_2(x)$$



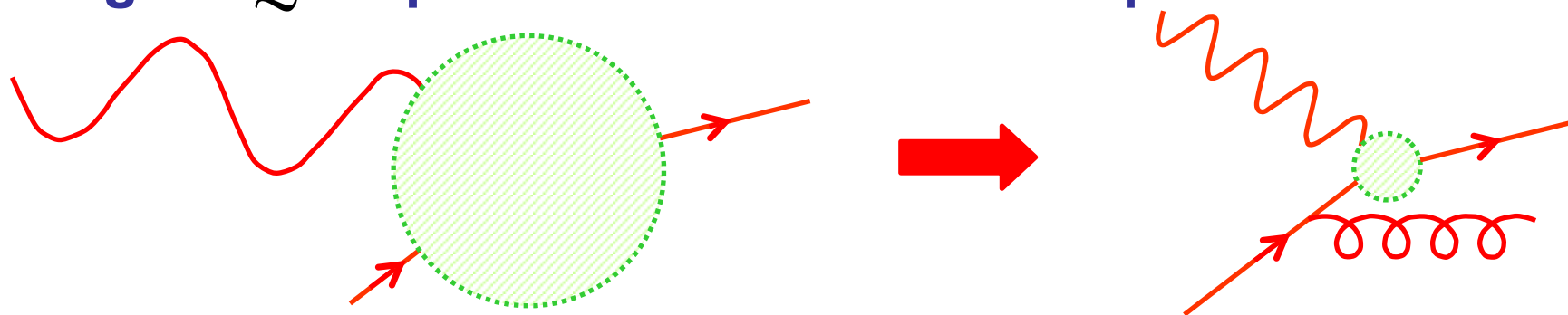
# Origin of Scaling Violations

★ Observe “small” deviations from **exact Bjorken scaling**  $F_2(x) \rightarrow F_2(x, Q^2)$



★ At high  $Q^2$  observe more low  $x$  quarks

★ “Explanation”: at high  $Q^2$  (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher  $Q^2$  expect to “see” more low  $x$  quarks



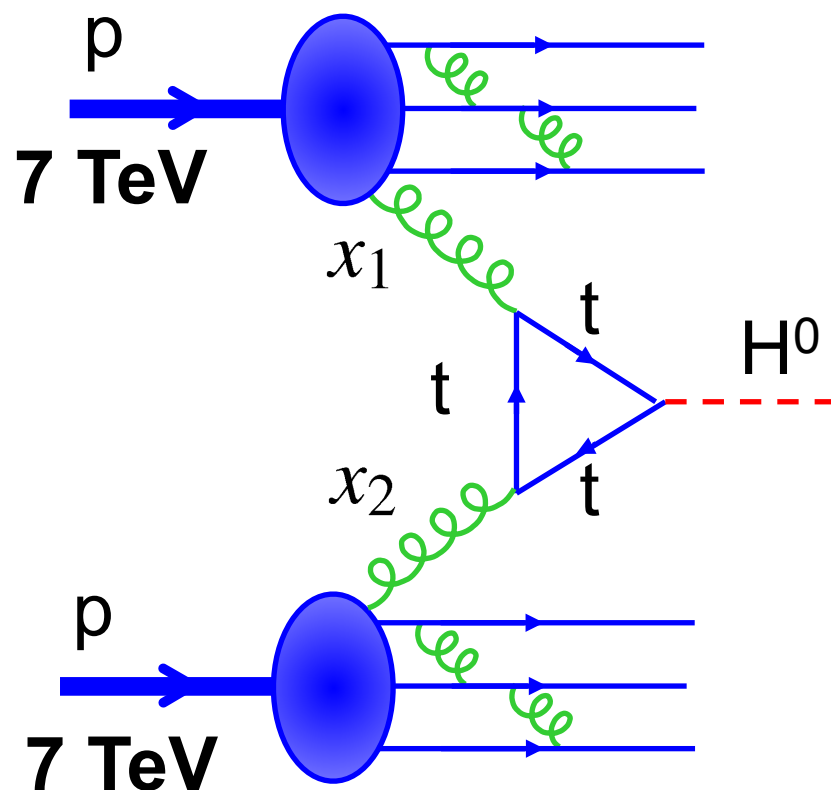
★ QCD cannot predict the  $x$  dependence of  $F_2(x, Q^2)$

★ But QCD **can** predict the  $Q^2$  dependence of  $F_2(x, Q^2)$



# Proton-Proton Collisions at the LHC

- ★ Measurements of structure functions not only provide a powerful test of QCD, the **parton distribution functions** are essential for the calculation of cross sections at  $pp$  and  $p\bar{p}$  colliders.
- **Example:** Higgs production at the Large Hadron Collider **LHC** ( 2009-)
  - The LHC will collide 7 TeV protons on 7 TeV protons
  - However underlying collisions are between partons
  - Higgs production the LHC dominated by “**gluon-gluon fusion**”

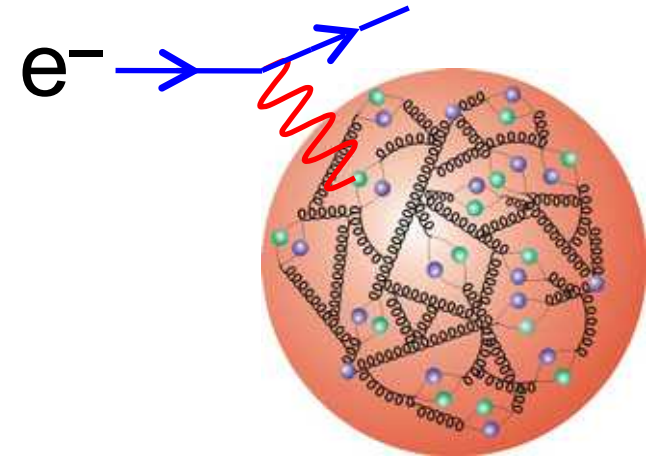


- Cross section depends on gluon PDFs

$$\sigma(pp \rightarrow HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \rightarrow H)dx_1dx_2$$

- Uncertainty in gluon PDFs lead to a  $\pm 5\%$  uncertainty in Higgs production cross section
- Prior to HERA data uncertainty was  $\pm 25\%$

# Summary



- ♦ At **very high** electron energies  $\lambda \ll r_p$  :  
the proton appears to be a sea of quarks and gluons.
- ♦ Deep Inelastic Scattering = Elastic scattering from the quasi-free constituent quarks
  - ➡ Bjorken Scaling  $F_1(x, Q^2) \rightarrow F_1(x)$  **point-like scattering**
  - ➡ Callan-Gross  $F_2(x) = 2xF_1(x)$  **Scattering from spin-1/2**
- ♦ Describe scattering in terms of parton distribution functions  $u(x), d(x), \dots$  which describe momentum distribution inside a nucleon
- ♦ The proton is much more complex than just uud - sea of anti-quarks/gluons
- ♦ Quarks carry only 50 % of the protons momentum – the rest is due to low energy gluons
- ♦ We will come back to this topic when we discuss neutrino scattering...