# 12. Air Gun Muzzle Velocity

## 12.1  Introduction

The object of this laboratory is not primarily to determine the velocity of an air gun pellet – sophisticated and much more precise methods could easily be conceived for that single purpose – but rather to demonstrate two different classic principles of measurement, and more generally also the following:

- To estimate errors from different primary sources.

- To work with cases of non-trivial error propagation.

- To establish a sound judgement of two important conservation laws in physics: conservation of energy, and conservation of momentum.

The equipment at hand is quite simple, with the expressed purpose to demonstrate that a clever physicist is always the better choice over sophisticated equipment. Two different experimental set-ups will be put to use:

In set-up A, an air-gun pellet is launched to hit a ballistic pendulum. The motion of the pendulum is observed and used to calculate the speed of the pellet.

In set-up B, a pellet is launched from an air gun over a well defined distance. The time of flight is registered electronically with a simple electronic chronometer.

## 12.2  Theory

**Theory section A: The Ballistic Pendulum.**

The design of the pendulum is shown in Fig. 12.1. In order not to complicate the evaluation of the motion, the pendulum bob is hanged in a parallelogram linkage made of two pairs of wires, whose mass is neglected. This arrangement is also known as *Ackley's ballistic pendulum*[1].

The pendulum bob is designed with a receptacle in the vertical part, containing a sticky substance, in which the pellet remains after impact. A pellet of mass $m_{\mathrm{K}}$, launched from the gun, hits the

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[1]In this manner there is no rotation of the bob to account for in the calculations,
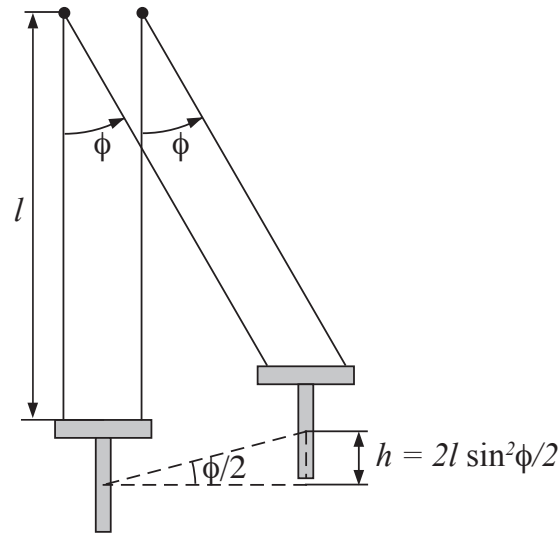
Figure 12.1: Conservation of total energy of the pendulum.

pendulum bob of mass $m_P$ at rest, and the two masses then travel with the same speed. Under these conditions, conservation of linear momentum before and after the impact of the pellet on the pendulum may be expressed as follows:

$$m_K \cdot v_K = ( m_K + m_P ) \cdot v_P \tag{12.1}$$

Here $v_K$ is the speed of the pellet prior to impact, and $v_P$ the linear speed of the pendulum with the pellet sticking, immediately after the impact.

The principle of the measurement is that the total energy of the oscillating pendulum is preserved: kinetic energy of the pendulum is completely translated into potential energy when the bob reaches its maximum height. Please note, that the energy of the pellet before and after the impact is not preserved; a substantial fraction of the initial kinetic energy of the pellet is expended in deformation upon impact into the bob. Our first object is to determine the maxim height, $h$, through the measured maximum angle of deflection of the bob[2].

