6. Friction and viscosity in gasses

6.1 Introduction

Similar to fluids, also for laminar flowing gases Newtons's friction law holds true (see experiment IR1). Using Newton's law the viscosity of air under normal pressure will be measured in the first part of the experiment. The viscosity will be determined by the friction force of two counter spinning disks between which a layer of air is enclosed.

In the second part of the experiment a vacuum pump will be used to reduce the air pressure between the two plates such that the mean free path of the air molecules is of the order of magnitude or larger than the distance between the two plates. It will be shown that in this case the model of air treated as a media with viscosity fails and that it is necessary to take into account the properties of the single molecules.

6.2 Theory

The experimental setup is shown in Fig. 6.1. Two flat, round disks I and II having the same radius R are mounted on a common rotation axis separated by a distance d. Disk II can be turned with negligible friction around this axis. On disk II a weight G is mounted at a distance l from the axis of rotation. Disk I will be driven by an electric motor to rotate with constant angular velocity ω.

The air between the two disks will be set in motion by the rotating disk I due to the resulting friction forces. This motion results in an torsional moment M_R onto disk II. The disk II will turn until the torsional moment M_G of the weight G compensates M_R. For a specific deviation angle α, M_G is given by:

$$M_G = -G \cdot l \cdot \sin\alpha \tag{6.1}$$

The torque M_R can be calculated from Newton's friction law. Since the distance between the two disk is small compared to the Radius R, the velocity profile between the two disks can be assumed to be linear (see Fig. 1 in experiment IR1). Hence, the force applied to disk II is given by:

$$F_R = \eta \cdot A \cdot \frac{v}{d}, \tag{6.2}$$

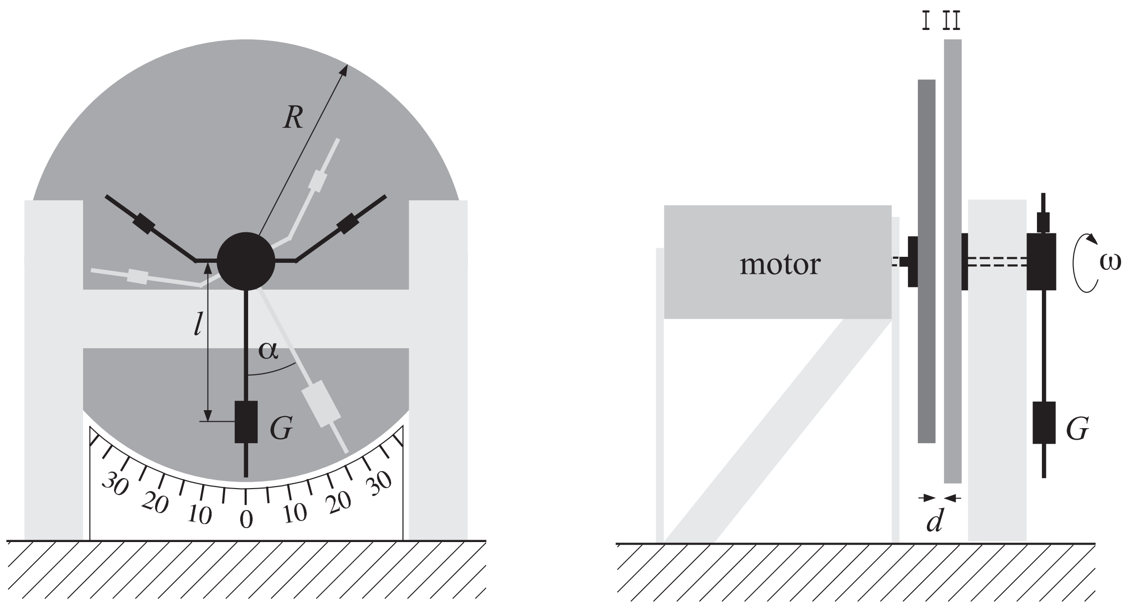

Abbildung 6.1: Experimental setup for the determination of the viscosity.

where the velocity v and hence F_R depend on the distance r from the axis of rotation as

$$v(r) = \omega \cdot r. \tag{6.3}$$

Thus the torque exerted in a circular ring with radius r and thickness dr can be calculated as

$$dM_R = r \cdot dF = r \cdot \eta \cdot dA \cdot \frac{v(r)}{d} = \eta \cdot \frac{2\,\pi \cdot \omega}{d} \cdot r^3\,dr. \tag{6.4}$$

The total torque M_R is obtained by integrating of the radius and yields:

$$M_R = \int_0^R dM_R = \eta \cdot \frac{2\,\pi \cdot \omega}{d} \cdot \int_0^R r^3\,dr = \eta \cdot \frac{\pi \cdot \omega \cdot R^4}{2\,d}. \tag{6.5}$$

In the equilibrium state the two torsional momenta compensate mutually $M_R = M_G$. Equating Eq. 6.1 and Eq. 6.5 and solving for η yields

$$\eta = \frac{2G \cdot d \cdot l}{\pi R^4} \cdot \frac{sin\alpha}{\omega} \tag{6.6}$$

If R, d, G, and l are known, η can be obtained by measuring α as a function of ω.

