# 5. Measurement of a magnetic field

## 5.1 Introduction

Magnetic fields play an important role in physics and engineering. In this experiment, three different methods are examined for the measurement of magnetic fields: the measurement of the Lorenz force acting on a conductor carrying an electrical current, the measurement of the voltage induced in a coil moving in the magnetic field, and the measurement of the transverse electric field generated in a current-carrying conductor (Hall effect).

## 5.2 Theory

### a) Magnetic fields

For the description of magnetic fields two fields are introduced, the magnetic field strength $\vec{H}$ and the magnetic flux density or induction $\vec{B}$. Both fields are closely related. In SI units, 1 A/m (ampere per meter) and 1 T (Tesla) are used for $H$ and $B$, respectively[1]. In vacuum the following relation holds:

$$\vec{B} \;=\; \mu_0 \cdot \vec{H} \tag{5.1}$$

with the so-called vacuum (magnetic) permeability $\mu_0 = 4\,\pi \times 10^{-7}$ Vs/Am. In matter the relation becomes:

$$\vec{B} \;=\; \mu_0 \cdot \left(\vec{H} + \vec{M}\right) \;=\; \mu\,\mu_0 \cdot \vec{H} \tag{5.2}$$

where

$$\vec{M} \;=\; (\mu - 1) \cdot \vec{H} \;=\; \chi \cdot \vec{H} \tag{5.3}$$

is the magnetization of the material, $\mu$ its relative magnetic permeability and $\chi = \mu - 1$ its magnetic susceptibility[2]. In general and in particular for crystalline solids, $\mu$ and $\chi$ are tensors, but for isotropic materials, where $\vec{M}$ is parallel to $\vec{B}$, both constants become scalars.

The values of $\mu$ and $\chi$ for some materials (assuming an isotropic relation) are given in Table 5.1. In general, we distinguish:

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[1]For the magnetic induction the former unit Gauss (1 G = $10^{-4}$ T) is often used.

[2]Both the quantities are defined as dimensionless material constants.

Table 5.1: Magnetic permeability $\mu$ and magnetic susceptibility $\chi$ for some materials (isotropic case, at $T = 300 \, \text{K}$).

| material | $\mu$ | $\chi$ | |
|----------|-------|--------|---|
| vacuum | 1 | 0 | — |
| air | 1.00000038 | $0.38 \times 10^{-6}$ | paramagnetic |
| aluminum | 1.0000208 | $20.8 \times 10^{-6}$ | paramagnetic |
| copper | 0.9999902 | $-9.8 \times 10^{-6}$ | diamagnetic |
| iron | $10 - 10000$ | 10 - 10000 | ferromagnetic |

- diamagnetic materials: $\mu < 1 \iff \chi < 0$
- paramagnetic materials: $\mu > 1 \iff \chi > 0$
- ferromagnetic materials: $\mu \gg 1 \iff \chi \gg 0$

For ferromagnetic materials $\mu$ is strongly dependent on the magnetic field strength $H$.

## b) Lorentz force

On a point charge $q$ moving with velocity $\vec{v}$ in a magnetic field, the action of the Lorentz force is described by the cross product

$$\vec{F} \;=\; q \cdot \vec{v} \times \vec{B} \tag{5.4}$$

Hence the Lorentz force is always perpendicular to the field lines $\vec{B}$ and to the velocity $\vec{v}$.

The Lorentz force on a part $d\vec{s}$ of a current-carrying conductor, in which the direction of $d\vec{s}$ is the same as the technical direction of the electric current, is given by:

$$d\vec{F} \;=\; I \cdot \vec{ds} \times \vec{B}. \tag{5.5}$$

Thus the force acting on a straight piece of a conductor with length $l$ and electric current $I$ in a homogeneous magnetic field is

$$\vec{F} = I \cdot \vec{l} \times \vec{B}, \tag{5.6}$$

in the direction perpendicular to the conductor and to the field lines according to the right-hand-rule of the cross product.