Immediately after the impact the following holds for the energies:

$$E_{kin} = \frac{1}{2} ( m_K + m_P ) \cdot v_P^2 \quad \text{und} \quad E_{pot} = 0 \tag{12.2}$$

At the point of maximum height $h$ the speed of the bob is zero:

$$E_{kin} = 0 \quad \text{and} \quad E_{pot} = ( m_K + m_P ) \cdot g \cdot h \tag{12.3}$$

Since the total energy must be conserved, we may write:

$$\frac{1}{2} ( m_K + m_P ) \cdot v_P^2 = ( m_K + m_P ) \cdot g \cdot h \tag{12.4}$$

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[2]Alternatively, the horizontal deflection of the bob may be directly measured.

from which $v_\mathrm{P}$ may be calculated, if the height $h$ is known:

$$v_\mathrm{P} = \sqrt{2\,g\cdot h} \tag{12.5}$$

The height $h$ may be found using simple trigonometric arguments, as in Fig. 12.1, from the maximum angle of deflection $\phi$ and the length of the pendulum $l$:

$$
\begin{aligned}
h &= l\cdot(1-\cos\phi) \tag{12.6}\\
&= 2\cdot l\cdot\sin^2\left(\frac{\phi}{2}\right); \tag{12.7}
\end{aligned}
$$

The definition of length $l$ is a matter of choice of geometry. Most practical appears to be to measure the distance to the top platform of the bob. The position of the pivot of the pendulum has to be determined together with the corresponding error.

Inserting Eq. 12.5 and 12.7 in Eq. 12.1 results in:

$$v_\mathrm{K} = \frac{m_\mathrm{P}+m_\mathrm{K}}{m_\mathrm{K}}\cdot\sqrt{4\,g\cdot l}\cdot\sin\frac{\phi}{2} \tag{12.8}$$

## Theory section B: The Electronic Chronometer.

The principles of this method was conceived by Kiryako ("Jerry") Arvanetakis in the 1950s, a consulting engineer under contract by NACA (later NASA).

Here we directly determine the speed $v$ of the pellet by measuring the length of a path travelled and the corresponding time of flight. The speed so determined is the average speed over the length of the path. Since air drag is roughly proportional to speed, the path has to be kept short if we want to come close to the muzzle speed:

$$v = \frac{\Delta s}{\Delta t} \tag{12.9}$$

The path $\Delta s$ is measured in some conventional way

Since the speed of the pellet is appreciable, and the path $\Delta s$ is short, the time of flight $\Delta t$ needs to be determined with some precision in order to keep the error down.

The electronic circuit of the chronometer is shown in Fig. 12.2. The breakers $S_1$ and $S_2$ are key elements here. The breakers defines the length of the path, one at each end. A breaker is implemented as a mesh of thin copper wire wound on a plexiglas frame, tightly enough so that a
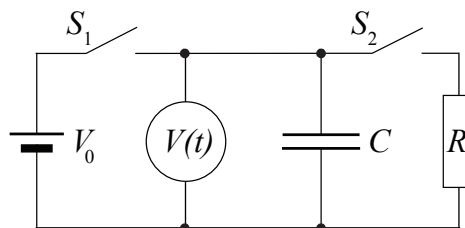


Figure 12.2: Principle of the electronic circuit of the chronometer.

pellet traversing the path will rip at least one strand of wire in the mesh. The wires of the meshes are connected as part of the chronometer circuit, as shown in Fig. 12.2 and 12.3

The circuit breakers $S_1$ and $S_2$ are closed as long as the wires are intact. In this situation, the capacitor $C$, which is the timekeeping element, is charged to the voltage $V_0$ set with the DC power supply.

When a pellet is launched from the gun, it enters the path by rupturing the wire of $S_1$. What remains of the circuit is in effect a discharge circuit, where the capacitor $C$ discharges through the resistance $R$. A well known first-order differential equation determines the exponential decay of the voltage over the capacitor with time (see appendix for a derivation).

$$V(t) = V_0 \cdot e^{-\frac{t}{RC}} \tag{12.10}$$

As the pellet arrives at the end of the path, defined by breaker $S_2$, the charge remaining in the capacitor when that wire is ruptured, results in a voltage $V_1$ over the capacitor:

$$V_1 = V_0 \cdot e^{-\frac{\Delta t}{RC}} \tag{12.11}$$

The time $\Delta t$ elapsed as the pellet travelled down the path between the breakers may be calculated from the corresponding change in voltage over the capacitor:

$$\Delta t = R \cdot C \cdot \ln \frac{V_0}{V_1} \tag{12.12}$$

As observed, the voltage $V_1$ of the capacitor after that circuit breaker $S_2$ has opened and terminated the discharge through $R$, does not remain constant as would be expected[3]. The reason may be found by inspection of Fig. 12.2: There is, in effect, still a discharge circuit left, consisting of the capacitor $C$ and the voltmeter. Although the electronic voltmeter has a high input resistance, of the order of 10 M$\Omega$, the capacitor will discharge through the instrument.

