4. Electron Charge-to-Mass Ratio

4.1 Introduction

In this laboratory, we study the interaction of electrons and a magnetic field, with the specific purpose to determine the electron charge-to-mass ratio $\frac{e}{m}$, sometimes referred to as the *specific charge* of the electron or the mass-normalized charge. Throughout this manual, we use the designation *magnetic field* for the field $\vec{H}$, and *magnetic induction* or simply *field* for the field $\vec{B}$.

4.1.1 Scientific background

While the charge of the electron, $-e$ [1] may be determined separately, as demonstrated in Millikan's oil droplet experiment, the electron mass m_e, because of its exceedingly low value[2], cannot be found in a direct experiment with satisfactory precision. The mass may be determined, though, if the charge-to-mass ratio is known. For convenience, we write in this text m for the electron mass.

In the beginning of the era of modern physics, in 1897, J.J. Thomson[3] with others performed measurements convincingly demonstrating that the previously observed *cathode rays*[4] were actually particles. These were later identified as electrons[5], and Thomson is now generally credited with their discovery. The 2006 CODATA[6] recommended value is $e/m = 1.758820150(44) \times 10^{11}$ C/kg. CODATA refers to this as the electron charge-to-mass *quotient*, but the term *ratio* is still in common use.

Methods to determine the charge-to-mass ratio often rely on the observation of the deflection of a charged particle in external magnetic or electric fields. A well known application of this principle is the mass spectrometer. The same principle can be used to extract information in experiments with rays of particles using the Wilson cloud chamber.

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[1]We adopt here the convention to designate e as the *elementary positive charge* to avoid confusion over its sign; CODATA 2011: $e = 1.602176565(35) \times 10^{-19}$ C.

[2]CODATA 2010 gives $m_e = 9.10938291(40) \times 10^{31}$ kg

[3]Sir Joseph John Thomson, OM, FRS (1856-1940), British physicist. Nobel laureate 1906.

[4]First observed in 1869 by German physicist Johann Hittorf, and named *Kathodenstrahlen* in 1876 by Eugen Goldstein

[5]The original name is retained, e.g. in the word Cathode Ray Tube (CRT).

[6]Committee on Data for Science and Technology

4.1.2   Tubes and electron guns

The 19th century experiments with cathode rays involved glass containers of different shapes and designs, but generally called tubes, of which an important example is the Crookes tube[7]. These tubes could be evacuated to a low pressure, and contained two separate electrodes connected to an external electric circuit: a negative electrode, *the cathode*, either cold or externally heated; and the electrically positive anode. Electrons produced at the cathode will be accelerated towards the anode by the electrostatic field.

Cathode rays were so named because they were early on observed to emanate from the negative electrode. The 'rays' themselves are invisible, but may be detected indirectly, either as a glowing light from the glass wall behind the anode, or, if the tube has a rarefied gas-filling, as a trace of light along the path of the rays, the colour of which depends on the atomic species of gas. In the first case, the visible light is a greenish fluorescence from the atoms of the glass wall being hit by the rays; in the latter, the light indicating the trace of the electrons is actually light from the gas atoms returning to a lower state after having been excited by the electron beam.

The production of the electron beam is of secondary interest in this lab, but nevertheless we need some understanding of the physics behind. A dedicated *electron gun* (also called electron emitter) is employed to generate an electron beam, see Fig.4.1. The cathode of the gun is an electrically heated metal filament fed by a separate external source of standard 6.3 V. Electrons are emitted from the surface of the cathode through *thermionic emission*, meaning that the thermal energy overcomes the binding energy of the electrons in the cathode, which is often coated with carbides or borides of transition metals to diminish the exit potential.

The free electrons are attracted by the anode and accelerated so that a collimated beam of electrons with approximately the same speed leaves through a small hole (aperture) in the anode. Often additional electrodes are provided for different purposes. Most common is the *Wehnelt cylinder*[8], which functions as an electrostatic lens, concentrating (focussing) the beam before it arrives at the anode. The cylinder is placed between the cathode and the anode. It is held biased to a variable, slightly negative voltage relative to the cathode, in order for it to repel electrons leaving the cathode in directions not parallel to the axis of the gun.

In the present laboratory, two different sets of equipment will be used: one with a magnetic field perpendicular to the beam of electrons; another with the magnetic field in parallel to the beam.