## c) Hall effect

Consider a conductor of cross-sectional area $A = d \cdot a$, which carries an electric current $I$:

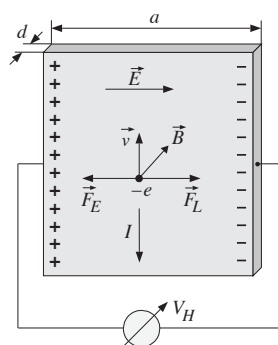$$I \;=\; j \cdot A \;=\; n \cdot e \cdot v \cdot d \cdot a \tag{5.7}$$

Figure 5.1: Sketch of the Hall-effect.

where $n$ is the carrier density, $e$ their charge and $v$ their drift velocity.

When the conductor is in a magnetic field, the field lines of which are perpendicular to the direction of the current, the moving charges experience the Lorentz force $F = e \cdot v_{\mathrm{d}} \cdot B$ transverse to their direction of motion, as represented in Fig. 5.1. Due to this force the charge carriers are deflected laterally and, as a consequence of a net charge separation, an electric field $\overrightarrow{E}$ is generated. Therefore, in equilibrium, the Coulomb force experienced by the charges $\overrightarrow{F_c} = e \cdot \overrightarrow{E}$ cancels the Lorentz force:

$$e \cdot E \; = \; e \cdot v \cdot B. \tag{5.8}$$

The transverse electric field can the be measured as a voltage across the conductor, the so-called Hall voltage $V_H$:

$$V_H \; = \; a \cdot E \; = \; a \cdot v \cdot B \; = \; \frac{I \cdot B}{n \cdot e \cdot d} \; = \; R_H \cdot \frac{I \cdot B}{d}. \tag{5.9}$$

The constant $R_{\mathrm{H}} = 1/(n \cdot e)$ is called the Hall constant of the material.

This effect is utilized in the Hall-sensors for high precision measurements of the magnetic fields. Since for a given magnetic field, the smaller the charge carrier density is the larger the Hall voltage, such probes often are realized with semiconductors.
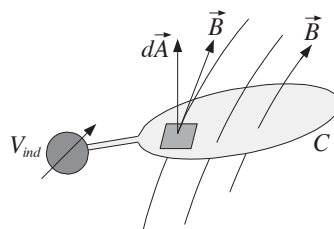
## d) Faraday's law of induction



Figure 5.2: Sketch representing Faraday's law of induction.

The magnetic flux $\Phi$ through a surface area $A$ is given by

$$\Phi \; = \; \int_A \overrightarrow{B} \cdot \overrightarrow{dA}. \tag{5.10}$$

Faraday's law of induction states that a temporal change of the magnetic flux through the area enclosed by a conductor loop $C$ (see Fig. 5.2) induces a voltage

$$V_{\text{ind}} \;=\; \oint_C \vec{E} \cdot \vec{ds} \;=\; -\frac{d\Phi}{dt}. \tag{5.11}$$
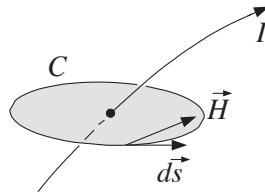
## e) Ampère's law



Figure 5.3: Sketch representing Ampère's law.

Every electric current produces a magnetic field, $\vec{H}$ in its vicinity. The Ampere's law states that the integral of the magnetic field along any closed curve $C$ equals the current flowing through every finite surface enclosed by this curve[3]

$$\oint_C \vec{H} \cdot \mathrm{d}\vec{s} = I \tag{5.12}$$

An example of the application of Ampere's law is given in the appendix of the experiment e/m, where it is used to calculate the magnetic field inside an infinitely long solenoid coil.

## f) Electromagnets

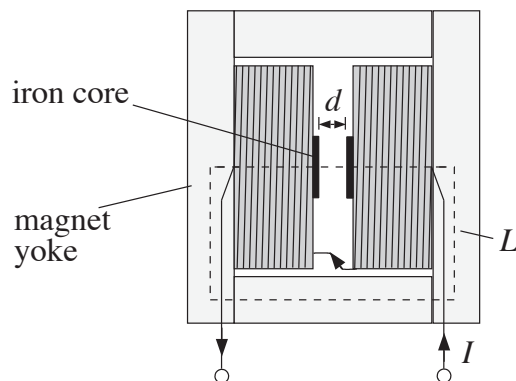A typical electromagnet is sketched in Fig. 5.4. It consists of a current-carrying coil wound around



Figure 5.4: Electromagnet.