6.3 Experimental Part

a) Viscosity of air at ambient pressure

In the first part the viscosity of air in ambient conditions is measured.

The experimental setup is already mounted and adjusted. The glass cover thus *should not be lifted* from the base plate. The dimensions R, d, G and l of the apparatus are listed at the experimental setup.

Die angular velocity ω of disk I is measured by means of a stroboscope. This device is similar to an electric flashlight for which the single light flashes repeatedly with a tunable frequency.

Hint: Always write down the chosen electrical voltage for the driving motor during the experiment, this makes it more convenient to reproduce the measurements.

On the back side of disk I a black field and a mirror are mounted. The rotating disk is illuminated by the stroboscope. If the rotation frequency of the disk is the same as the repetition rate of the light flashes, the disk will be illuminated always at the same position and seems to stop. This is called a static picture of the disk. It is most easily observable by the position of black field or the mirror.

- In order to measure ω, change the frequency of the light flashes until you observe a static picture of the disc. You can read off the rotational frequency of the disc in units of “Pulses per Minute” directly from the calibrated scale of the stroboscope. What do you observe if the flash frequency is an integer multiple of the rotation frequency or *vice versa*?

- When measuring the angular velocity do you thus have to scan through the frequencies of the stroboscope from low to high or from high to low values?

- Measure at a pressure of 1000 mbar (atmospheric pressure or about 730 Torr) the angle α of disk II as a function of the angular frequency ω of disk I. Repeat these measurements with increasing or decreasing angular velocity to avoid hysteresis effects and take the average over these measurements .

The resulting curve is shown in Fig. 6.2. For low velocities α increases linearly as expected according to Eq. 6.6 with increasing ω (for small angles α the approximation $\sin\alpha \approx \alpha$ can be made). Above the critical velocity v_{krit} the gas flow becomes turbulent, hence the friction forces, and the torsion angle α increases *faster* than linearly with increasing velocity.

- Determine the best line fit for at least five data points below the critical velocity v_{krit} and determine the viscosity η of air from the slope of the line. Estimate the error on η.

- Estimate from the progression of the curve the critical velocity v_{krit} and from this calculate the critical Reynolds number Re_{krit} (see experiment IR1).

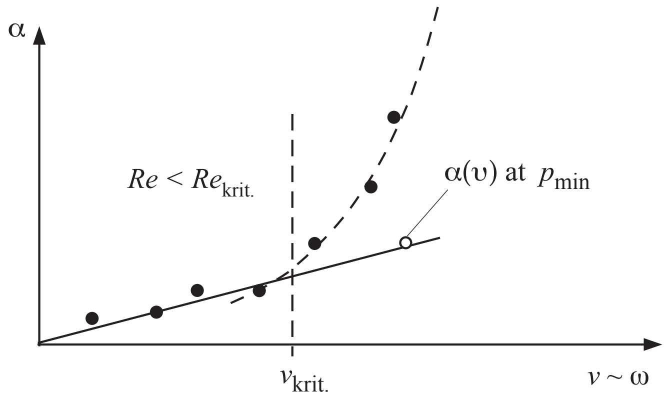

Abbildung 6.2: Dependency of α on the rotational speed v as expected according to Eq. 6.6.

b) Investigation of the viscosity of air at low pressure

- Set the rotation frequency ω of disk I to a value below the critical velocity v_{krit}. Turn on the vacuum pump and keep track of the torsion angle of disk II while evacuating the vacuum vessel.
- You will observe that the viscosity of air *does not* depend on the pressure down to very low pressures. Use the discussions in the Appendix to estimate at which pressure a drop in viscosity would be observed for the experimental setup. At $p = 10^{-6}$ bar and $T = 20°$C the mean free path of N_2- and O_2- molecules is approximately 7 cm.
- After the minimal pressure is reached, increase the rotation frequency ω to a value above the value found for the critical velocity v_{krit} at ambient pressure. Mark the measured torsion angle α in the $\alpha(v)$-diagram from the first part of the experiment. Extrapolate the best fit line from the first part and explain why the measurement at minimal pressure lies on this line (see Appendix).