4.1.3   Beam parallel to a magnetic field

In this experiment, also called the longitudinal arrangement of field and beam, we put to use a cathode ray tube of a type previously commonplace in electronic oscilloscopes, shown in Fig. 4.4 inside the coil that produces the field parallel to the axis of the tube. The electron gun is mounted at the narrow end of the tube, which opens in a shape resembling a circular cone towards the wider, approximately flat end, coated with a fluorescent material. The CRT is provided with pairs

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[7]Sir William Crookes, OM, FRS (17 June 1832 – 4 April 1919) chemist and physicist.
[8]Invented 1902 to 1903 by German physicist Arthur Rudolph Berthold Wehnelt.

Abbildung 4.1: Electron gun in cathode ray tube installation. The electron beam leaves to the left. Clearly seen is the cathode with three holes - there are three separate cathodes incorporated in this design - the Wehnelt cylinder and the anode with its single hole. The wedge shaped structure to the left is a pair of electrostatic deflection plates. The scale of the picture is such that the horizontal section shown has an actual length of about 5 cm.(from Wikipedia: *willemdd*)

of electrostatic plates, allowing the beam to be deflected by transversal electrostatic fields, though only horizontal deflection is used in this lab. The trace of the beam may be observed on the screen as it strikes the fluorescent coating.

4.1.4   Beam perpendicular to a magnetic field

Alternatively, this is also the *transversal* arrangement of beam and field. The tube used here is sometimes known as the Perrin tube[9]. The electron beam comes from a gun mounted in a neck of a spherical tube, and directed through the centre to become available for experiments. Since the pressure in the tube is so low that almost no interaction between the gas molecules and the electrons takes place, there is no visible trace of the beam, except for the screen inserted vertically in the centre of the tube with an inclination of 15° to the beam. The beam leaves the anode through a horizontal slit and is thus wide enough to excite a trace along the entire screen, which is covered on the reverse with a fluorescent coating and has a printed cm-scale grid (graticule) on the front.

4.2   Theory

4.2.1   Accelerated electron beams

For simplicity, we let the electron have the positive charge e, and adjust the relationships accordingly. With U_A the accelerating voltage between cathode and anode of the electron gun, the speed of the electron with charge e may be calculated from the conservation of energy: electrical potential energy dispensed between cathode and anode must equal the kinetic energy gained by an electron

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[9]Jean Baptiste Perrin (30 September 1870 – 17 April 1942). French physicist and Nobel laureate in 1926.

as it leaves the gun through the anode aperture.

$$\frac{1}{2} m \cdot v^2 \;=\; e \cdot U_A \;.$$ (4.1)

Solving for the speed v then gives:

$$v \;=\; \sqrt{\frac{2 \, e \cdot U_A}{m}} \quad .$$ (4.2)

Within narrow limits, which depend on the statistical processes involved in the thermal emission at the cathode, and stray fields in the electron gun, all the electrons of the beam will have approximately the same speed.

4.2.2 Motion of an electron in a homogeneous perpendicular magnetic field

In order for the electron beam to travel in a homogeneous field $\vec{B}$, the spherical tube is positioned symmetrically between the coils of a Helmholtz pair, with their common axes running through the centre of the tube, such that the field is perpendicular to the electron beam. A charge e with velocity $\vec{v}$ moving in the field $\vec{B}$, is subject to the Lorentz force $\vec{F}_L = e \cdot \vec{v} \times \vec{B}$. The force is always perpendicular to the propagation of the electron and thus no work is done and no kinetic energy lost or gained.

For the special case of an electron moving at speed $\vec{v}_s$ perpendicularly to the field lines of a homogenous field $\vec{B}$, the magnitude of the Lorentz force is $F_L = e \cdot v_s \cdot B$ constant. The path of the electron becomes a circle with radius r:

$$\frac{m \cdot v_s^2}{r} \;=\; e \cdot v_s \cdot B$$ (4.3)

or explicitly

$$r \;=\; \frac{v_s}{(e/m) \cdot B} \quad .$$ (4.4)

The time T required for one complete turn in the circle is accordingly:

$$T \;=\; \frac{2\pi \cdot r}{v_s} \;=\; \frac{2\pi}{(e/m) \cdot B} \;,$$ (4.5)

which apparently is independent of the speed of the electron. To illustrate this, we may imagine three electrons in a plane perpendicular to a field $\vec{B}$, see Fig. 4.2. The electrons start at the common point P at time $t = 0$, but in different directions and with different speed. Equation 4.4 tells us that the electrons follow circular paths with different radii and Eq. 4.5 that they all require the same time T for a full turn in the circle, and meet again at point P at the same time $t = T$ later. Since $\vec{F}_L = e \cdot \vec{v} \times \vec{B}$, the Lorentz force vanishes for electrons moving strictly parallel to the field lines. In this case there would be no interaction with the field that could be used to determine e/m.

