3. Elastic Collision

3.1 Introduction

A collision is an interaction of two bodies during a short period of time, while momentum and energy are being exchanged. It is called an *elastic* collision if no energy in the production of heat, irreversible deformations, electronic excitations or other non-kinematic effects is used up. During the collision, there are complicated forces at work between the two bodies. The laws of conservation of momentum, energy, and (under certain circumstances) angular momentum allow a satisfying determination of the behaviour of the interacting bodies after the collision. In this experiment, this will be illustrated in the elastic collision of two balls.

In the first part of the experiment, a ball's kinetic energy, after rolling down an inclined plane, that can be transmitted onto a ball at rest is determined. The second part concerns itself with verifying the law of conservation of momentum by measuring the momentum after the collision in relation to the scattering angle for an initially resting ball. The result is then compared with theory.

3.2 Theory

a) Kinetic Energy of a Rolling Ball

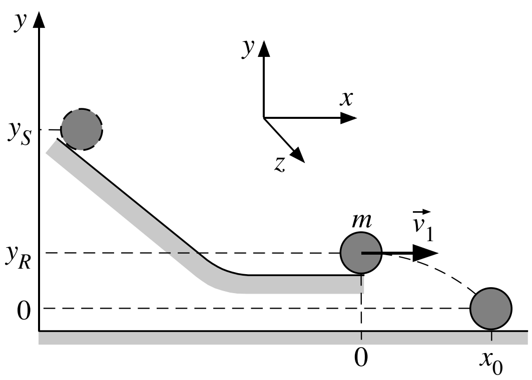

Figure 3.1: Experimental set-up

For a ball rolling down an inclined plane, as shown in fig. 3.2, the law of conservation of energy allows the calculation of the kinetic energy E_k at the end of the ramp if friction loss is ignored, while the potential energy E_p and rotational energy E_r are known:

$$E_k = E_p - E_r \tag{3.1}$$

$$\frac{1}{2} m \cdot v_1^2 = m \cdot g \cdot (y_S - y_R) - \frac{1}{2} I_0 \cdot \omega^2 \tag{3.2}$$

With parameters as follows:

m = Mass of the ball
v_1 = Absolute value of the ball's velocity
g = Gravitational acceleration = $9.81\,\mathrm{m/s^2}$
$y_S - y_R$ = Difference in height between starting point and end of ramp
I_0 = Moment of inertia of the ball
ω = Angular velocity of the ball

If r is the radius of the ball and the ball is in direct contact with the underlay in one point only, then:

$$v_1 = \omega \cdot r \tag{3.3}$$

With

$$I_0 = \frac{2}{5} m \cdot r^2 \tag{3.4}$$

follows from eq. 3.2 the absolute value of velocity at the end of the ramp:

$$v_1 = \sqrt{\frac{10}{7} g \cdot (y_S - y_R)} \tag{3.5}$$

In this experiment, the ball is guided by a V-shaped chute (see fig. 3.2). While rolling, there is direct contact with the underlay in two points. Instead of rolling on a great circle with radius r, the ball rolls on two respective parallel circles with radius $r' = r \cdot \sin\alpha$. For this experiment $\alpha = 60°$. Correspondingly, the condition of rolling becomes:

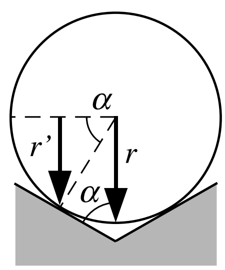

Figure 3.2: Explanation of radius r'

$$v_1 = \omega \cdot r' = \omega \cdot r \cdot \sin\alpha \tag{3.6}$$

thus

$$v_1 = \sqrt{\frac{1}{k} \cdot g \cdot (y_S - y_R)} \tag{3.7}$$

with

$$k = \frac{1}{2} + \frac{1}{5 \cdot \sin^2 \alpha} \tag{3.8}$$

After leaving the ramp and until the impact, the ball is falling during a time

$$\tau = \sqrt{\frac{2\, y_R}{g}} \tag{3.9}$$

in the gravitational field of the earth and in the process covers the horizontal distance

$$x_0 = \tau \cdot v_1 = \sqrt{\frac{2}{k} \cdot y_R \cdot (y_S - y_R)} \tag{3.10}$$

From the measurement of x_0, the velocity of the ball when leaving the ramp can be determined und compared with the theoretical value of eq. 3.7. When determining the values of y_S and y_R, it is important to bear in mind that during the impact the *center of mass* of the ball and not the point of impact is found in the $y = 0$ plane.