6.4 Appendix

Pressure dependency of the gas viscosity

In a gas the molecules move with a high velocity and collide with each other and the walls of the container. This motion is chaotic and has not preferred direction. Internal friction only results from a position dependent net mean drift velocity v superimposed on the chaotic movement.

As an example consider a gas enclosed by two parallel plates which move with a velocity v with respect to each other (see Fig. 1, experiment IR1). If the pressure of the gas is small such that the mean free path $\bar{l}$ between two collisions of the molecules is larger than the distance of the two plates, the molecules from the non-moving plate (mean velocity $v = 0$) reach the moving plate without colliding with other molecules (see Fig. 6.3). They will stick to the wall for a short moment and get a small amount of momentum. Eventually, on the plate a shear $\tau = F/A$ is exerted, which corresponds to the transferred momentum per unit of time and area.

$$\tau \propto Z \cdot m \cdot v \tag{6.7}$$

Here m is the molecule mass and Z the number of molecules which hit the plate per second and unit area. Z is proportional to the molecule density which is proportional to the pressure p. Thus, $\tau \propto p$ and since $\eta \propto \tau$ the relation holds: $\eta \propto p$.

The mean free path $\bar{l}$ of the gas molecules is inversely proportional to the particle density and to the pressure p. If the pressure is increased such that $\bar{l} \ll d$, the molecules reaching the moving plate do not directly come from the non-moving plate but from a gas-layer at an average distance $\bar{l}$ from the wand in which they encountered collisions and acquired the mean velocity at this position:

$$v(z = d - \bar{l}) = v \cdot \left(1 - \frac{\bar{l}}{d}\right) \tag{6.8}$$

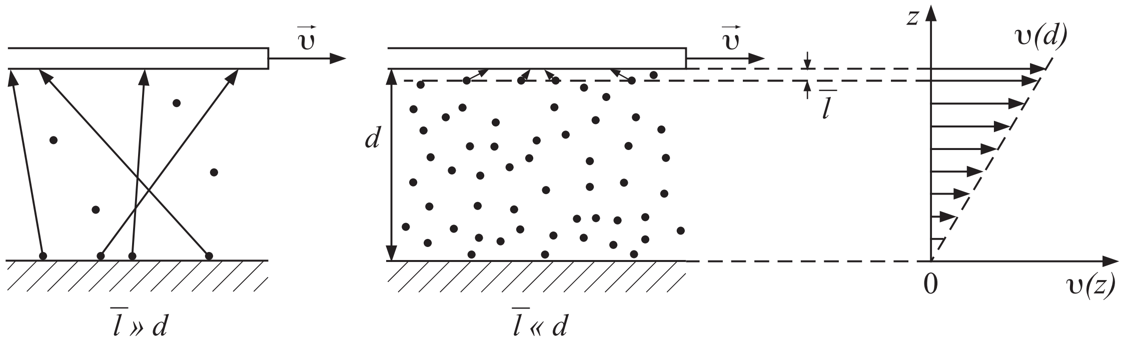

Abbildung 6.3: Motion of the molecules between two plates for $\bar{l} \gg d$ (left panel) and $\bar{l} \ll d$ (right panel).

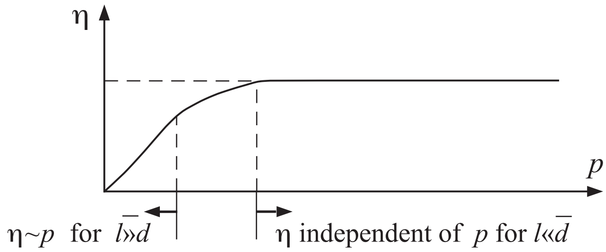

Abbildung 6.4: Pressure dependency of the viscosity.

The momentum transferred to the moving plate yields:

$$m \cdot \left(v - v(z = d - \bar{l})\right) = m \cdot v \cdot \frac{\bar{l}}{d}. \tag{6.9}$$

Using Eq. 6.7 we obtain

$$\tau \propto Z \cdot m \cdot v \cdot \frac{\bar{l}}{d} \quad \text{for} \quad \bar{l} \ll d \tag{6.10}$$

Since $Z \propto p$ and $\bar{l} \propto 1/p$, the shear on the plate and thus the viscosity of the gas do not depend on the pressure at low particle density or pressure. Summarizing, a pressure dependence of the viscosity as shown in Fig. 6.4 is expected.