If, however, the electron is moving with velocity $\vec{v}$ at some angle φ to the direction of a homogenous field, then decomposing $\vec{v}$ into components parallel and perpendicular to the field allows us to device a method for the determination of e/m. No Lorentz force is acting on the component $v_p = v \cdot \cos \varphi$ parallel to the field lines, and consequently the component v_p of the electron remains unaffected.
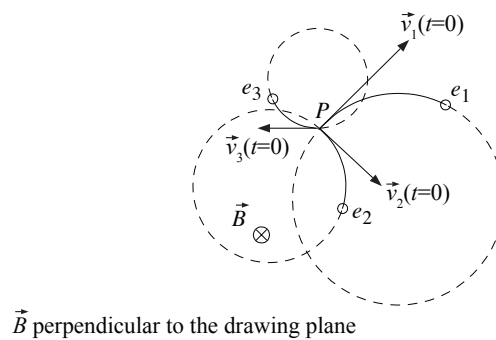
$\vec{B}$ perpendicular to the drawing plane

Abbildung 4.2: Circular traces of three electrons in a field $\vec{B}$ perpendicular to the plane of the paper, starting at point P in different directions with different speed.

In a plane perpendicular to the field, the tangential component of the motion with speed $v_s = v \cdot \sin \varphi$ is subject to a Lorentz force that guides the electron on a circular path with a constant radius, according to Eq. 4.4. Figure 4.3 shows that the combined motion is a helical path with axis parallel to the field and with constant radius. In a distance d in the direction of the field, given by

$$ d \;=\; v_p \cdot T \;=\; \frac{2\,\pi \cdot v_p}{(e/m) \cdot B} \tag{4.6} $$

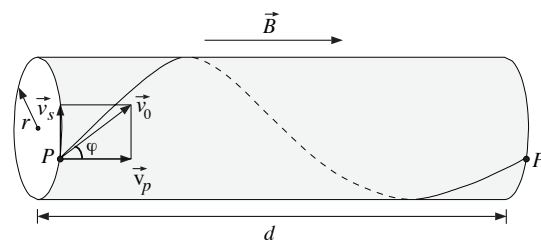the electron completes a full circular turn perpendicular to the field.

Abbildung 4.3: Path of an electron in a homogenous field $\vec{B}$, starting at an angle φ at point P and completing one turn of a circle.

4.3   Experimental

The experimental work requires skillful adjustments and careful reading of voltages and currents. In some instances, to enhance precision, current is read from an analogue galvanometer with a narrow strip of mirror below the tip of the pointer. The position of the pointer should be read when the pointer obscures its own reflection. In this manner the reading becomes parallax free, and a major source of error eliminated.

4.3.1   Determination of e/m with a field $\vec{B}$ parallel to the electron beam

In the actual set-up, a cathode ray tube is mounted on a horizontal rail that allows a coil of copper wire winding (solenoid) to be slid over the tube, as shown in Fig. 4.4. A field of homogeneous magnetic induction $\vec{B}$ may thus be applied parallel to the axis of the tube.
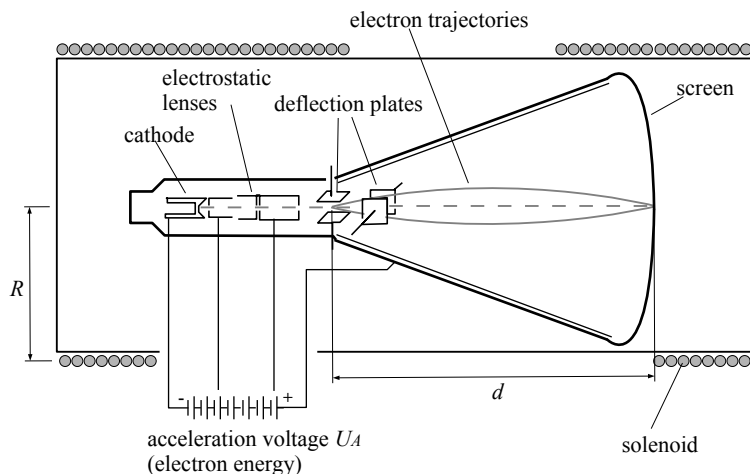
The dimensions are as follows:

solenoid:

$$l = 395 \pm 1 \text{ mm},$$
$$R = 39 \pm 1 \text{ mm},$$
$$609 \text{ windings},$$

electron path $d = 75 \pm 2$ mm.