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[3]This is actually an application of the Stokes' theorem $\oint_C \vec{V} \cdot \vec{ds} = \int_{A_C} \left( \vec{\nabla} \times \vec{V} \right) \cdot d\vec{A}$, where $A_C$ is any surface the border of which is given by the closed curve $C$. Therefore the current density $\vec{j} = \vec{\nabla} \times \vec{H}$.

two iron cores (the poles). In the air gap between the two pole a magnetic field is generated, which is homogeneous when the air gap between the two poles is narrow in comparison to its cross-sectional dimensions. The dashed line in Fig. 5.4 represents the closed path, along which Ampere's law is evaluated:

$$H_E \cdot L + H_L \cdot d = N \cdot I \tag{5.13}$$

where $H_L$ is the magnetic field in the air gap and $H_E$ denotes the magnetic field in the iron cores. At the surface of the iron cores the normal component of the magnetic flux density must be continuous, i.e.. $B_L = B_E$. Using Eqn. 5.2 we obtain

$$H_L = \mu_E \cdot H_E \tag{5.14}$$
$$= \frac{N \cdot I}{L/\mu_E + d}. \tag{5.15}$$

Using the approximation $L/\mu_E \ll d$ leads to

$$H_L = \frac{N \cdot I}{d} \tag{5.16}$$
$$B_L = \frac{\mu_0 \cdot N \cdot I}{d} \tag{5.17}$$

Without the iron cores $B_L$ would be significantly smaller yielding

$$B_L = \mu_0 \cdot H = \frac{\mu_0 \cdot N \cdot I}{L} \tag{5.18}$$

In an electromagnet with an iron core the magnetic field $H$ is always proportional to the current, $I$, flowing through the coil. In contrast to that, the magnetic induction $B$ follows an hysteresis curve as shown in Fig. 5.5 . For large values of $H$ the magnetic induction reaches a saturation value. Due to the magnetization the value of $B$ does not go back to zero when the current is reduced again to zero (thus $H = 0$) after the saturation value of $B$ was reached. The value $B_R$ is the residual or remanent magnetic induction when $I = 0$ and $H = 0$. $H_K$, called the coercitive field, is the magnetic field strength that must be produced in order to bring the induction field back to zero.
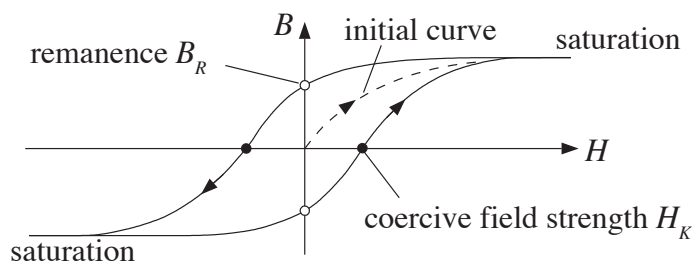


Figure 5.5: Hysteresis curve.

## 5.3  Experimental part

In this experiment the magnetic induction in the air gap of an electromagnet is measured as a function of the flow through the solenoid coil current $I_M$ in three different ways: by measuring the Lorentz force acting on a current carrying conductor, through the induced voltage in a loop and by means of a Hall probe.

