Top quark mass effects in diphoton production at NNLO in QCD

Matteo Becchetti

Università di Bologna

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In collaboration with R. Bonciani, L. Cieri, F. Coro, F. Ripani

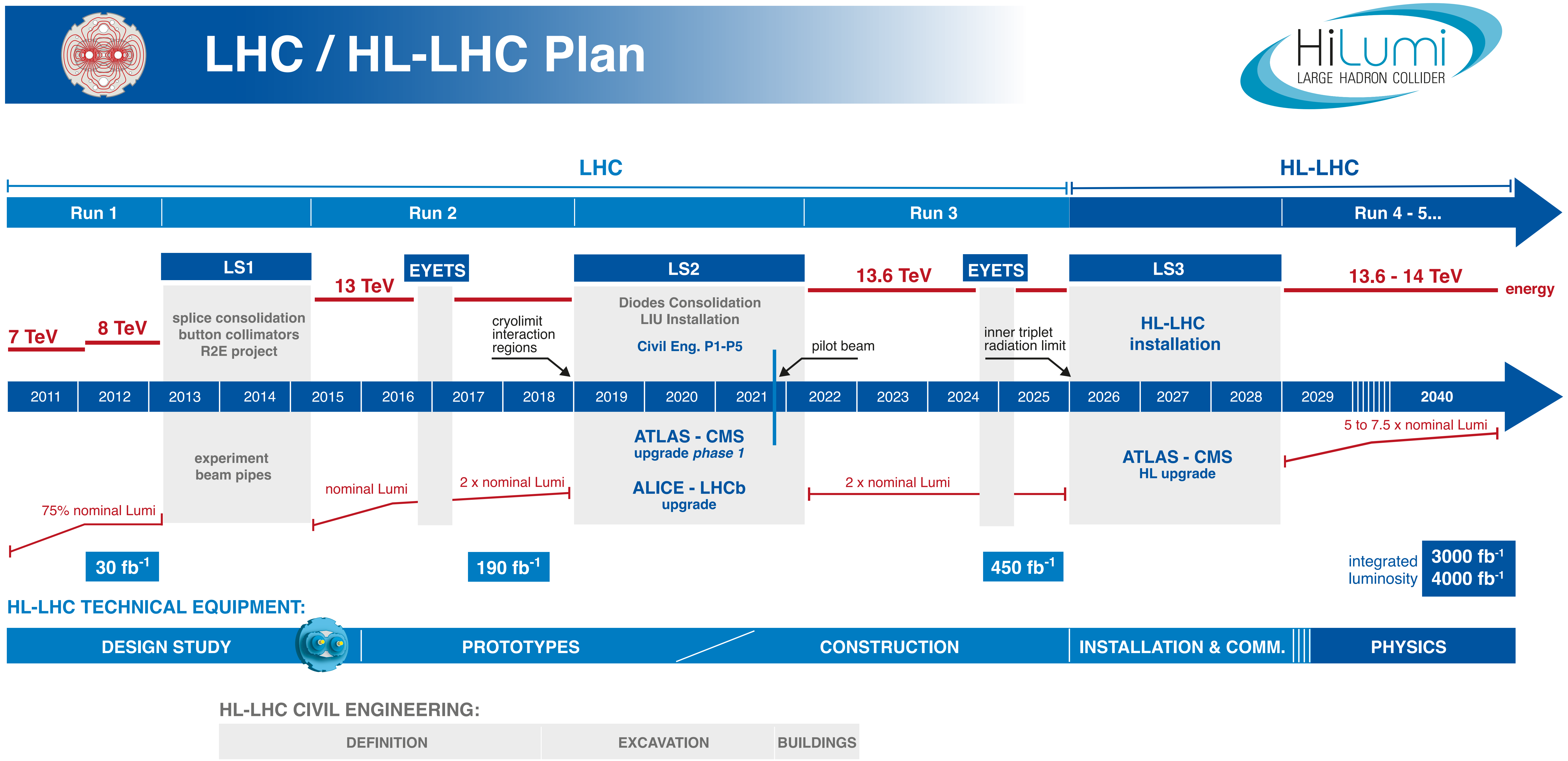
Outline

- Introduction
- Two-loop Amplitude for the quark annihilation channel
- Phenomenology
- Conclusion and Outlook

Introduction

Motivation

High-Luminosity LHC Plan



- Experimental precision $\sim \mathcal{O}(1\%)$ for many observables

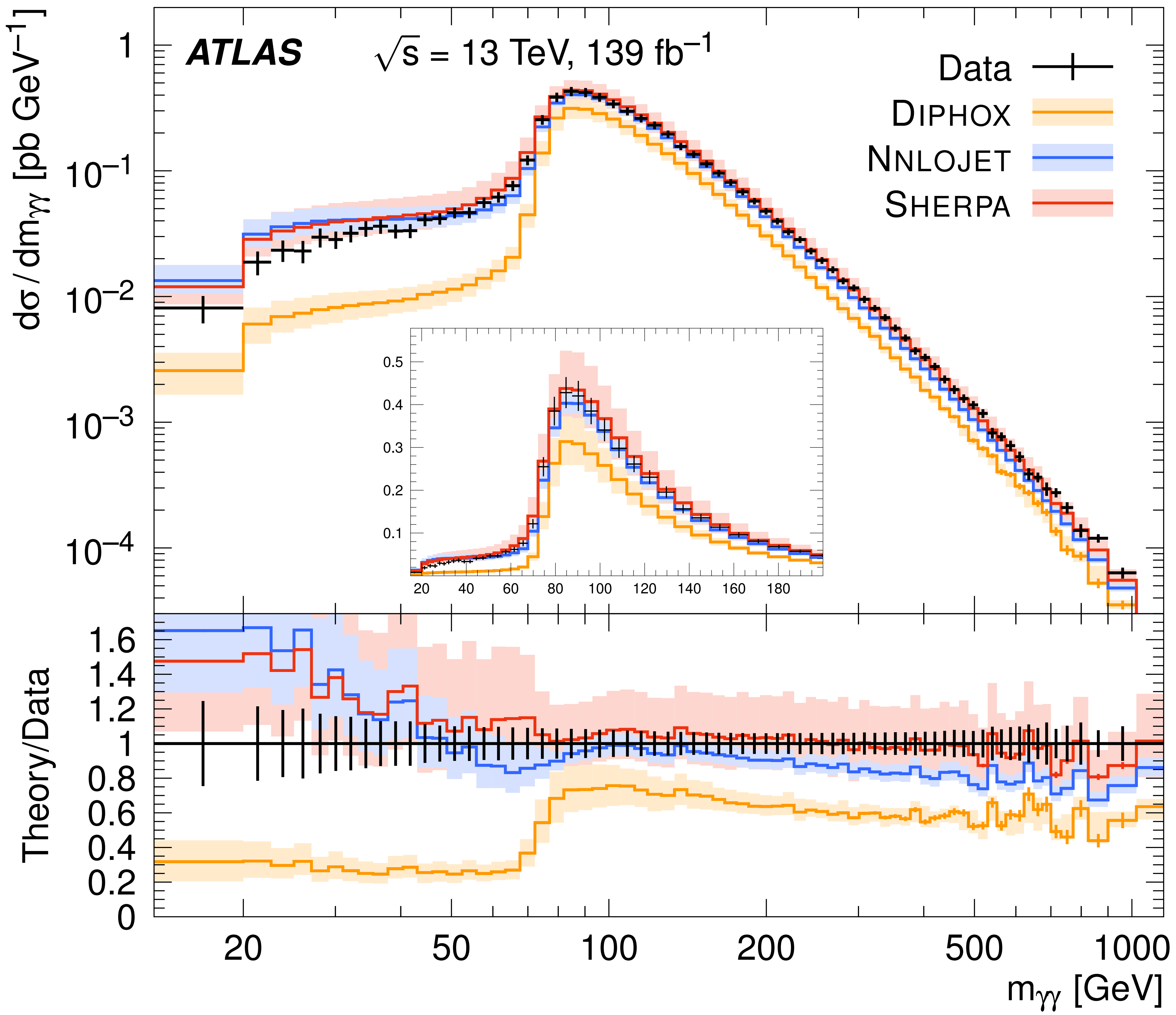
- NNLO QCD Corrections required to reduce theoretical uncertainty

Diphoton Production

- It is a probe for the SM, and check of the validity of pQCD
- Important channel for Higgs studies
- Irreducible background for NP searches
- Possible alternative channel for measuring top quark mass

[Kawabat, Yokoya ’17; Dugad, Jain, Mitra, Sanyal, Verma ’18]

- Interesting framework to asses size of massive corrections at NNLO



[ATLAS (2017) arXiv:2107.09330]

State of the Art

- NNLO QCD corrections with five light quarks flavours
  [Catani, Cieri, de Florian, Fererra, Grazzini ’12 ’18]
  [Campbell, Ellis, Li, Williams ’16]
  [Schuermann, Chen, Gehrmann, Glover, Hofer, Huss ’22]