Abbildung 4.4: Longitudinal section through the oscilloscope tube inside the coil producing a longitudinal field. The indirectly heated cathode is indicated together with the anode acceleration voltage circuit.

The beam of electrons that leaves the electron gun travels on a straight line parallel to the axis of the tube, and is not influenced by the presence of a field $\vec{B}$. The electrons leaving the electron gun have the same speed v (within narrow limits) determined in Eq. 4.2 by the acceleration voltage U_A. The small aperture at the gun exit may be considered to be a common point from which the electrons start, corresponding to the point P in Fig. 4.3.

With a pair of plates just after the anode, the beam may be deflected in a horizontal electrostatic sinusoidal AC field, which becomes visible at the fluorescent screen as a straight line, symmetrical about the centre point, and of a length proportional to the amplitude of the applied voltage. In effect, this allows the user to let the electron travel with a small angle φ relative to field $\vec{B}$, equivalent to the situation discussed above. The difference is only that, for practical reasons, we now have not

a single constant angle φ that would correspond to single point on the fluorescent screen, but a time dependent distribution of angles $\pm\varphi$, corresponding to the line. The angles φ may be considered to be small and therefore $\cos\varphi \approx 1$ and thus $v_p = v \cdot \cos\varphi \approx v$, independent of the exact value of φ.

As the field strength B gradually increases, the trace of the beam, the straight line on the screen, graduately rotates and decreases in length, because the angular velocity is independent of the electron speed and of the radius of the trace of the electron as can be seen from Eq. 4.5. All traces will meet again in a single point (image of the effective electron source) provided that the field strength B corresponds to an integer number of circles in the plane perpendicular to the magnetic field lines. To put it differently, for the correct value of field strength B, such that Eq. 4.6 is satisfied, we expect to see a single point on the screen. We may then replace v_p in Eq. 4.6 with v from Eq. 4.2 and solve for the ratio e/m. This method was first used by H. Busch in 1922.[10]

$$e/m \;=\; \frac{8\,\pi^2 \cdot U_A}{B^2 \cdot d^2} \;. \tag{4.7}$$

The control box of the CRT, marked *PS zu KO-RÖHRE*, has a few rather obvious controls. From left to right: a red pilot light and a white toggle switch for the electric power, a black radio knob for beam focus, a toggle switch marked *Ablenkung* for the fixed amplitude horizontal deflection of the beam (50 Hz sinusoidal AC voltage applied to the horizontal plates), and below another black knob for adjusting the intensity of the beam. The calculation of the solenoid field B is explained in an appendix at the end of this manual.

- Ensure that the CRT is situated roughly in the center of the solenoid.

- Switch on the high-voltage supply (Heinzinger) for the cathode (electron acceleration). By pressing *Preset* the current limit can be adjusted (2 mA). The voltage should be zero. Press *HV on* and increase the voltage to about 400 V, switch on deflection. Adjust focus and intensity, such that for $B = 0$ a sharp, horizontal trace is seen on the screen Limit the intensity in order not to damage the fluorescent coating of the tube.

- Turn on the power supply of the solenoid. The voltage limit should be set to about 40 V; use the current control (big center knob) to vary the current in the solenoid. From time to time check that the coil does not become excessively hot (the outside should not feel hot to the touch of your hand). Increase the current and describe how the trace on the screen changes.

- Increase the solenoid current and find the value for which deflected electrons are refocussed. Determine the uncertainty of this subjective procedure. Make notes for the report about your observations.

- Choose six values for the acceleration voltage ranging from 350 V to 600 V. For each value determine the current needed to refocus the electron beam (again: only shortly apply high currents and ensure that the current does not exceed 4 A!).