b) Velocities of the Collision Members after the Collision

The law of conservation of energy and momentum in a collision of two balls with the masses m_1 and m_2 is as follows:

$$\frac{1}{2} m_1 \cdot v_1^2 + \frac{1}{2} m_2 \cdot v_2^2 = \frac{1}{2} m_1 \cdot {v_1'}^2 + \frac{1}{2} m_2 \cdot {v_2'}^2 \tag{3.11}$$

$$m_1 \cdot \vec{v}_1 + m_2 \cdot \vec{v}_2 = m_1 \cdot \vec{v}_1' + m_2 \cdot \vec{v}_2' \tag{3.12}$$

Where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the balls before the collision, while $\vec{v}_1'$ and $\vec{v}_2'$ are the velocities after the collision.

If two balls have the same mass $m = m_1 = m_2$ and the second ball is at rest before the collision, the law of conservation of energy is simplified to:

$$v_1^2 = {v_1'}^2 + {v_2'}^2 \tag{3.13}$$

Following the Pythagorean theorem, the velocity vectors after the collision thus have to be perpendicular to each other. With the conventions defined in fig. 3.3 we can derive the different velocity components: If the collision happens in the $x - z$ plane, the velocity of the first ball before the collision has only a component in the x-direction. It follows for the x- and z-components of velocity:

$$\text{x-components}: \quad v_1 = v_1' \cdot \cos\varphi_1 + v_2' \cdot \cos\varphi_2 = v_1' \cdot \sin\varphi_2 + v_2' \cdot \cos\varphi_2 \tag{3.14}$$

$$\text{z-components}: \quad 0 = v_1' \cdot \sin\varphi_1 - v_2' \cdot \sin\varphi_2 = v_1' \cdot \cos\varphi_2 - v_2' \cdot \sin\varphi_2 \tag{3.15}$$

with $\varphi_1 + \varphi_2 = \frac{\pi}{2}$ as sketched in fig. 3.3.

Solving eq. 3.15 for v_1' and inserting it in eq. 3.14 leads to the formula of v_2' as a function of scattering angle φ_2:

$$v_2' = v_1 \cdot \cos\varphi_2 \tag{3.16}$$

This means, as can be seen in fig. 3.4, that the tip of the velocity vector $\vec{v}_2'$ for an arbitrary scattering angle φ_2 is always located on a circle with a diameter defined by vector $\vec{v}_1$.

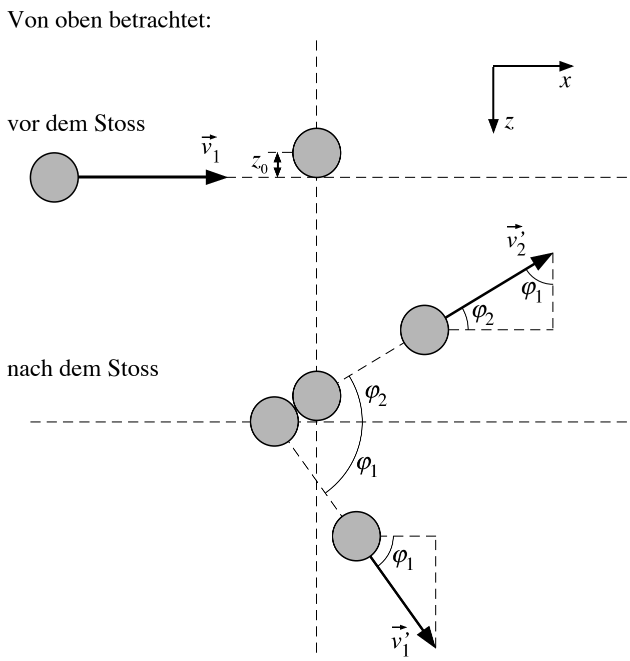

Figure 3.3: Definition of scattering angles φ_1 and φ_2.

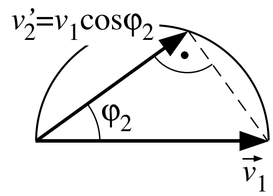

Figure 3.4: Relation between v_2' and φ_2

3.3 Experimental Part

a) Measurement of the Velocity of the Rolling Ball

The holder for the second ball (see fig. 3.5) will not be used before the second part of the experiment. Make sure that it is removed for this part of the experiment. The ball would hit the holder in its path and would thus be diverted.