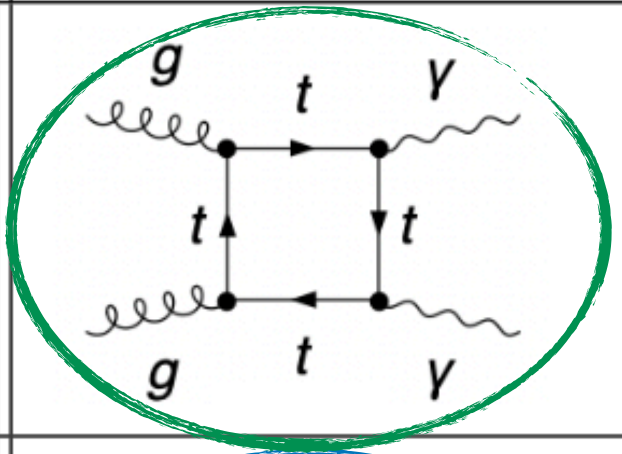
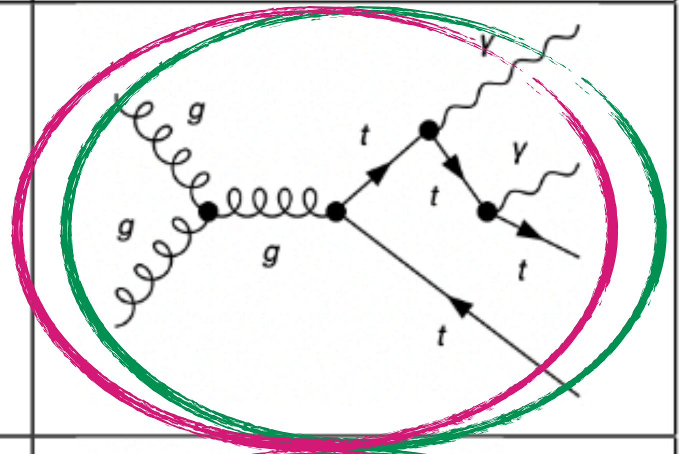
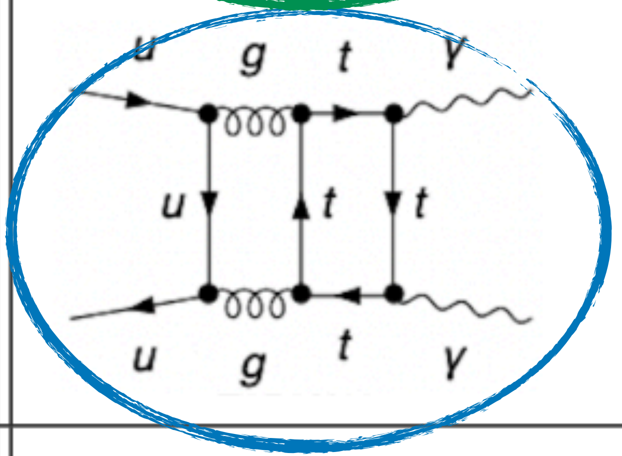
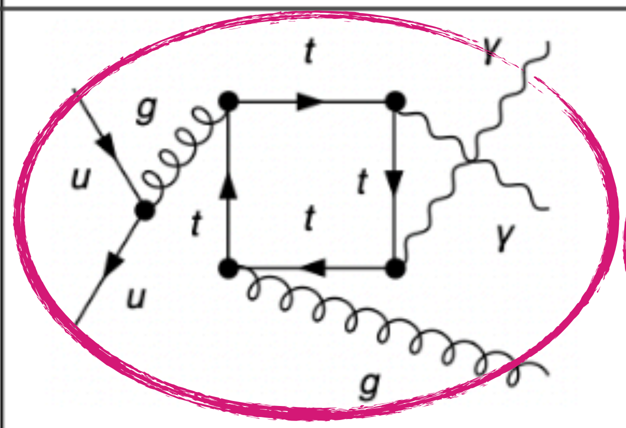
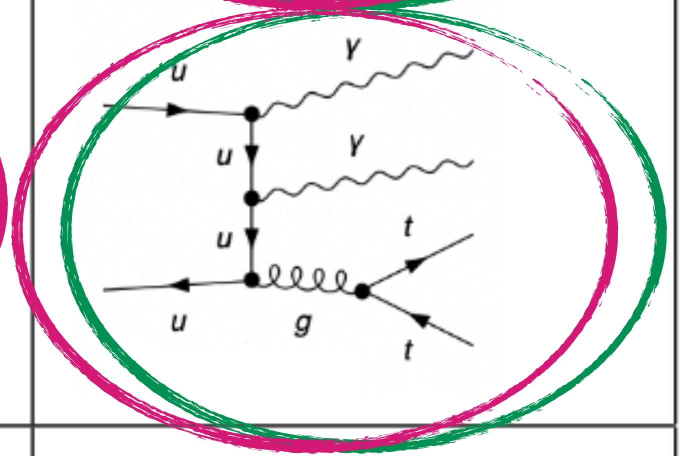
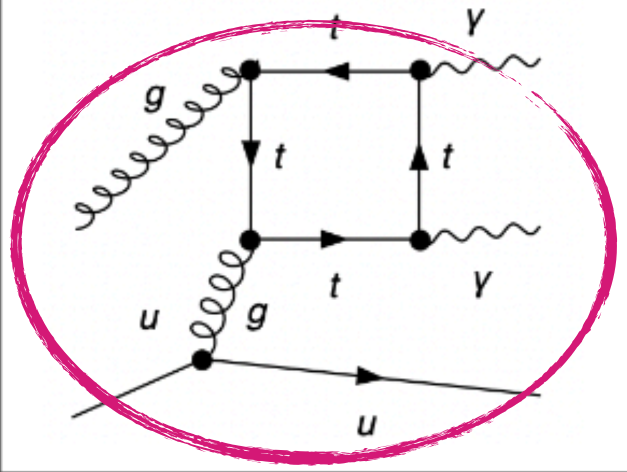
- Necessary scattering amplitude elements for N3LO analysis (massless case)
  [Bern, De Freitas, Dixon; Coal, Chakraborty, Gambuti, von Manteuffel, Tancredi; Chawdhry, Czakon, Mitov, Poncelet; Agarwal, Buccioni, von Manteuffel, Tancredi; Badger, Gehrmann, Marcoli, Moodie]

- First-order Electroweak\QED corrections
  [Cieri, Sborlini ’21; Binoth, Guillet, Pilon, Werlen ’00; Chiesa, Greiner, Schoenherr, Tramontano ’17]

- NLO top quark mass effects in the gluon fusion channel
  [Maltoni, Mandal, Zhao ’19; Chen, Heinrich, Jahn, Jones, Kerner, Schlenk, Yokoya ’20]

Massive Contributions at NNLO

Massive corrections $\mathcal{O}(\alpha_s^2)$

Channels	$\gamma\gamma$	$\gamma\gamma j$	$\gamma\gamma jj$
gg		✗	
$q\bar{q}$			
qg	✗		✗

- ★ Double-Virtual Contribution [MB, Bonciani, Cieri, Coro, Ripani ’23]
- ★ One-loop box Contribution [Campbell, Ellis, Li, Williams ’16]
- ★ Real-Virtual Contribution
- ★ Double-Real Contribution

qT Subtraction Scheme

- For the production of a singlet-colour system F in hadron collision

$$d\sigma^{F}_{(N)NLO}\big|_{q^T\neq 0} = d\sigma^{F+jets}_{(N)LO}$$

- Singular behaviour of the cross section for the system F+jest, at qT=0, known

$$d\sigma^{CT} = d\sigma^{F}_{(LO)} \otimes \Sigma^{F}(q_T/Q)$$

$$\Sigma^{F}(q_T/Q) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n}\sum_{k=1}^{2n}\Sigma^{F(n;k)}\frac{Q^2}{q_T^2}\log^{k-1}\frac{Q^2}{q_T^2}$$

- Singular behaviour from resummation of logarithmic contributions at small transverse momentum

[Parisi, Petronzio '79; Collins, Soper, Sterman '85; Catani, de Florian, Grazzini '00]

qT Subtraction Scheme

- NNLO cross section in qT subtraction scheme [Catani, Grazzini '07]

$$\mathrm{d}\,\sigma^{\gamma\gamma}_{NNLO} = \mathcal{H}^{\gamma\gamma}_{NNLO} \otimes \mathrm{d}\,\sigma^{\gamma\gamma}_{LO} + \left[\mathrm{d}\,\sigma^{\gamma\gamma+jets}_{NLO} - \mathrm{d}\,\sigma^{CT}_{NLO}\right]$$

Hard virtual function

NLO cross section for diphoton plus jet

Counterterm for small-qt singularities

- NLO (massless) cross section for diphoton plus jet known [Del Duca, Maltoni, Nagy, Trocsanyi '03]
- NLO (massless) counterterm known [Del Duca, Maltoni, Nagy, Trocsanyi '03]
- One-loop and two-loop (massless) Hard Function contributions known [Balazs, Berger, Mrenna, Yuan '98] [Anastasiou, Glover, Tejeda-Yeomans '02] [Catani, Cieri, de Florian, Ferrera, Grazzini '14]
- We include massive contribution to the cross section at NNLO

Hard Function quark annihilation channel

- All-order relation

$$\mathcal{H}^{q\bar{q},\gamma\gamma} = \frac{|A^{(\mathrm{fin})}_{q\bar{q},\gamma\gamma}|^2}{|A^{(0)}_{q\bar{q},\gamma\gamma}|^2}$$

Finite reminder

Born Amplitude

- Perturbative expansion

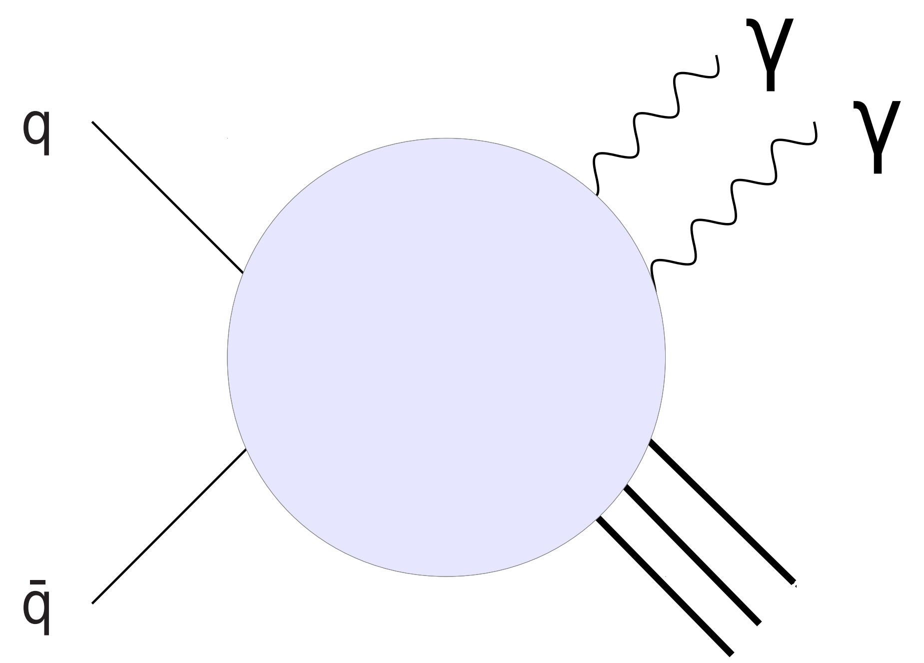
$$\mathcal{H}^{q\bar{q},\gamma\gamma} = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{q\bar{q},\gamma\gamma}_{NLO} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{q\bar{q},\gamma\gamma}_{NNLO} + \cdots$$

