2. Imaging by lenses

2.1 Introduction

Optical lenses and lens systems form the basis for numerous imaging devices that are used in science, technics, and everyday life. Examples include glasses, magnifying glasses, microscopes, telescopes, and photo cameras. Essential properties of lenses and lens systems can be described by the simple rules of geometrical optics. These are generally valid if the distances of a considered problem are large in comparison to the wavelength of visible light and the wave character of light can be neglected.

In this experiment, fundamental properties of thin lenses and lens systems, such as the focal length and the reproduction scale, are studied.

2.2 Theory

a) Refraction on a spherical joint face

The simplest optical imaging system consists of a spherical joint face with radius of curvature r between two media with the different indices of refraction n_1 and n_2. The image B of a point G lying on the optical axis can be reconstructed using close-to-axis light rays, as shown in Fig. 2.1. The law of refraction states

$$\frac{\sin\,\varphi_1}{\sin\,\varphi_2} = \frac{n_2}{n_1} \tag{2.1}$$

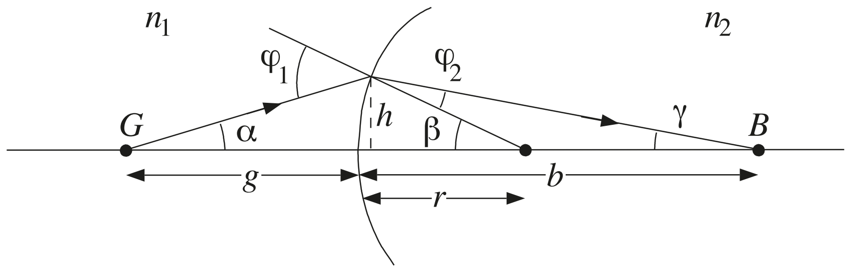

Figure 2.1: Imaging of a point on a spherical joint face.

where α and β are measured relative to the normal on the joint face. Using Fig. 2.1, then $\varphi_1 = \alpha+\beta$ and $\varphi_2 = \beta - \gamma$. For close-to-axis rays, these angles are small, such that

$$\frac{n_2}{n_1} = \frac{\sin(\alpha+\beta)}{\sin(\beta-\gamma)} \approx \frac{\alpha+\beta}{\beta-\gamma} \tag{2.2}$$

or

$$n_1 \cdot \alpha + n_2 \cdot \gamma = (n_2 - n_1) \cdot \beta \tag{2.3}$$

With $h/g = \tan\,\alpha \approx \alpha$, $h/r = \tan\,\beta \approx \beta$ and $h/b = \tan\,\gamma \approx \gamma$ follows the imaging equation

$$\frac{n_1}{g} + \frac{n_2}{b} = \frac{n_2 - n_1}{r} \tag{2.4}$$

The position of the image point is therefore completely determined by the two indices of refraction and the radius of curvature of the joint face.

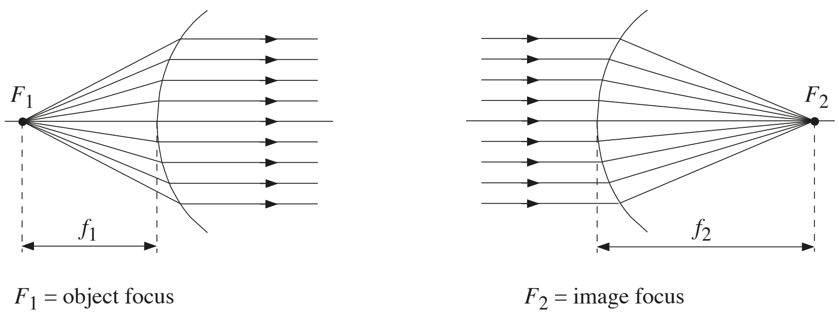

Figure 2.2: On the definition of the focal points.

From Eq. 2.4, two interesting special cases are apparent (see Fig. 2.2):

- The object distance g is equal to the focal length f_1 on the object side if the image point is infinitely far away/the image distance is infinite ($b = \infty$). This is the case if all rays coming out of the object point become parallel to the optical axis

$$g = \frac{n_1 \cdot r}{n_2 - n_1} = f_1 \tag{2.5}$$

- The image distance b is equal to the focal length f_2 on the image side if the object point is infinitely far away/the image distance is infinite ($g = \infty$). This is the case if incoming rays parallel to the axis are focused in the image point

$$b = \frac{n_2 \cdot r}{n_2 - n_1} = f_2 \tag{2.6}$$

With these definitions follows

$$\frac{f_2}{f_1} = \frac{n_2}{n_1} \tag{2.7}$$

and the imaging equation can be written as

$$\frac{f_1}{g} + \frac{f_2}{b} = 1 \tag{2.8}$$

The imaging properties of the spherical joint face can thus also be fully described by the two focal lengths f_1 and f_2.

The image construction for an extensive object is illustrated in Fig. 2.3.