- Calculate for each current value the field strength B inside the solenoid using Eq. 4.15 and plot B^2 as function of voltage U_A.

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[10]H. Busch, Physik Zeitschrift **23**, S. 438 ff. (1922); see as well H.V. Neher, Amer. J. Phys. **29, 471 (1961).**

- Fit a line to the data points and determine e/m from its slope using Eq. 4.7. Calculate the error on e/m.

4.3.2   Determination of e/m using a field $\vec{B}$ perpendicular to the electron beam

A beam of electrons that enters perpendicularly into a homogeneous field of induction $\vec{B}$ with speed v, describes a circle with radius Eq. 4.4. Eliminating the speed from Eq. 4.2 we obtain

$$e/m \;=\; \frac{2\,U_A}{r^2 \cdot B^2} \;.\tag{4.8}$$

The specific charge of the electron, e/m, is thus given by the radius of curvature of its circular path r, the acceleration voltage U_A and the magnetic induction B.

The experimental set-up is outlined in Fig. 4.5. A pair of Helmoltz coils (not shown) is positioned such that a horizontal field acts perpendicular to the beam. The geometry and the pertinent field calculations are explained in the appendix
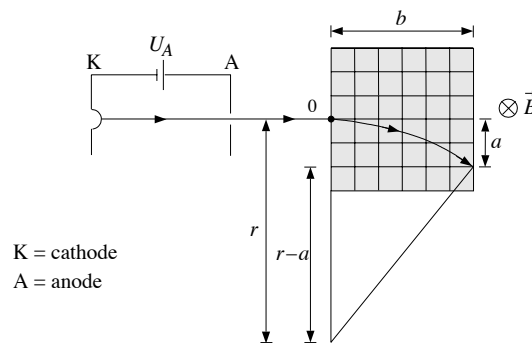

Abbildung 4.5: Principles of the experiment in the Perrin tube, showing in the plane of the paper the electron gun and a beam of electrons bent into a circular path by a perpendicular magnetic induction $\vec{B}$.

- Qualitatively investigate the independent influence of the acceleration voltage and the current through the Helmholtz coils

- Set the acceleration voltage to 2000 V and adjust the coil current such that the trace goes through the point of the graticule with co-ordinates (10,+2). The beam is assumed to pass horizontally through the origin. Read the corresponding coil current from the amperemeter.

- Calculate the magnetic induction B for the midpoint between the coils, using Eq. 4.20 (appendix). The radius of the Helmholtz coils is $R = 6.8\,$cm (estimate the error), the number of turns $N = 320$.

- Calculate the radius of curvature r of the trace using the co-ordinates a and b given in Fig. 4.5, using the equation of the circle $(r - a)^2 + b^2 = r^2$. We may solve for the radius of curvature to obtain

$$r = \frac{a^2 \,+\, b^2}{2\,a} \;.\tag{4.9}$$

Estimate the primary errors and calculate the error of the radius using error propagation.

- The assumption that the beam passes through the origin of the graticule in parallel to the grating lines is generally not true, which is liable to introduce a systematic error in the measurement. It may be removed by reversing the direction of the field, thus making the circle bend in the other direction. This operation does not, however, remove the random errors in the reading of the co-ordinates, which has to accounted for properly.

- Repeat the previous determination of the radius of the circle in the new orientation: adjust the current so that the trace passes through (10,-2) and read the current as before.

- Calculate the magnetic induction, again using Eq. 4.20, and the mean value of the current. Then calculate the ratio e/m according to Eq. 4.8.

- Repeat the procedure above with an acceleration voltage of 3000 V, or, if this cannot be attained, a value for which the reading on the voltmeter of the supply is stable.

- Calculate the error of the measurement of e/m, using estimated errors of the measured parameters and error propagation for Eq. 4.8.

- Calculate the mean of the two separately obtained values of the ratio e/m.

4.4   Appendix

a) The magnetic induction of a solenoid.