- The two-loop massive amplitude in quark annihilation channel is IR finite, after UV regularisation

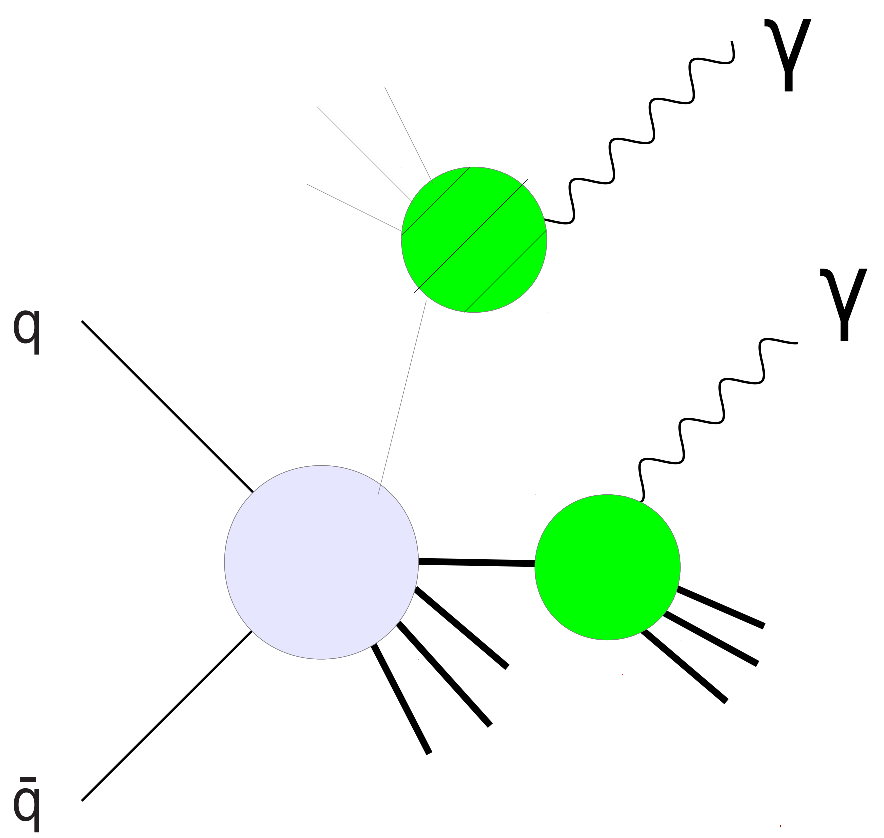
- The two-loop massive corrections can be included by simple addition in the two-loop Hard Function

Photon Isolation Criteria

Direct Component: photon production from hard interaction

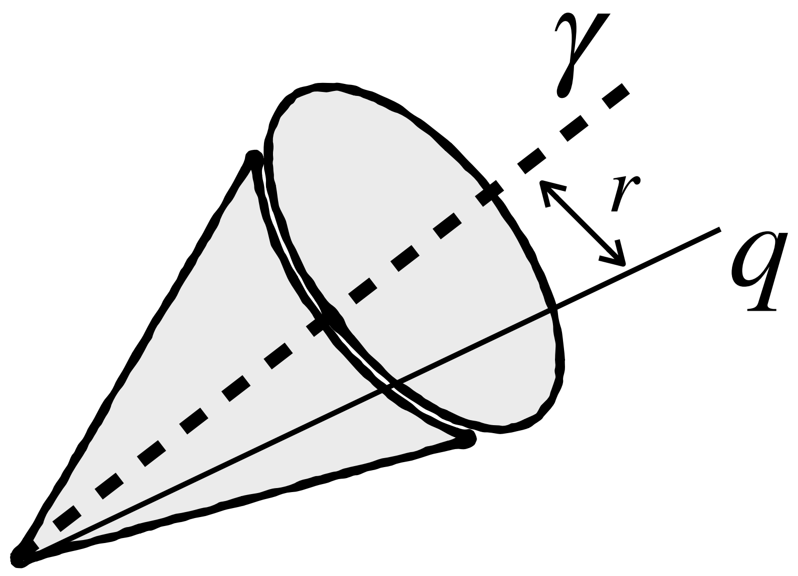


Fragmentation Component: photon production from non-perturbative fragmentation of hard Parton



- Experimentally photons have to be isolated
- Isolation reduces fragmentation component

We use smooth cone isolation [Frixione '98]



$$\sum_{r<R} E_T^{had} \leq \epsilon p_{T_\gamma} \chi(r;R)$$

$$\chi(r;R) = \left(\frac{r}{R}\right)^{2n}$$

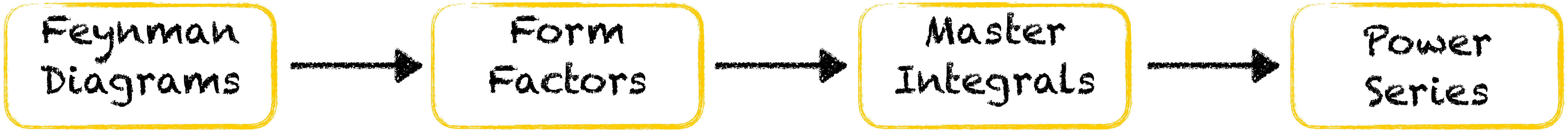
IR-Safe Cross Section

r->0: No fragmentation Component

Two-loop Amplitude for the quark annihilation channel

Strategy of the Computation

- Missing ingredient for a complete NNLO analysis of diphoton production with top quark mass dependence
- Analytic structure of Feynman integrals involves elliptic geometries
- We evaluate numerically the integrals using power series expansion technique [Moriello '18]



Form Factors Decomposition

- We consider scattering amplitudes for diphoton production in quark annihilation channel

$$\mathscr{A}_{q\bar{q},\gamma\gamma}(s,t,m_t^2) = \sum_{i=1}^{4} \mathscr{F}_i(s,t,m_t^2)\bar{v}(p_2)\Gamma_i^{\mu\nu}u(p_1)\epsilon_{3,\mu}\epsilon_{4,\nu}$$

- The amplitude can be decomposed as sum of four independent tensor structures

$$\Gamma_1^{\mu\nu} = \gamma^\mu p_2^\nu, \quad \Gamma_2^{\mu\nu} = \gamma^\nu p_1^\mu, \quad \Gamma_3^{\mu\nu} = p_{3,\rho}\gamma^\rho p_1^\mu p_2^\nu, \quad \Gamma_4^{\mu\nu} = p_{3,\rho}\gamma^\rho g^{\mu\nu}$$

- We compute the two-loop form factors contribution coming from diagrams with heavy quark loops

$$\mathscr{F}_i = \mathscr{F}_i^{(0)} + \left(\frac{\alpha_S^B}{\pi}\right)\mathscr{F}_i^{(1)} + \left(\frac{\alpha_S^B}{\pi}\right)^2\mathscr{F}_i^{(2)} + \cdots$$

Massive Quark Contribution

- Contribution from heavy quark loops appear at order $\mathcal{O}(\alpha_S^2)$

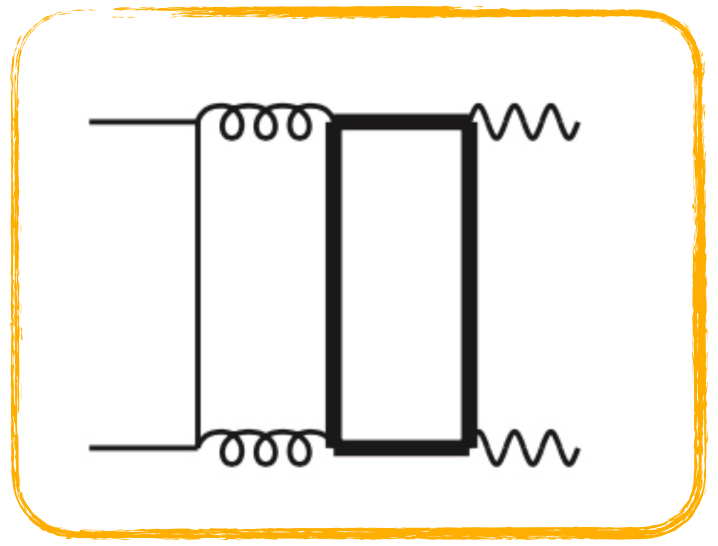
$$\mathcal{F}^{(2)}_{i,\mathtt{top}} = 4\pi\alpha_{em}\delta_{kl}C_F\left[Q_q^2\mathcal{F}^{(2)}_{i,\mathtt{top};0} + Q_t^2\mathcal{F}^{(2)}_{i,\mathtt{top},2}\right]$$