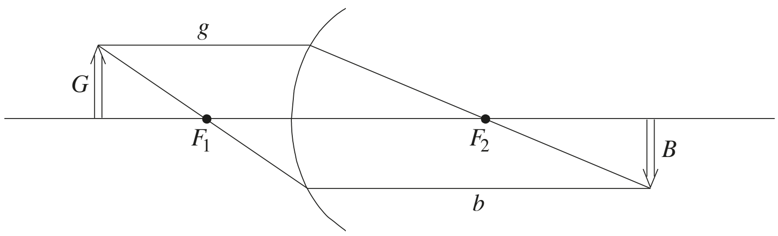

Figure 2.3: Image construction on a spherical joint face.

b) Imaging of optical systems

An optical system (telescope, microscope, ...) consists of multiple lenses that are arranged on an optical axis. Its imaging properties can be determined by looking at the system as a sequence of spherical join faces between two media of different indices of refraction. If the focal lengths on the object and image side of each of these joint faces are known, the image can be constructed through repeated use of the above mentioned technique. Starting with the object and the first joint face on the object side, the created image of a joint face is then treated as the object of the next joint face each time. The image of the last joint face is then the image of the optical system as a whole.

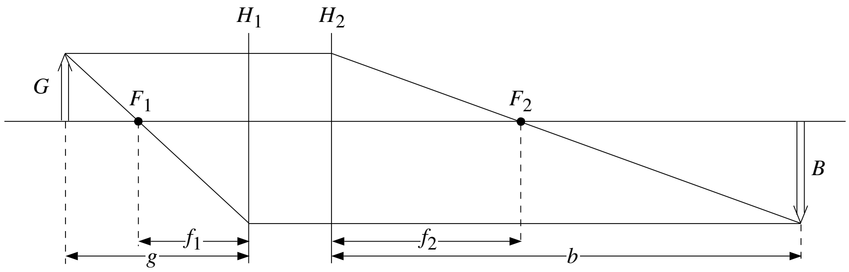

Figure 2.4: Image construction with the help of principal planes.

A much easier method for image construction is shown in Fig. 2.4. One can show that an arbitrary lens system can be described completely by specifying a focal point on the object and on the image side as well as two principal planes. Paraxially incoming rays are refracted on the principal plane H_2 on the image side, crossing in the focal point F_2. Rays coming from the focal point F_1 on the object side are refracted on the object side principal plane H_1, emerging paraxial. This optical path however is an observation abstraction; the true process is much more complicated.

- With a so-called thin lens, both principal planes coincide with the lens plane. For its image construction, the so-called center point beam can be used (cf. beam L_1 in Fig.. 2.6).
- With thick lenses or lens systems, the principal planes can lie outside of the lens or the lens system, as illustrated by several examples in Fig. 2.5

The imaging equation (Eq. 2.8) applies for thick lenses and lens systems as well. The object length g and the object side focal length f_1 are measured from the object side principal plane, the image length b and the image side focal length f_2 from the image side principal plane. If the object G

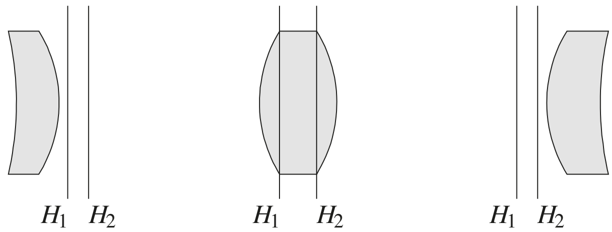

Figure 2.5: Position of the principal planes for varying lens thickness.

and the image B are inside the same medium, the object and image side focal lengths are equal ($f_1 = f_2 = f$). The imaging equation then simplifies to:

$$\frac{1}{g} + \frac{1}{b} = \frac{1}{f} \tag{2.9}$$

and the reproduction scale of the optical system is given by

$$\frac{B}{G} = -\frac{b}{g} \tag{2.10}$$

(Also compare to construction of the image in Fig. 2.4).

c) Real and virtual images

Fig. 2.6 illustrates the construction of real and virtual images using the example of a thin converging lens. A real image arises if light rays actually intersect in a point. A virtual image arises if only the backwards extensions of the real light rays intersect in a point. Real images can be made visible on a screen, virtual images cannot.

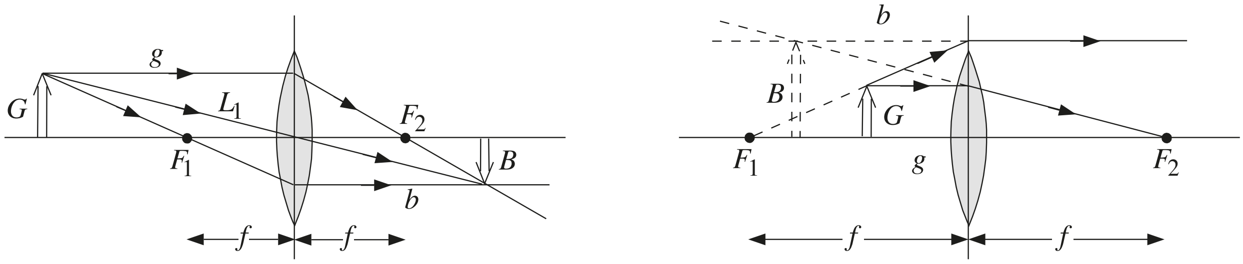

Figure 2.6: Real (left) and virtual (right) image of a thin converging lens.