In the experiment using a longitudinal field, the magnetic induction $\vec{B}$ is produced with a cylindrical solenoid. The magnetic field $\vec{H}$ in the coil may be calculated using the law of Ampère[11]:

$$\oint_{\Gamma} \vec{H} \cdot d\vec{s} \; = \; \sum_{i}^{N} I_i \qquad\qquad (4.10)$$

In words, this relationship tells us that the integral along a closed path of integration Γ in the magnetic field, is equal to the sum over the N enclosed currents $\sum_{i}^{n} I_i$. The integration is performed with the infinitesimal path element of integration $d\vec{s}$. Inside an infinitely long solenoid the field is approximately homogeneous and parallel to the axis, while the field vanishes outside the solenoid.
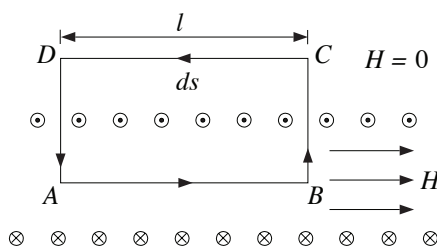

Abbildung 4.6: Longitudinal section of the solenoid to aid in the calculation of the magnetic field $\vec{H}$. Shown is a closed rectangular path of integration $\Gamma : ABCD$.

To calculate the magnetic field H inside the solenoid, the path of integration Γ should be chosen to render the most convenient evaluation, in this case the rectangle $ABCD$ in Fig. 4.6. The path is assumed to enclose N turns of wire, each carrying the same current I. On the right of Eq. 4.10 the sum becomes $N \cdot I$. On the left we evaluate the path integral over the four linear segments, and readily find that only AB contribute: the field for the segment CD is zero and the segments BC and DA are perpendicular to the field so that the scalar product $\vec{H} \cdot d\vec{s}$ vanishes. The left side of Eq. 4.10 thus becomes $H \cdot l$, and the magnetic field H inside the infinitely long solenoid:

$$H = \frac{N \cdot I}{l} \qquad\qquad (4.11)$$

A, exact calculation of the field along the symmetry axis in a distance x from one end of the solenoid of finite length l and radius R yields:

$$H = \frac{N \cdot I}{2l} \left(\frac{x}{\sqrt{x^2 + R^2}} + \frac{l - x}{\sqrt{(l - x)^2 + R^2}} \right) \qquad\qquad (4.12)$$

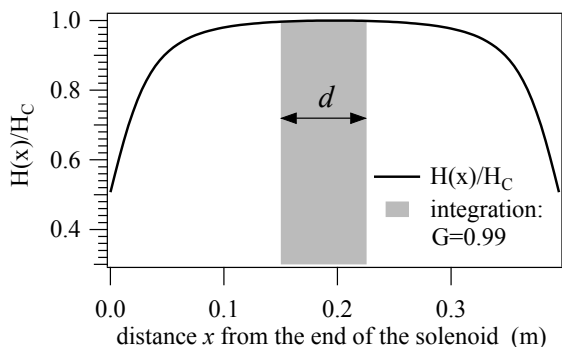[11]André-Marie Ampère (1775 − 1836), French physicist and mathematician.

Abbildung 4.7: Magnetic field strength with respect to H_C in the center of the solenoid as function of distance x from one end of the solenoid. The grey shaded area denotes the trajectory of the electrons and, hence, the range of integration for the calculation of the correction factor $\overline{G}$.

In the present case, the length d of electron trajectory is short compared to the length of the solenoid and close to the center of the latter, as depicted by the grey shaded area in Fig. 4.7. The field variations felt by the electrons are, thereby, small with respect to the mean field strength. Thus, the inhomoheneity of the magnetic field may be lumped into an average field strength $\overline{H} \approx \overline{G} \cdot H_C$. Calculating H_C from Eq. 4.12 and $\overline{G}$ by integration over the path length d yields finally:

$$\overline{H} = \overline{G} \cdot H_C = \overline{G} \cdot \frac{N \cdot I}{\sqrt{l^2 + 4R^2}} \quad \text{with } \overline{G} = 0.99. \tag{4.13}$$

The magnetic induction $\vec{B}$ in vacuum is then obtained as $\vec{B} = \mu_0 \cdot \vec{H}$, using the vacuum permeability μ_0 (also known as the *magnetic constant*)[12].