$$C_F = \frac{N_c^2 - 1}{2N_c}$$

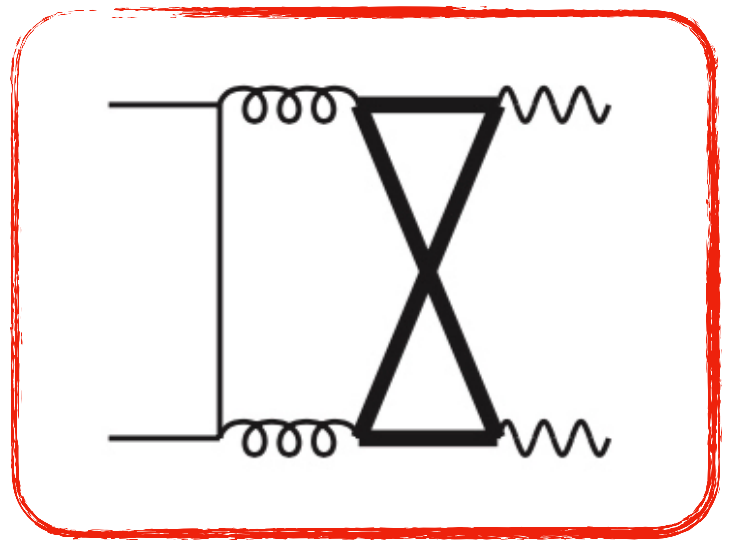
Q_q Light-quark electric charge

Q_t Heavy-quark electric charge

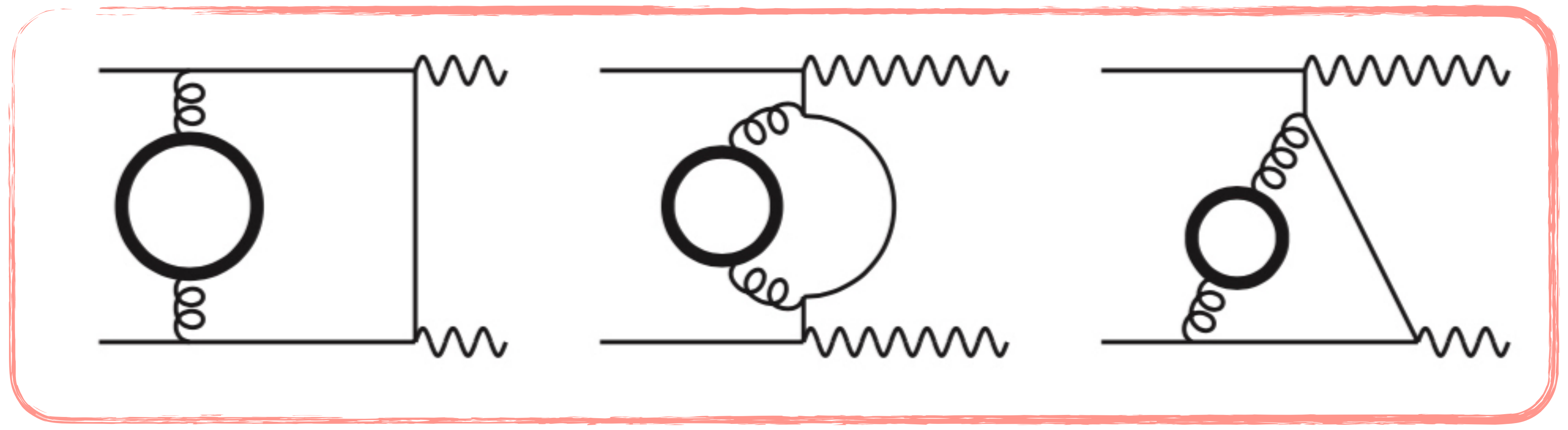
- Three different kind of contributions



PLA



NPL



PLB

UV and IR structure

- Since diagrams with a heavy quark loop start contributing at two loop, $\mathscr{F}^{(2)}_{i,\mathtt{top}}$ does not have IR singularities
- All the divergences are of UV origin
- Renormalisation is performed in a mixed scheme

$$\mathscr{F}_i^R = Z_q \mathscr{F}_i \left(\alpha_S^B \to Z_{\alpha_S} \alpha_S \right)$$

On-shell Scheme

Light-quark $\overline{\text{MS}}$

Heavy-quark OS

$$\mathscr{F}^{(2)R}_{i,\mathtt{top}} = \mathscr{F}^{(2)}_{i,\mathtt{top}} + \delta Z_q^{(2)} \mathscr{F}_i^{(0)} + \delta Z^{(1)}_{\alpha,N_h,OS} \mathscr{F}_i^{(1)}$$

$$Z_q = 1 + \left(\frac{\alpha_S}{\pi}\right) \delta Z_q^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \delta Z_q^{(2)} + \cdots$$

$$Z_{\alpha_S} = 1 + \left(\frac{\alpha_S}{\pi}\right) \left(\delta Z^{(1)}_{\alpha,N_l,\overline{MS}} + \delta Z^{(1)}_{\alpha,N_h,OS} \right) + \cdots$$

Scalar Integrals Topologies

- Form factors written as linear combination of 72 Master integrals (MIs)

- MIs can be cast into three independent scalar integrals topologies

$$\mathcal{I}_{\text{topo}}(n_1,\ldots,n_9) = \int \frac{\mathcal{D}k_1 \mathcal{D}k_2}{D_1^{n_1} D_2^{n_2} D_3^{n_3} D_4^{n_4} D_5^{n_5} D_6^{n_6} D_7^{n_7} D_8^{n_8} D_9^{n_9}}$$

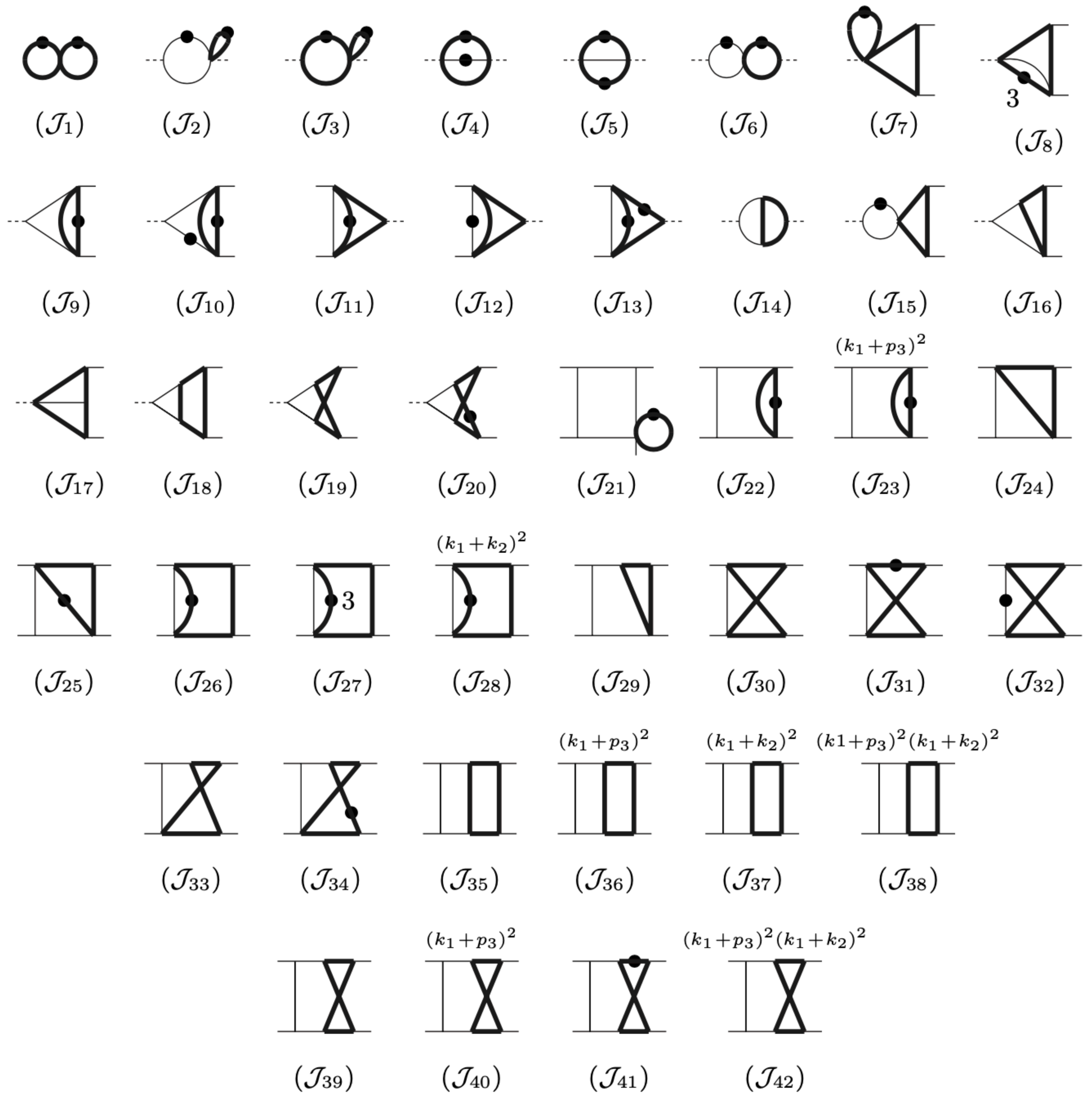
- Two Planar Topologies, PLA and PLB, analytically computable in terms of Multiple Polylogarithmic functions

[Aglietti, MB, Bonciani, Caron-huot, Ferroglia, Henn, Mastrolia, Penin, Remiddi,...]