### a) Measurement of the Lorentz force on a current-carrying conductor

In the first part, the magnetic flux density is to be measured by measuring the Lorentz force acting on a current-carrying conductor. The experimental setup is sketched in Fig. 5.6.
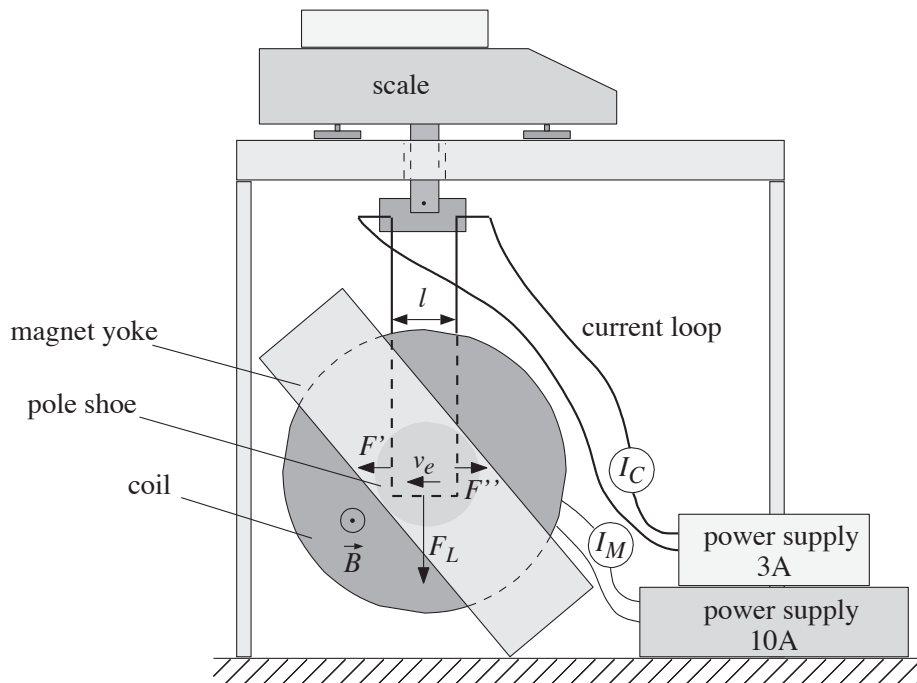


Figure 5.6: Experimental apparatus used for measuring the Lorentz force acting on the wire.

A rectangular conductor loop, through which the current $I_C$ flows, is placed between the poles of the electromagnet, such that the field lines of the magnetic field are perpendicular to the loop. The forces, $F'$ and $F''$, acting on the vertical segments of the rectangular loop cancel each other. The Lorenz force $F_L$ acting on the horizontal segment of the loop of length $l$ is measured directly using a scale. From this the magnetic induction field is determined by

$$F_L = l \cdot I_C \cdot B. \tag{5.19}$$

At the beginning of the experiment, the current loop is mounted by the assistant on the hook of the scale and oriented so that the horizontal segment is exactly centered between the two poles. This ensures that there is no resultant torque acting on the scale during the experiment.

- Connect the current loop to the power supply.

- Tare the balance off by pressing the TARE button. The scale should now display 0 g.

- Set the magnet current $I_M$ for about one second to 10 A, and then turn it back to 8 A. This ensures that the magnetization is saturated, and the measurement starts on a well defined point on the hysteresis curve (Fig. 5.5).

- Measure the force indicated by the scale as function of $I_M$, while reducing the solenoid current in steps of 1 A from 8 to 0 A.

- Calculate the magnetic induction, $B$, according to Eqn. 5.19 and plot it as a function of the solenoid current $I_M$.

- Calculate and plot the error bars in the graph using the error margins given in the lab.

- Compare the measured values of $B$ versus $I_M$ with the expected relation.

## b) Measurement of the voltage induced in a test coil

In the second part of the experiment, the magnetic induction is determined by the measurement of the voltage induced in a probe coil from the magnetic field center of the electromagnet. The experimental setup is sketched in Fig. 5.7. A coil with $N$ turns and cross-sectional area $A$ is located at first in the magnetic field between the poles of the magnet and is then taken out to the field-free space outside the magnet.
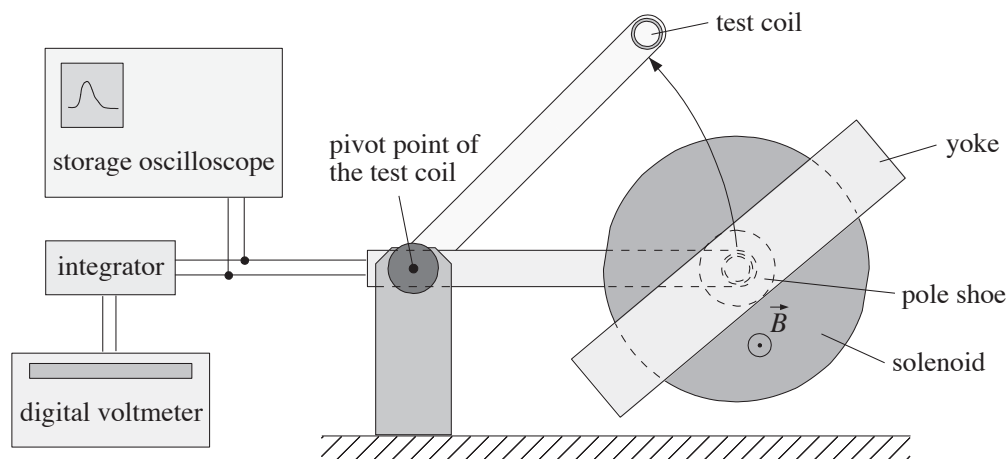