Since reading the voltage $V_1$ remaining at the end of the flight path directly from the instrument could only be made with considerable uncertainty, we chose a method to reconstruct the voltage through retrograde extrapolation, utilising observations of the decaying voltage for about two minutes after circuit breaker $S_2$ has opened. This method is much faster, and yields a more precise error, than just repeating the experiment several times. The discharge through the instrument obeys the same exponential law as before: Eq. 12.10. The data should therefore be plotted on semi-logarithmic graph paper for evaluation.

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[3]A capacitor does discharge on its own, but much slower than what is observed here.

## 12.3   Experimental

### Introduction about air guns.

An air gun is a device that fires projectiles (pellets) by means of an over-pressure from compressed gas released behind the pellet. Air guns are in this sense different from firearms, which internally burns a propellant to achieve the pressure required to launch the projectile.

Nevertheless, air guns are not toys, and must be handled with due caution. Some models can attain projectile muzzle speeds comparable to those of a firearm, and some air guns are indeed used for small-game hunting[4]. A proper backstop is required, in particular for indoor use. The shooting range must be prepared against ricocheting projectiles. A hearing protection is recommended. It may not be obvious, but the initial impact of the pressure release at the instance of firing can be intense enough to inflict lasting hearing damage.

An expended or damaged pellet must never be inserted since it is certain to have suffered small deformations that may cause it to jam in the barrel or tumble in flight because of inefficient rifling.

### The Spring Loaded Air Gun.

In experiment B, a simple *spring-piston* single-shot breech-loaded air gun with the break barrel serving as a cocking lever is employed. The single cocking stroke is performed as the hinged barrel is forced downward in an arc until it locks, and the force of the cocking spring is released. The pellet is then inserted into the back of the still open barrel. Spring-piston guns, as the name implies, operate by means of a coiled steel spring loading the piston, contained within the compression chamber that opens directly into the back of the barrel. Cocking the barrel causes the piston assembly to compress the spring until a small hook on the rear of the piston engages the sear; pulling the trigger releases the sear and allows the spring to decompress, pushing the piston forward, thereby compressing the air in the chamber directly behind the pellet. Once the air pressure is high enough to overcome any static friction and/or barrel restriction holding the pellet, the pellet moves forward, propelled by an expanding column of air. All this takes place in a fraction of a second, during which the air undergoes adiabatic heating to several hundred degrees and then cools as the air expands.

Spring-piston guns have a practical upper muzzle speed limit of about 370 m/s for .177 cal (4.5 mm) pellets. Higher velocities tends to cause unstable pellet flight and loss of accuracy. Drag increases rapidly if a pellet is pushed past the speed of sound. A general consensus holds that velocities about 270 m/s) offer an ideal balance between power and pellet stability.

### The Carbon Dioxide Gun.

In experiment A, a single action, carbon dioxide driven gun of rather more sophisticated design is used. There is no piston compressing air at launch. The gas expanding behind the pellet, is drawn from a small pressure container with liquefied $CO_2$ inserted in the back of gun. Carbon

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[4]Also military versions have been developed.

dioxide is available at equilibrium pressure[5] as long as liquid is still left in the container. The firing mechanism is made by pushing forward, as a unit, the two knobs, one at each side of the chamber. This action also releases the loading chamber to pop up at the top of the gun, into which the pellet is inserted. Pushing down the chamber closes the gun and makes it ready for launch.