- Non Planar Topology, NPL, analytic structure described by elliptic geometry

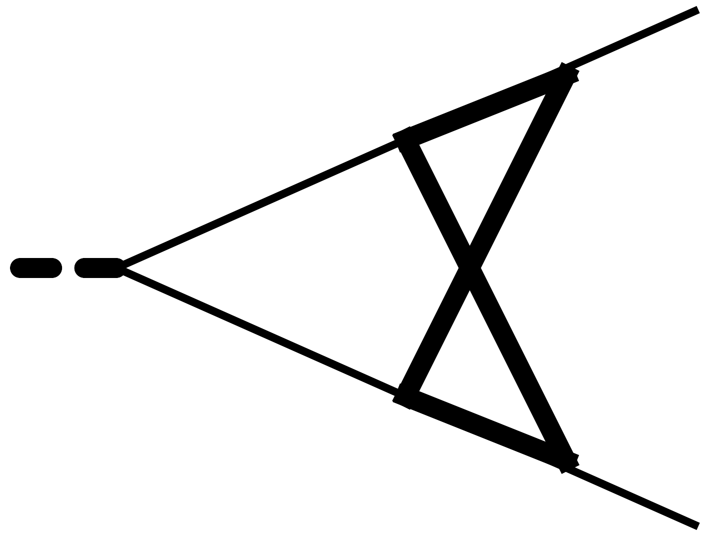
Only numerical evaluation

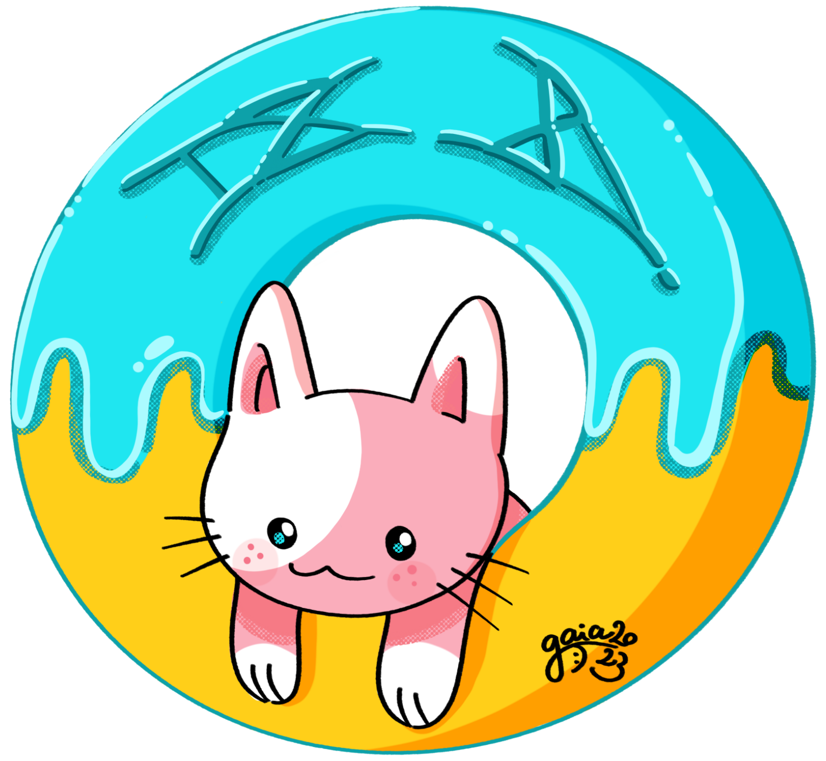
[Maltoni, Mandal, Zhao '18; Chen Heinrich, Jahn, Jones, Kerner, Schlenk et al. '20]



Elliptic Sectors

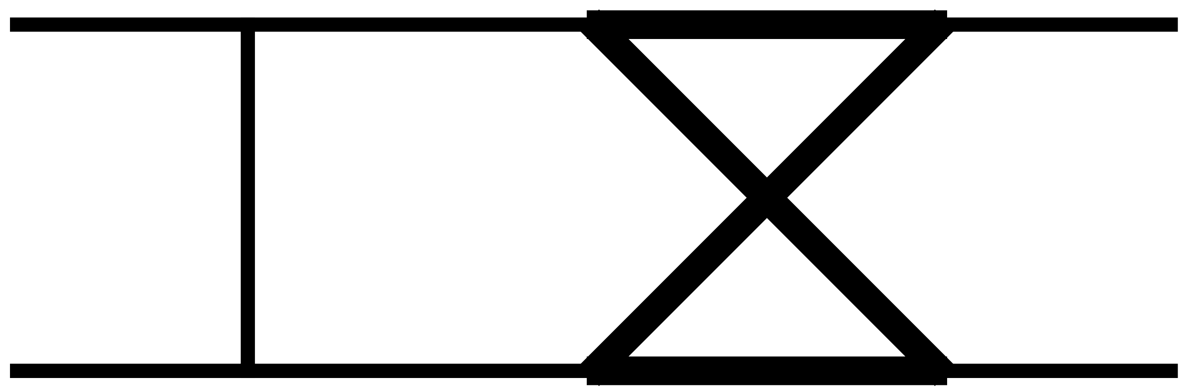
Non-Planar Triangle





[G.Fontana]

Non-Planar Double-box



- Analytic solution written in terms of elliptic generalisation of MPLs (eMPLs)

[von Manteuffel, Tancredi '17]

[Broedel, Duhr, Dulat, Penante, Tancredi '18]

- Geometry of the analytic structure described by the elliptic curve

$$y_T^2 = z(z+s)(z-a_+)(z-a_-)$$

$$a_\pm = \frac{1}{2}\left(-s \pm \sqrt{s(s+16m_t^2)}\right)$$

- Analytic solution currently unknown

- Geometry of the analytic structure described by the elliptic curve

$$y_{db}^2 = (z+t)(z+s+t)(z-b_+)(z-b_-)$$

$$b_\pm = \frac{1}{2}\left(-s - 2t \pm \sqrt{s(s+16m_t^2)}\right)$$

Differential Equations

- We compute the MIs by means of differential equations method (DEQs)

$$\mathrm{d}\,\vec{f}(\vec{x},\epsilon) = \mathrm{d}\,A(\vec{x},\epsilon)\,\vec{f}(\vec{x},\epsilon)$$

- Vector of kinematics invariants

$$\vec{x} = \{y, z\}, \quad y = \frac{s}{m_t^2}, \quad z = \frac{t}{m_t^2}$$

- Analytic Boundary conditions fixed at $\vec{x}_0 = \{0{,}0\}$

- Semi-Analytic solution obtained through generalised power series expansion

[Moriello '18]

- DEQs for planar topologies in canonical form

$$\mathrm{d}\,\vec{f}_P(\vec{x},\epsilon) = \epsilon\;\mathrm{d}\,A_P(\vec{x})\,\vec{f}_P(\vec{x},\epsilon)$$

- DEQs for nonplanar topologies in split form

$$\mathrm{d}\,\vec{f}_{NP}(\vec{x},\epsilon) = \epsilon\;\mathrm{d}\,A_{NP}(\vec{x})\,\vec{f}_{NP}(\vec{x},\epsilon) + \mathrm{d}\,\tilde{A}_{NP}(\vec{x},\epsilon)\,\vec{f}_{NP}(\vec{x},\epsilon)$$

d-Logarithmic Contribution

Elliptic sectors Contribution

Generalised Power Series Evaluation

- We exploit the Generalised Power Series method as implemented in DiffExp [Hidding '20]

Series Solution around singular points of DEQs

$$\vec{f}(t,\epsilon) = \sum_{k=0}^{\infty} \epsilon^k \sum_{i=0}^{N-1} \rho_i(t) \vec{f}_i^{(k)}(t), \quad \rho(t) = \begin{cases} 1, & t \in \left[t_i - r_i, t_i + r_i\right) \\ 0, & t \notin \left[t_i - r_i, t_i + r_i\right) \end{cases}, \qquad \vec{f}_i^{(k)}(t) = \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{N_{i,k}} c_k^{(i,l_1,l_2)} \left(t - t_i\right)^{\frac{l_1}{2}} \log(t - t_i)^{l_2}$$

- The method does not depend on the functional space of the solution
- Numerical evaluation of MIs in whole phase-space
- Suitable for phenomenological applications

Numerical Evaluation

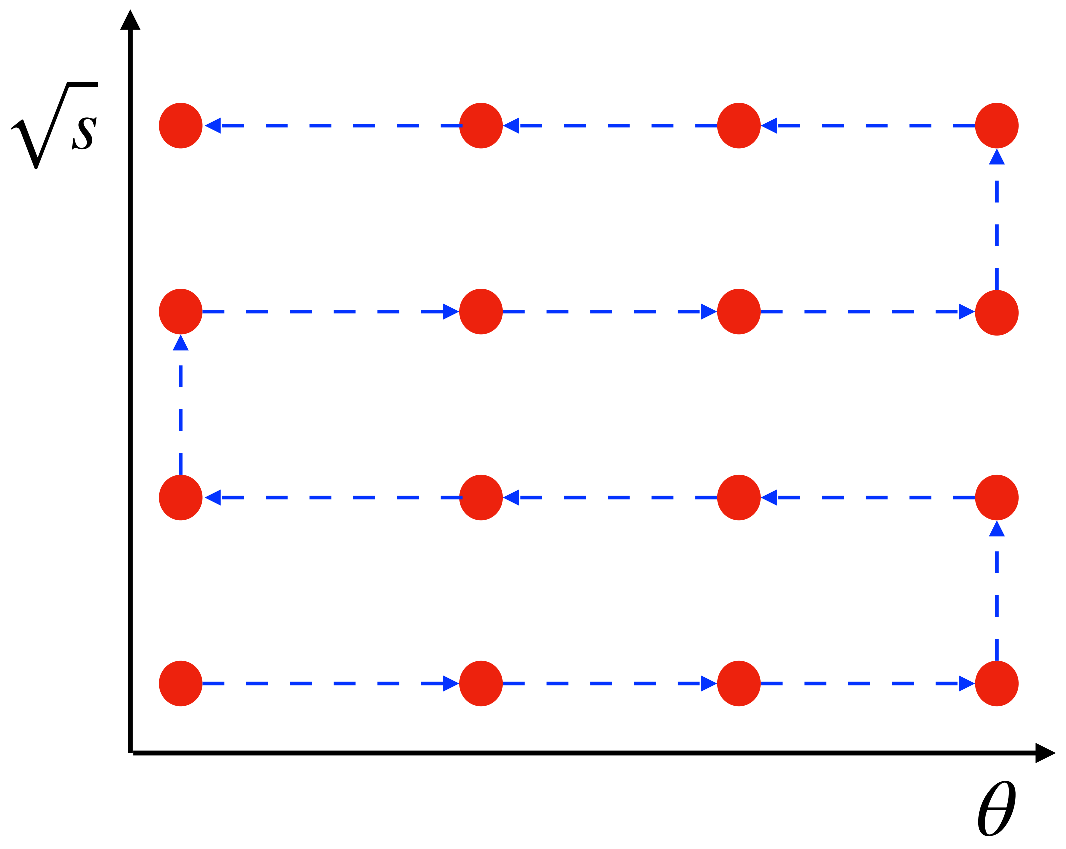
- We evaluate the MIs directly in the **physical phase-space region**

$$s > 0, \quad t = -\frac{s}{2}(1-\cos(\theta)), \quad -s < t < 0$$

- We build a numerical grid for the Hard function

$$-0.99 < \cos\theta < 0.99, \quad 8\,\mathrm{GeV} < \sqrt{s} < 2.2\,\mathrm{TeV}$$