Figure 5.7: Experimental setup for magnetic field measurement using a moving test coil.

According to the law of induction, a voltage is induced during the motion of the coil out of the field between the pole shoes:

$$V_{\text{ind}}(t) \ = \ -N \cdot A \cdot \frac{dB}{dt}. \tag{5.20}$$

Integrating over the time and considering $B(t=0) = B$ and $B(t=\tau) = 0$ we obtain the following relation for the voltage pulse:

$$\int_0^\tau V_{\text{ind}}(t)\,dt \ = \ -N \cdot A \cdot \int_{B(t=0)}^{B(t=\tau)} dB \ = \ -N \cdot A \cdot B. \tag{5.21}$$

The integral of the induced voltage has to be measured with an integrator circuit, which is described in the appendix of this manual.

- The probe-coil must be disconnected before the calibration of the integrator or should not be connected until the end of the calibration!

- Turn on the integrator and wait about 5 minutes, until it has warmed up reaching its operating temperature.

- Determine together with the assistant the transfer constant of the integrator using the method described in the Appendix. **Caution: the settings on the main unit supply should be changed by the assistant only!**

- For this purpose an harmonic alternating voltage amplitude $V_{\mathrm{rms}} \approx 1$ V at a frequency of 50 Hz is fed into the input of the integrator. The input and output voltages are measured by the digital voltmeter (range 2 $V_{\mathrm{AC}}$) and the parameters GAIN adjusted so that both are equal. Equation (5.32) then yields:

$$RC \;=\; \frac{1}{100\,\pi}\ \mathrm{s} \;\;\approx\;\; \frac{1}{314}\ \mathrm{s} \tag{5.22}$$

- Put the test coil between the pole shoes, set the solenoid current $I_M$ to 4 A and reset the integrator by actuating the ZERO button.

- Swing out the sample coil of the magnetic field. Observe the waveform of the induced voltage on the oscilloscope and read its integral from the integrator.

- Repeat the experiment several times, where you can quickly swing the coil each time differently from the magnetic field. Observe how the pulse shape and integral value of the voltage pulse depend on the velocity of the coil.

- Bring the electromagnet into saturation, as in the first experiment, by adjusting the magnet current for about one second to 10 A and then immediately reduce to 8 A.

- Measure the voltage pulse reduction at the integrator as function of the solenoid current $I_M$ in steps of 1 A from 8 A down to 0.

- Calculate the magnetic induction $B$ for each point. According to Eqn. 5.30 (see Appendix) and Eqn. 5.21

$$V_A \cdot R \cdot C \;=\; N \cdot A \cdot B \tag{5.23}$$

and finally

$$B \;=\; \frac{V_A \cdot R \cdot C}{N \cdot A}. \tag{5.24}$$

Plot the values of $B$ as a function of the solenoid current $I_M$ including the error bars. Perform the error calculation the specifications on the device.

## c) Measurement of the B-field using a Hall probe

In the third part of the experiment, the magnetic induction is to be determined by means of a Hall probe. The device used consists of the Hall probe itself and a calibrated display device on which the measured magnetic induction can be read directly.

*Attention: When handling the Hall probe use extreme caution, since it is a fragile and very expensive instrument.*

- Calibrate the Hall probe for two well-known magnetic fields ($B = 0$ and $B = 0.55$ T). The instructions for this are available in the laboratory.

- Mount the Hall probe in the center between the pole shoes of the electromagnet. Make sure that the flat side of the probe is exactly parallel to the poles edges.