The firing mechanism incorporates valve, hammer, and bolt in a single tube. The bolt is connected to the hammer. The cocking action pulls the hammer forwards and compresses the internal hammer spring, which is caught and locked in place by the sear as the hammer and spring mechanism reaches the far end of travel.

When the sear is released by the trigger, the hammer is pushed by the spring until it hits the valve. A small amount of gas is then released from the $CO_2$ container into the internal bolt chamber, and the valve closed by the valve spring. The ensuing burst of gas channels out of the front end of the bolt, and propels the pellet down the barrel.

**Pellet Ammunition.**

As opposed to firearms, air-guns generally use a slightly undersized projectile that is designed to obturate under pressure and seal the bore. The diabolo pellet is a drag stabilised waisted projectile, also known as *wasp waist pellet*. It is available in a variety of head styles, and features a hollow tapered skirt from the waist back. The skirt of the pellet is thin, made of a malleable material, usually lead, which flares out to fit the bore and provide a good seal when pressurised in the gun, thus also engaging the rifling. In a smoothbore barrel, the skirt will still flare to provide a tight seal.

**Experiment A: The Ballistic Pendulum.**

The evaluation follows the procedure described in the theory-section, where the maximum deflection angle is inserted into Eq. 12.8.

Because of the erratic motion of the pendulum bob immediately after impact, It is difficult to read the maximum deflection angle. In addition, since the damping because of air drag is substantial, considering the projected area of the pendulum bob, the maximum angle observed for the first swing does not corresponds to the kinetic energy of the pendulum at the instance of impact. We try to solve both these problems with data analysis. For the first 10 periods, read the maximum deflection angle. Then read only for the 20th, 30th till the 100th completed swing.

There is a reason for this scheme: The set-up does not allow much freedom of adjustment. The pellet does not hit the pendulum in the centre of percussion[6], and probably also not in the vertical centre-line. The consequences of this is easily seen in the complicated set of oscillatory modes that are excited immediately after impact. The energy that goes into these modes comes from the

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[5]At ambient temperature, the equilibrium pressure is 6-7 MPa, which renders this type of gun only a medium power weapon. It is common as paint-ball marker.

[6]For a pendulum, also known as the *centre of oscillation*. The center of percussion is the point where a perpendicular impact will produce translational and rotational forces that perfectly cancel for a given pivot point. Corresponds to the *sweet spot* for a baseball bat or a sword.

impact and therefore corrupts the measurement. The unwanted modes appears to die away after about ten periods of oscillation.

Assuming that the damping of the pendulum is determined only by air drag, with an opposing force proportional to speed, i.e. $F = -c \cdot v$, where $c$ is the viscous damping coefficient, the equation of motion is a second order differential equation:

$$\frac{d^2\varphi}{dt^2} + 2\,\zeta\omega_0\,\frac{d^2\varphi}{dt^2} + \omega_0^2\,\varphi = 0 \tag{12.13}$$

the solution of which is a sinusoidal with and exponential pre-factor. Here $\zeta$ is known as the *damping ratio*, $\omega_0$ the frequency of the corresponding simple harmonic oscillator (undamped oscillation):

$$\zeta = \frac{c}{2\,m\omega_0}; \tag{12.14}$$

$$\omega_0 = \sqrt{\frac{g}{l}} \tag{12.15}$$

Plotting the decay with time of the maximum angle of deflection on semi-logarithmic graph paper would yield a straight line if viscous damping of the main pendulum oscillation mode, would the equation be an exact description of the pendulum motion. As we learnt above, there are additional modes active at the beginning, which we cannot account for with our simple model. In addition, viscous damping is not the only effect: The four wires employed in the parallelogram suspension are subject to friction that we cannot neglect, at least not for smaller angles, for which the damping because of drag becomes relatively less important. We therefore can expect to find only an approximate correspondence with a straight line. This is not, however, of great importance, since we attempt graphical evaluation to find the true maximum angle of deflection, and in particular the error in the determination. It is left to the student to devise a suitable method for accomplishing this.