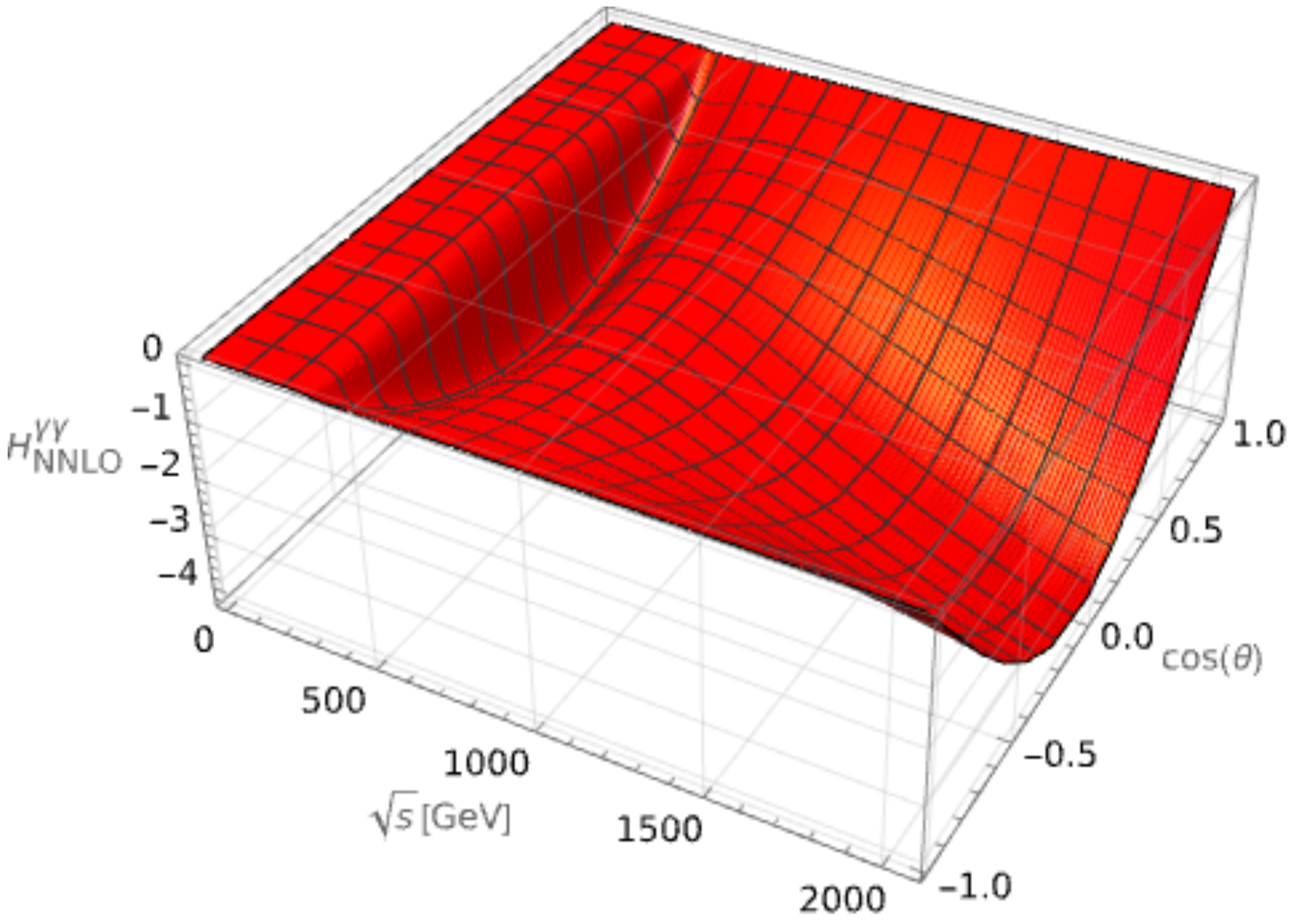
$$p_{i,j} := \begin{cases} s_i = s_0 + (s_f - s_0)\frac{i}{572} \\ t_j = -\frac{s_i}{2}(1-\cos\theta_j), \quad \cos\theta_j = \cos\theta_0 + (\cos\theta_f - \cos\theta_0)\frac{j}{23} \end{cases}$$



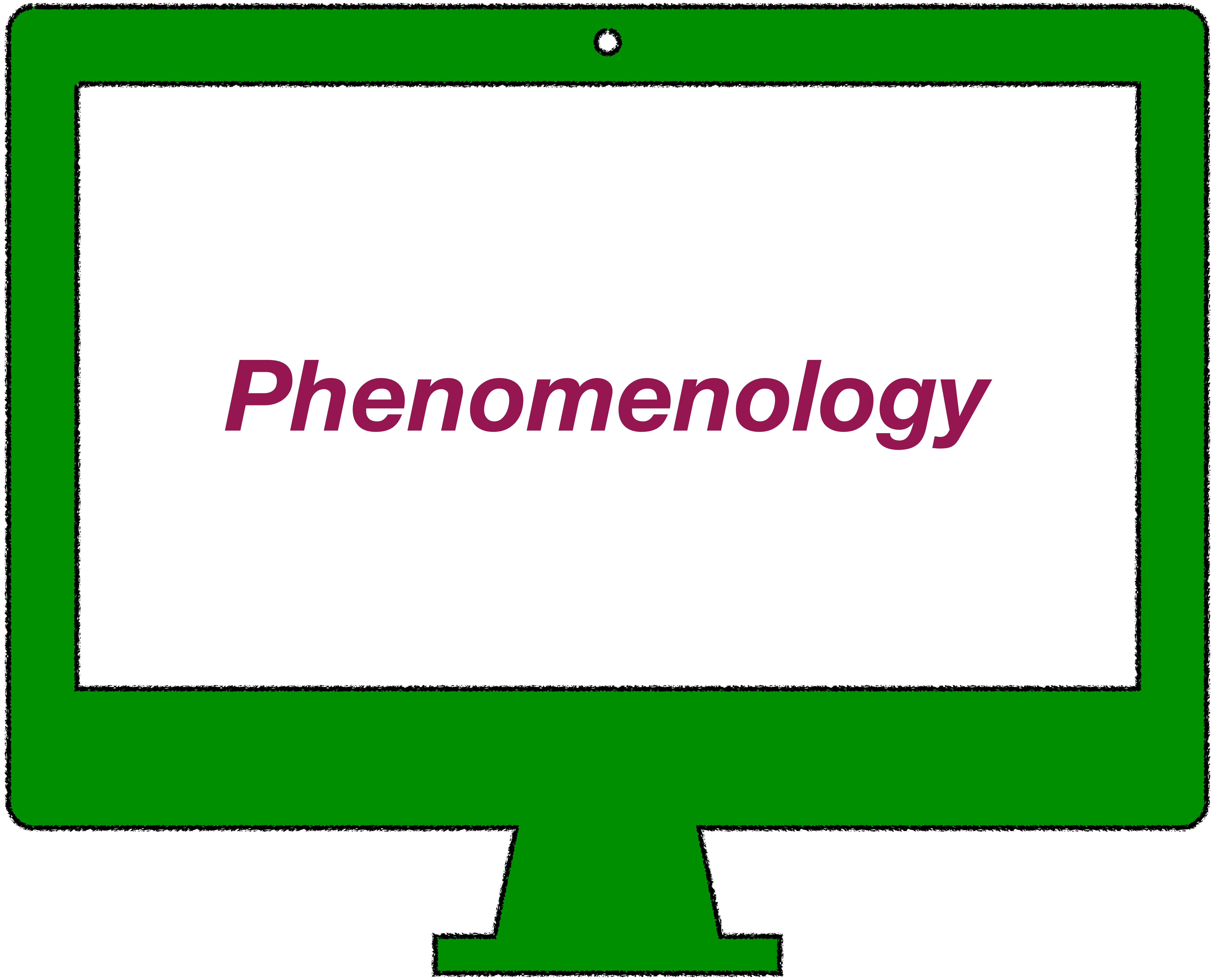
Evaluation time 13752 points★

Planar Topology: O(2.5h)

Non-Planar Topology: O(10.5h)



★ Single core laptop


Phenomenology

Framework

- Massive corrections encoded in new version of 2γNNLO [Catani, Cieri, de Florian, Ferrera, Grazzini '16]

- We exploited fast integration routines of the DYTurbo framework [Camarda et al. '20]

Kinematical parameters

- $\sqrt{s} = 13 TeV$
- Photon transverse momentum: $p_{T_\gamma}^{hard} \geq 40 GeV, \quad p_{T_\gamma}^{soft} \geq 30\, GeV$
- Photon rapidity: $|y_\gamma| < 2.37$ Excluding $1.37 < |y_\gamma| < 1.52$