- Bring the electromagnet as in the first part of the experiment into saturation by adjusting the magnet current for about one second to 10 A and then immediately reduce to 8 A.

- Measure with the Hall probe and plot the induction field $B$ as a function of the solenoid current, $I_M$, while reducing the current from 8 A down to 0 A in steps of 1 A.

- Determine and plot the measurement uncertainty as error bars (the error of the probe amounts to $\pm 1.5\%$ of the measuring range, the error of the amplifier and display amounts to $\pm 0.25\%$ of the measuring range).

- Finally, compare the results obtained in the three parts of the experiment.

## 5.4   Appendix

The core of the integrator used in the second part of the experiment is realized with an operational amplifier whose basic circuit is shown in the left part of Fig. 5.8. An operational amplifier can be represented as an amplifier with external input impedance $Z_E$ and a feedback impedance $Z_R$. An operational amplifier operating in inverting mode changes the sign of the output voltage $V_A$ with respect to the voltage input $V_E$. Furthermore, the input impedance of the amplifier itself as well as its open loop gain $G_0$, that is the gain without feedback, can be very large.
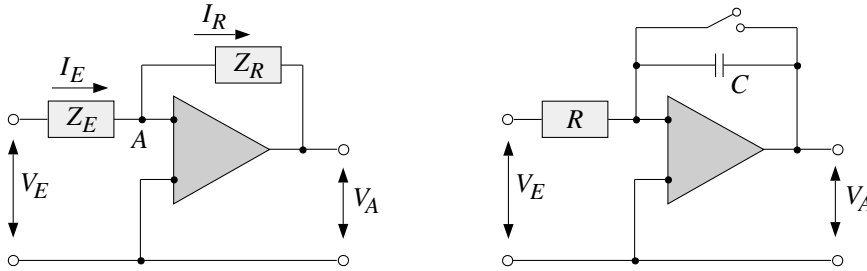


Figure 5.8: Operational amplifier.

Since the internal input impedance is much larger than the "external impedances" of the circuit for this kind of applications, the input bias current into the ampifier from node $A$ can be neglected. With this approximation it is possible to consider only the current flowing trough the feedback impedance, $Z_R$, and through the input impedance $Z_E$. Then, from Kirchhoff's node law applied to the node $A$ it follows that

$$I_E - I_R \;=\; \frac{V_E}{Z_E} - \frac{V_R}{Z_R} \;=\; 0. \tag{5.25}$$

The amplifier works in such a way that the voltage between the two input terminals is minimized or ideally zero. Therefore, in our example the node $A$ will be on electric ground. Hence, with feedback we obtain for the voltage gain $G$

$$|G| \;=\; \frac{V_A}{V_E} \;=\; \frac{Z_R}{Z_E}. \tag{5.26}$$

Thus, the gain is solely determined by the external impedances $Z_E$ and $Z_R$.

If we specifally choose $Z_E = R$ and $Z_R = 1/(i\omega C)$ (see the circuit in the right panel of Fig. 5.8), it follows that

$$I_E \;=\; \frac{V_E}{R} \tag{5.27}$$

$$I_R \;=\; C \cdot \frac{dV_A}{dt} \tag{5.28}$$

$$\Rightarrow \; \frac{V_E}{R} \;=\; C \cdot \frac{dV_A}{dt}. \tag{5.29}$$

Integration over time yields

$$V_A \;=\; \frac{1}{RC} \cdot \int_0^\tau V_E \, dt \;+\; V_E(t=0) \tag{5.30}$$

The output voltage $V_A$ is therefore equal to the time integral of the induced voltage pulse at the input when the input voltage $V_E(t = 0)$ is set to zero by short-circuiting the feedback capacitor right before the arrival of the input pulse.

The time constant $RC$ can be determined by applying an harmonic AC voltage of known amplitude $V_E$ and known frequency $\nu$ at the input of the operational amplifier and adjusting the resistance $R$ to obtain unitary gain. Then we have

$$\left| \frac{V_A}{V_E} \right| \;=\; \frac{1}{\omega \cdot RC} \;=\; 1 \tag{5.31}$$

and, thereby,

$$RC \;=\; \frac{1}{\omega} \;=\; \frac{1}{2\,\pi \cdot \nu}. \tag{5.32}$$