The angle corresponding to the kinetic energy of the pendulum bob at the instance of impact may be found with retrograde extrapolation to the time of impact from a series of successive readings of maximum deflection angles; this procedure corrects for the damping during the first swing.

You may want to start the experiment by firing the empty gun, in order to demonstrate the presence an appreciable muzzle wind, quite capable of setting the pendulum in motion. Think about this as a source of error that you may want to compensate for.

For the measurement, follow the points outlined below:

- Check that the pendulum bob is correctly hanged, and that it is possible to read the angle relative to the reference scale.

- Start the pendulum motion by hand, and observe the oscillation. The bob should move in the plane of the projectile trajectory, i.e. as the gun is pointed.

- Launching a test pellet may be desired next. Try to identify difficulties. The bob should not oscillate around the four wires, but this is almost impossible to avoid, and may be compensated with the extrapolation procedure. If necessary, try to adjust the pendulum.

- Weigh the pellet that is going to be used.

- Cock and load the gun. Plan the measurement. Try out a scheme with your lab team that allows you to read the maximum angle and take notes for as many oscillation as possible like suggested above.

- Draw a diagram of the deflection angle and time. In a semi-logarithmic diagram data should approximately appear on a straight line, as was outlined above.

- Using retrogade-extrapolation, determine the angular deflection that would correspond to the energy at the instance of impact.

- Weigh the pendulum bob with the pellet still sticking. The sum of the masses is needed, and if the two masses are determined separately, the total error becomes larger[7]. After weighing, the pellet should be removed from the bob, and discarded in the small container for waste lead metal.

- Calculate the speed of the pellet $v_K$ according to Eq. 12.8.

- Calculate the linear speed of the pendulum $v_P$ immediately after impact using Eq. 12.5.

- From $v_K$ and $v_P$ so determined, calculate the kinetic energy of the pellet before impact, and that of the pendulum with pellet after the impact.

There is no potential energy involved in the calculation of impact energy, but still the kinetic energy is not preserved. On inspection of the pellet after the impact (after weighing), we find that the pellet is deformed. Total energy must be preserved, but at impact, some of it is spent on deformation of the pellet, of which most is expended as heat. Remaining energy goes into kinetic energy that we calculated for the pendulum and bullet after impact. We must be mindful in the interpretation of this observation. The deformation of the pellet does depend on the choice of pellet, but the apparent loss of energy only depends on the restrictions of the situation given: The fact that the pellet moves with the same speed as the pendulum bob after impact, and that the pendulum was at rest before impact.

Show algebraically that under these conditions the kinetic energy cannot be preserved. You may wish to utilise the following steps:

- Write an expression for the kinetic energy before the impact into the resting pendulum.

- Write an expression for the energy of the pendulum bob and pellet travelling together after impact.

- Use the fact that momentum has to be preserved during impact; solve for the speed of the pellet, and insert in the expression for the kinetic energy of the pellet.

- Write an expression for the difference $Q$ in kinetic energy before and after impact. Manipulate this expression to show that the difference cannot be zero.

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[7]Separate weighing afterwards is not recommended, since some of the sticky substance might be lost
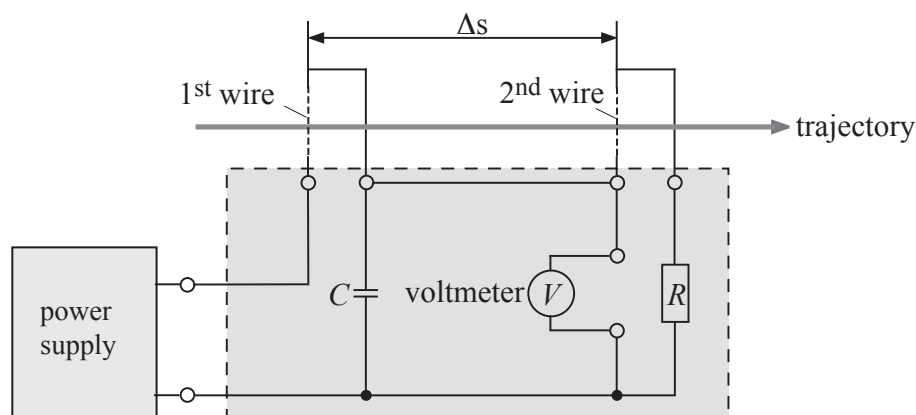
Figure 12.3: Set-up for the chronometer measurement. Open circles represents banana sockets.