ATLAS Collaboration Parameters

Isolation cone parameters

$$E_T^{had}(r) \leq \epsilon p_{T_\gamma} \chi(r;R)$$

$$\chi(r;R) = \left(\frac{r}{R}\right)^{2n}$$

$$R = 0.4$$

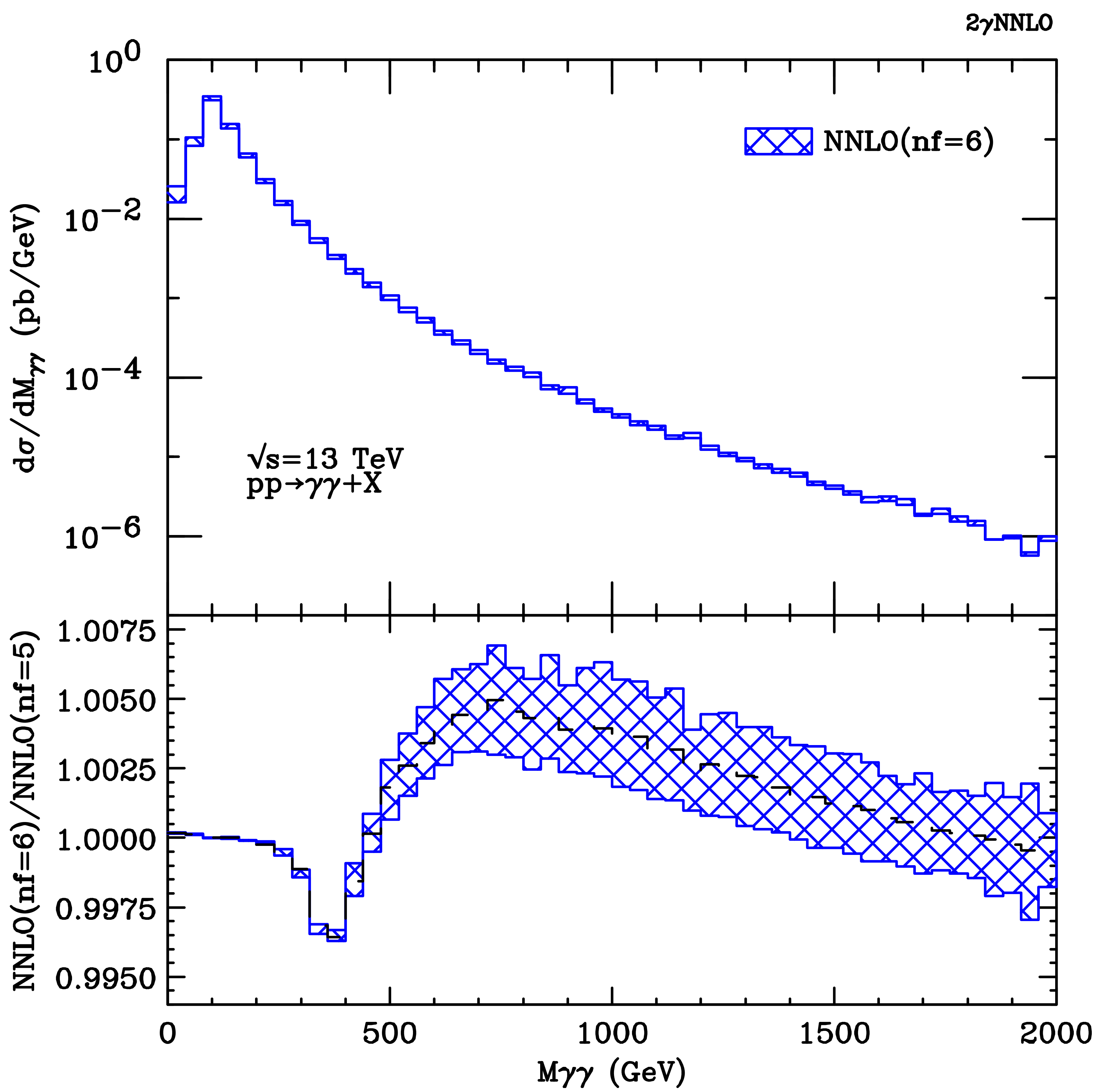
$$\epsilon = 0.09$$

$$n = 1$$

Scales choice

- $\mu \equiv \mu_F = \mu_R = M_{\gamma\gamma}$
- Theoretical uncertainty: seven-point variation scale by factors $\{1/2, 2\}$

NNLO Invariant Mass Distribution



Lower Panel: ratio between fully massive and massless NNLO

$M_{\gamma\gamma} \leq 2m_t$

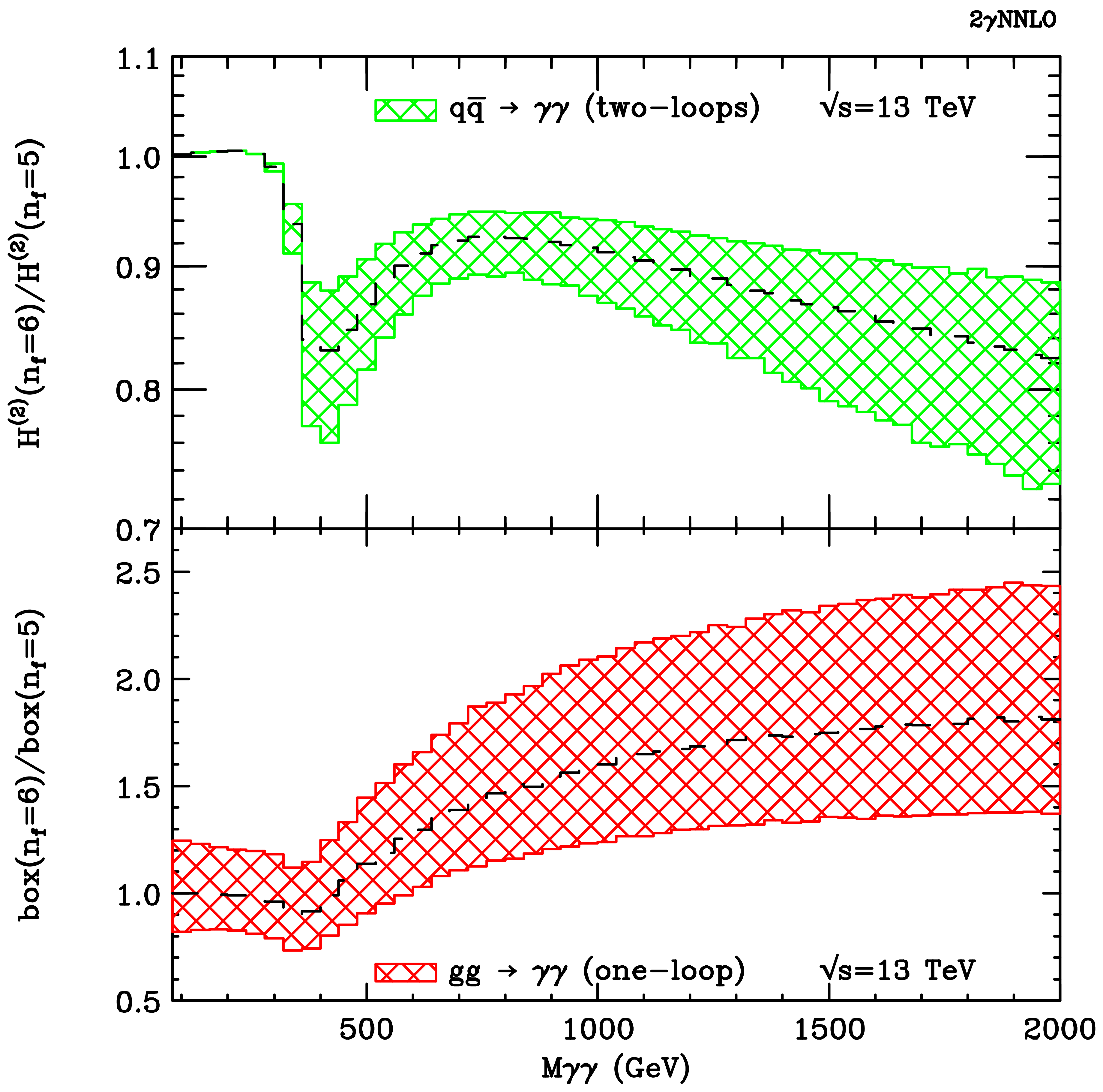
Massive corrections smaller than massless one. Peak at top-quark threshold

$M_{\gamma\gamma} > 2m_t$

Massive corrections larger than massless one. Maximum deviation at 2.3 times top-quark threshold

Effect of massive corrections: deviation from massless one in the range [-0.4%,0.8%]

Hard Function



Upper Panel: ratio between fully massive and massless NNLO

Smaller, in whole invariant mass range, than the massless one

Lower Panel: ratio between one-loop box and massless NNLO

One-loop box asymptotically behaves as a 6 light quark contribution

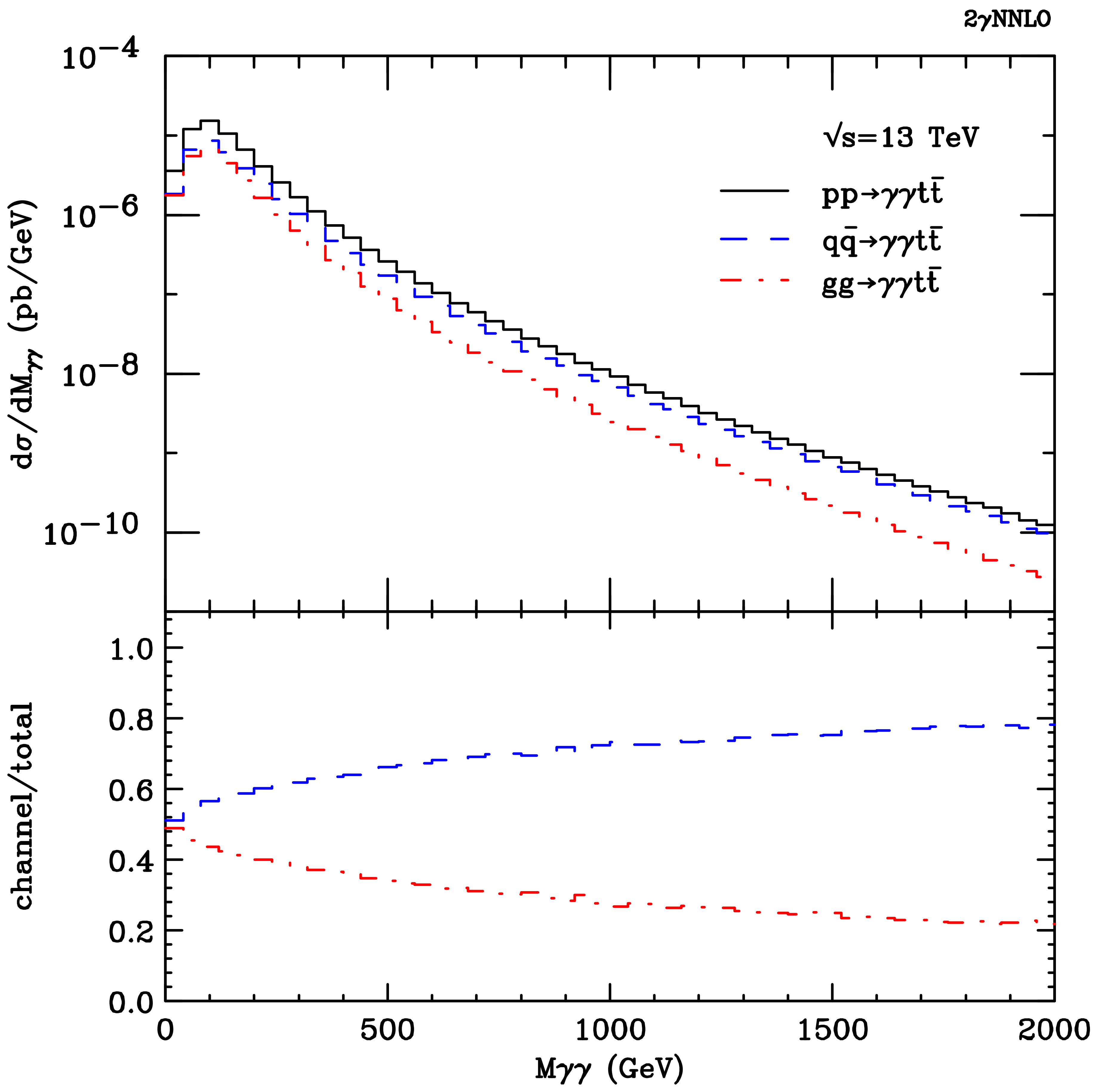
$$(\sum_{nf=6} e_q^2)^2/(\sum_{n_f=5} e_q^2)^2 = 225/121 = 1.8595...$$