Attach onto the impact plane some pressure-sensitive paper on which the ball leaves a dark mark during the impact. The mark is larger and more visible if the surface of the impact plane is lined with a soft material. Make sure that the paper cannot move.

Check with a water spirit level that the ramp and the impact plane are horizontally aligned. If that is not the case, use one of the available adjusting screws to first align the impact plane and then the ramp horizontally.

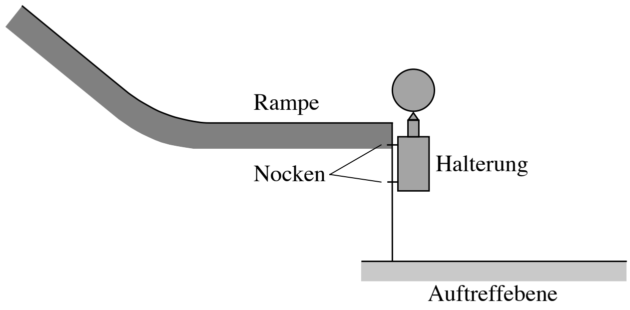

Figure 3.5: The holder for the second ball (second part of the experiment)).

- Let the ball roll down the ramp and measure the distance x_0 that the ball has travelled after leaving the ramp until the impact happens. Repeat this experiment five times. Calculate the mean and the error of the measurement (estimate the errors of the used quantities) and compare the result with the expected value of eq. 3.10. The angle α of the used ramp is 60° with a tolerance of 1%.
- Determine from the measurement result the velocity of the ball at the exit point of the ramp.
- Answer the following questions:
  - How would the result change if the ramp and/or the plane of impact were not horizontal?
  - The chute, in which the ball is rolling down, is covered with a plastic coating to allow a sufficiently high friction coefficient for the ball to roll and not to slide. What effect would a sliding ball have?

b) Verification of the Law of Conservation of Momentum in the Collision

Attach the holder (see fig. 3.5) for the second ball. Make sure that the cams (bumps) of the holder fit into the corresponding holes to achieve a firm hold. The holder can be used for the larger and, if installed in reverse, also for the smaller balls. But the larger balls give better results.

- Why do the smaller balls give worse results?

For this part of the experiment, a ballstopper can be mounted, that can hold the balls in place right after the collision. In the measurement for large scattering angles however, it will have to be removed again.

Check the vertical placement of the equipment: Put one ball on the end of the ramp and a second of same size on the holder and verify using the water spirit level that both balls have the same height.

First, test the case of a central collision. Using the micrometer gauge adjustment knob, adjust the horizontal position of the ball holder in relation to the ramp to achieve an impact parameter between the two balls of $z_0 = 0$ (the impact parameter z_0 is the distance between the two balls perpendicular to the direction of motion of the first ball before the collision, see fig. 3.3).

- Let the ball roll down the ramp and hit the second ball of same size located on the holder at rest. Measure the distance x_0 that the second ball travels.
- Repeat this experiment five times and determine the mean and the error of the measurement
- By what distance should the location of impact of the pushed ball be ahead of the first experiment if the entire momentum is transmitted from the first ball to the second ball? Compare the result of your measurement with your prediction.

Now, the relation between scattering angle and the velocity of the pushed ball (eq. 3.16) will be analysed. As the time of flight of the ball from collision to impact solely depends on the height traveled (and is thus independent of horizontal velocity), velocity vectors are projected onto locations in this experimental set-up. So if the tips of velocity vectors are located on a circle (see fig. 3.4), then the locations of impact form a circle, too.

- Vary the impact parameter z_0 in steps from 1 mm of $z_0 = -2\,\text{cm}$ to $z_0 = +2\,\text{cm}$ and do an individual measurement for each setting. Note down for every location of impact of the pushed ball the respective value of the impact parameter set on the micrometer gauge.
- Make sure that the measurement points are actually on a circle using the circle form at your experiment station. Are significant offsets of the circular form measureable? If so, how can you explain them?
- Determine the diameter of the circle that is formed by the measurement points and use it to calculate the velocity v_1 and the corresponding error.
- Depict the scattering angle φ_2 as a function of the impact parameter z_0. What relation $\varphi_2(z_0)$ do you suspect? Illustrate the two balls and their collision and indicate how the velocity vectors of the pushing ball before the collision can be deconstructed into the two velocity vectors after the collision. Draw the resulting function $\varphi_2(z_0)$ in the measured diagram. Does the suspected function correspond to the measurement?