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{V} \cdot \text{s}}{\text{A} \cdot \text{m}} = 1.256637061... \times 10^{-6} \frac{\text{V} \cdot \text{s}}{\text{A} \cdot \text{m}} \tag{4.14}$$

The magnetic induction B in vacuum on the symmetry axis of a solenoid eventually becomes:

$$B = \frac{\mu_0 \cdot \overline{G} \cdot N \cdot I}{\sqrt{l^2 + 4R^2}} \tag{4.15}$$

b) The magnetic induction of a pair of Helmholtz coils

Two coils of electrical wire may be arranged such that a nearly uniform magnetic field is attained in the volume between them, a principle developed by the German physicist Hermann Ludwig Ferdinand von Helmholtz (August 31, 1821 – September 8, 1894). More specifically, a *Helmholtz pair* consists of two identical thin, but not necessarily circular coils, positioned symmetrically on each side of the experimental area along a common axis, and separated by a distance d equal to the radius R of the coils. The two coils are electrically connected in a manner such that the same electrical current runs through both, and that the field of each has the same direction, Fig. 4.8

The magnetic induction in the region between the two coils may be calculated using the law of Biot[13] and Savart[14]:

$$d\vec{B} = \mu_0 \cdot \frac{I}{4\pi} \cdot \frac{d\vec{s} \times \vec{r}}{r^3} . \tag{4.16}$$

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[12]Alternative SI-Units are H m^{-1}, or N A^{-2}, or T m/A, or Wb/(A m)

[13]Jean-Baptiste Biot (1774 – 1862) French physicist, astronomer, and mathematician

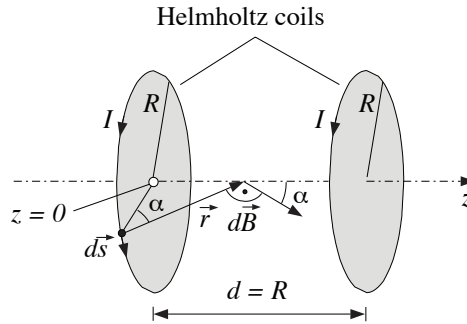[14]Félix Savart (1791– 1841) French physicist, professor at Collège de France in 1836

Abbildung 4.8: The relevant geometry for a set-up with a Helmholtz pair of radius R and distance $d = R$. The z axis defines the symmetry of the pair; the origin is in the centre of the left coil.

Finding a general solution valid for the entire volume is a rather involved procedure. We therefore restrict the calculation to the component along the symmetry axis.

$$dB_z \;=\; \mu_0 \cdot \frac{I}{4\,\pi} \cdot \frac{ds}{r^2} \cdot \cos\,\alpha \tag{4.17}$$

From figure 4.8 we find the relations $r^2 = R^2 + z^2$ and $\cos\alpha = R/r$. Integrating over the N turns of the left coil, we obtain the contribution to the field from the left coil at position z on the axis of symmetry:

$$B_l \;=\; \mu_0 \cdot \frac{N \cdot I}{4\,\pi} \cdot 2\,\pi\,R \cdot \frac{R}{(R^2 + z^2)^{3/2}} \;=\; \frac{\mu_0 \cdot N \cdot I}{2} \cdot \frac{R^2}{(R^2 + z^2)^{3/2}} \tag{4.18}$$

The analogous calculation of the contribution B_r from the right coil in the same system of coordinates requires that we replace z with $d - z = R - z$. The total field is then found by a summation of the two contributions:

$$B_z \;=\; \frac{\mu_0 \cdot N \cdot I}{2} \cdot \left| \frac{R^2}{(R^2 + z^2)^{3/2}} + \frac{R^2}{(R^2 + (R - z)^2)^{3/2}} \right| , \tag{4.19}$$

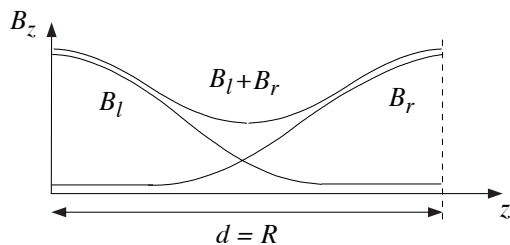which is shown in Figure 4.9 as a function of z.

Abbildung 4.9: Magnitude of the field component B_z between a Helmholtz pair, on the axis of symmetry, according to Eq. 4.19. The fields B_r and B_l are the contributions from the right and the left coil, respectively.

For the midpoint on the symmetry axis between the coils, cf Fig. 4.8, we find, with $z = d/2 = R/2$ in Eq. 4.19:

$$B_z\,(z = R/2) \;=\; \frac{\mu_0 \cdot N \cdot I}{R} \cdot \frac{8}{5\sqrt{5}} \; . \tag{4.20}$$