Size of both ratios around negative peak: -15%

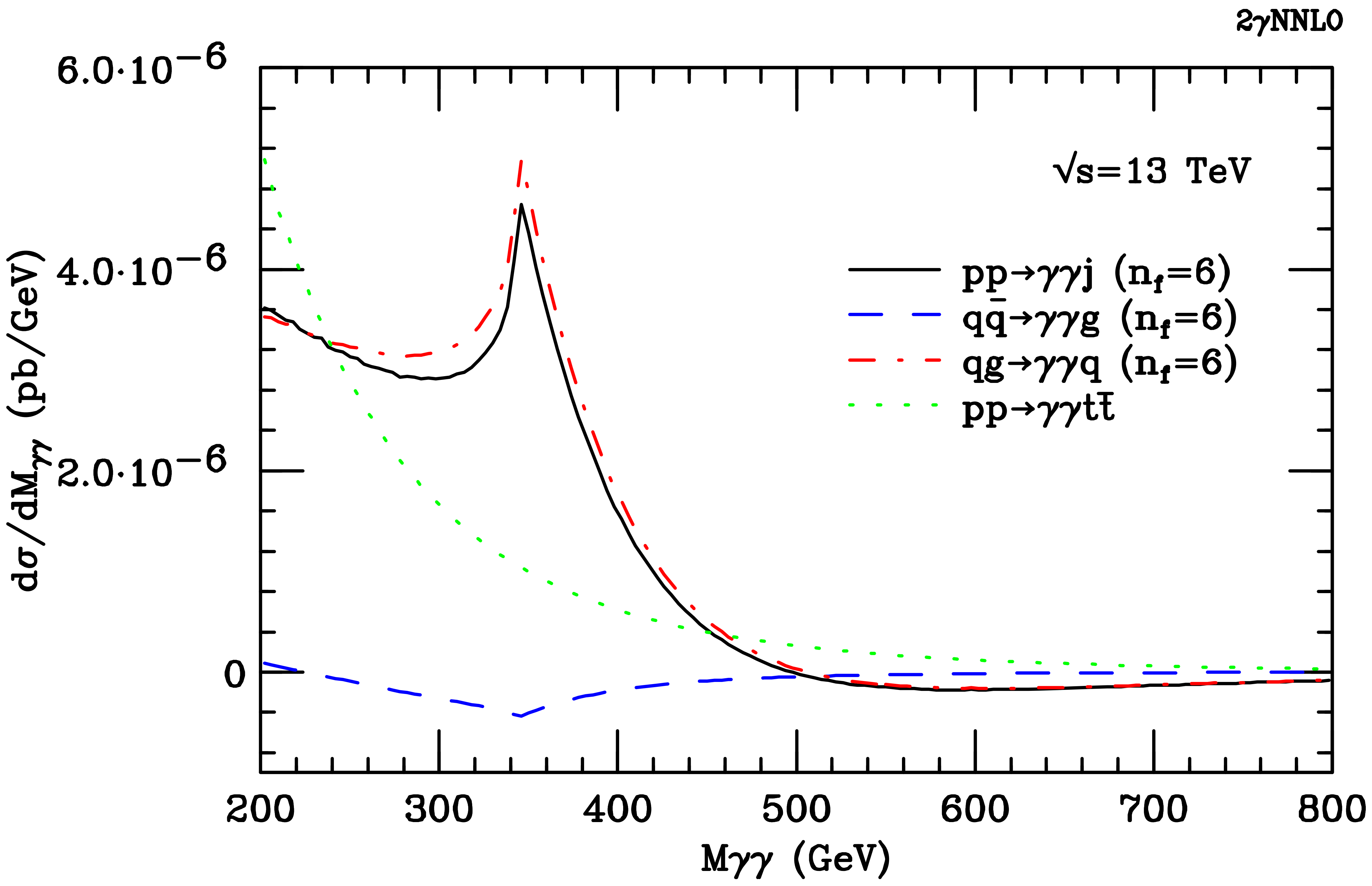
Most sizeable massive contributions at NNLO

Double-Real and Real-Virtual Contributions

Double-Real

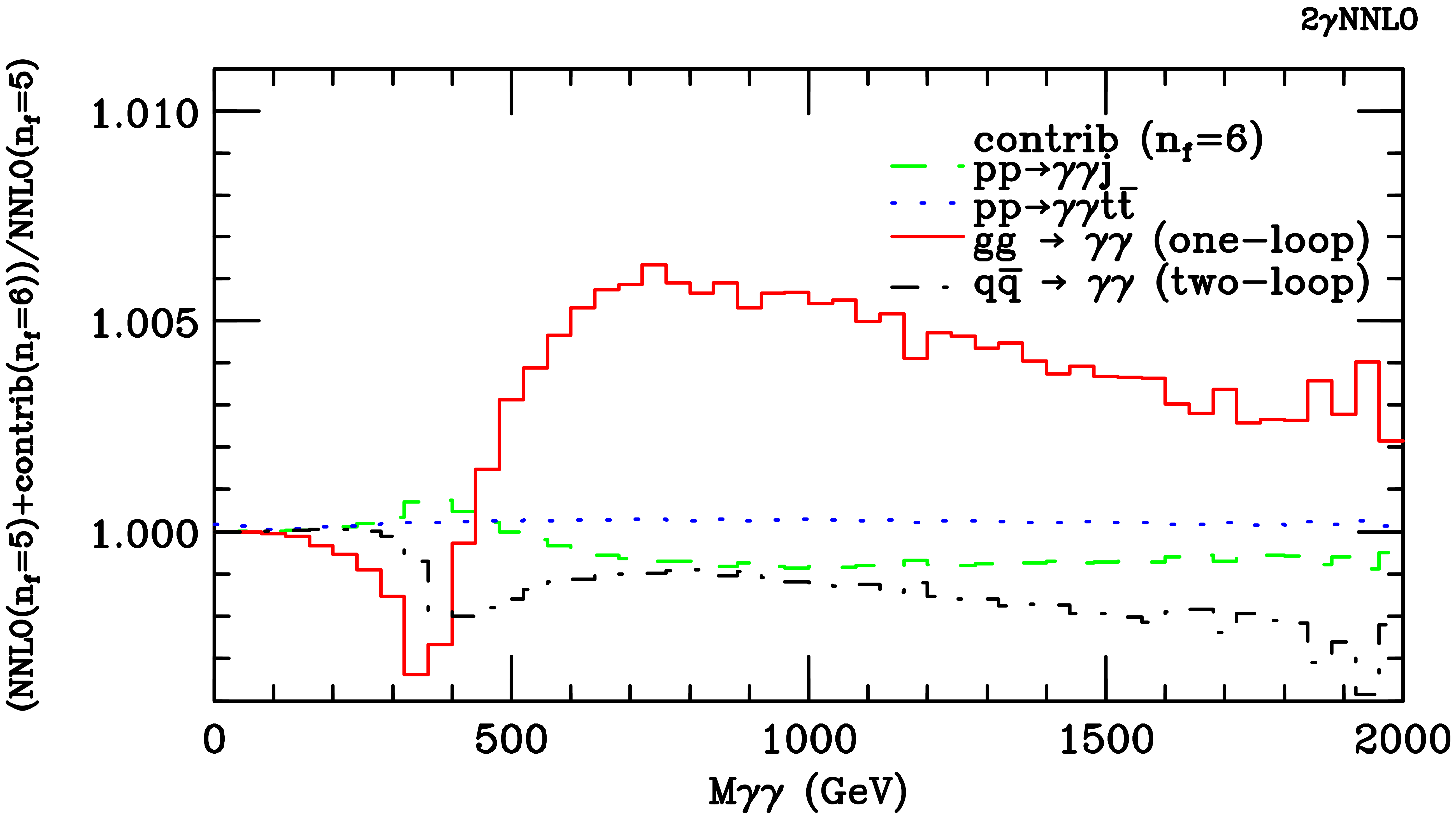


Real-Virtual



Summary of Massive Contributions

Ratios of each massive contributions with respect to massless NNLO



Two-loop quark annihilation and one-loop box dominant massive contributions

Real-Virtual Contribution subdominant. It reduces the size of the negative peak at the top threshold

Size of massive Double-Real Contribution tiny and not relevant for phenomenology

Conclusion and Outlook

Conclusion and Outlook

- We computed the fully massive corrections to diphoton production at NNLO
- The two most significant contributions are the one-loop box and the two loop qq channel
- Massive corrections relevant at the top quark threshold but also for large values of the invariant mass

Future Developments

- Constraints on top quark mass
- Inclusion of partial N3LO massive contributions
- qT Resummation

Thank you for your attention!