- Derive an expression, in the masses $m_K$ and $m_P$, for the fraction of the energy $Q$ of the total energy. Compare with the result of the measurement.

## Experiment B: The Electronic Chronometer.

The set-up is shown in Fig. 12.3. The larger, grey rectangle represents a pre-wired circuit board of plexiglas with banana sockets for connecting the two wire meshes that make up the circuit breakers, the digital voltmeter, and the power supply. The two components, the capacitor $C$ and the discharge resistor $R$ are soldered onto the circuit board. The component values of $R$ and $C$ are determined upon inspection.

Preparations:

- Wind copper wire on the frames. Leave about $20\,\text{cm}$ of free wire at both ends of the frame. The wire is covered with isolating lacquer that has to be removed with a flame and a knife[8] for a length of about 2-3 cm.

- Mount the frames in their holders, and connect the wires as follows: Pass the bare copper wire under the screw-down banana sockets, but do not wind around the seating pin of the sockets; also, do not use the hole in the seating pin. Tighten the sockets. Check connection of the frame with an ohmmeter connected to the banana sockets. If the isolation was not correctly removed, or the sockets not sufficiently tightened, the instrument indicates a broken circuit (or the highest resistance it can show)[9].

- Connect all other leads with banana connectors.

- Call the lab assistance for a final check-up.

Make ready to start the measurement:

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[8]Squeezing the seared wire between your thumb nail and top of a finger, and pulling, is also effective.
[9]The resistance for a good connection is only a fraction of an Ohm.

- Turn on the power supply and set a voltage to some suitable value, high enough to allow as many digits as possible to be read[10].

- As a final check, measure the voltage over the discharge resistance $R$ (it should read the same value as the power supply).

- Use a stop-watch, or equivalent, as master clock, starting at the time the pellet is launched. Then, for each ten seconds, read the voltage over the capacitor for 100 s.

- When everything is ready, cock the gun, load the pellet, launch the pellet and start keeping time and reading voltages.

- Determine errors where appropriate. Inspect the path length. What are the sources of error? Where are the positions of the circuit breakers? Do we know if a wire on the front side or the back side of the mesh frame was first to break?

- Draw a diagram of voltage against time on semi-logarithmic graph paper. Use retrograde extrapolation to determine the voltage at the time the pellet trips the second breaker $S_2$. In particular, also determine the error from the graph.

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[10]In order to reduce the relative error.

## 12.4 Appendix

**a) Derivation of Eq. 12.10**

A capacitor with capacity $C$, stores charge $Q$ if a voltage $V$ is applied to it:

$$Q = C \cdot V \tag{12.16}$$

If a circuit with resistance $R$ is connected in series with a capacitor, a time dependent discharge current $I(t)$ flows, while the voltage decreases and the capacitor discharges:

$$I(t) = \frac{V(t)}{R} \tag{12.17}$$

Using Eq. 12.16 and 12.17 we have, with $I = dQ/dt$:

$$\frac{V(t)}{R} = \frac{d}{dt}(C \cdot V(t)) \tag{12.18}$$

and finally:

$$\frac{dV}{dt} - \frac{1}{RC} \cdot V = 0 \tag{12.19}$$

This first order differential equation in $V(t)$ may be directly integrated to yield a solution representing the exponential decay of the voltage over the capacitor:

$$V(t) = V_0 \cdot e^{-\frac{t}{RC}} \tag{12.20}$$

where the constant of integration $V_0$ is determined by the initial conditions, namely $V_0 = V(0)$, the voltage of the power supply.