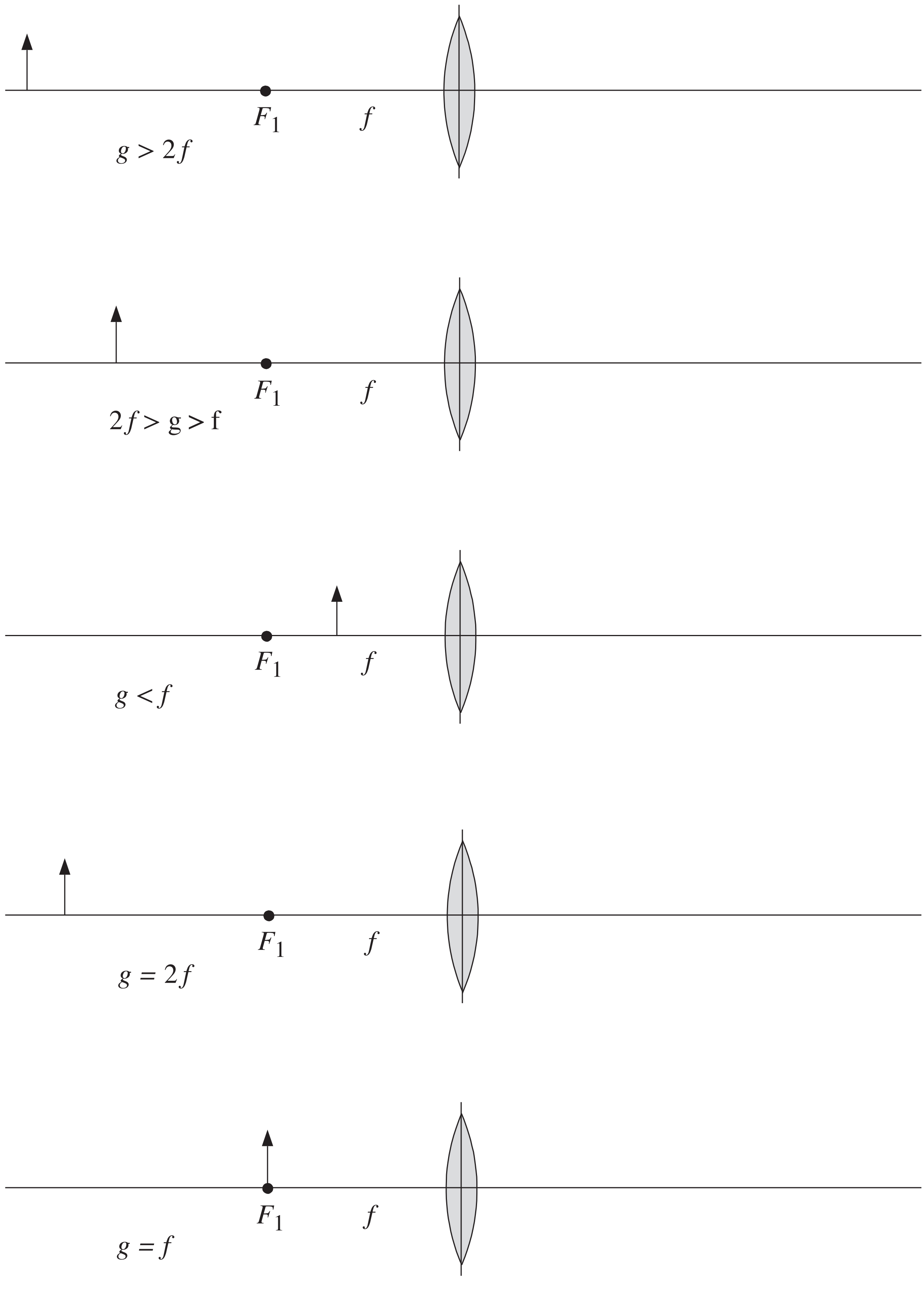

Figure 2.7: Exercise on the image construction of a thin converging lens.

2.3 Experimental Part

In the first part of the experiment, the focal lengths of two thin lenses are to be determined. In the second part, an object is to be reproduced with a given magnification/focal length. In the third part, the focal points and the principal planes of a lens system are to be determined.

As a preparation for the actual experiment, image construction will be practised initially on a thin converging lens using the given examples in Fig. 2.7.

- Construct the image of the arrow for each of the five examples and determine the reproduction scale for each of the figures.

a) Determination of the focal length of thin lenses

The focal lengths of a thin converging lens and of a thin diverging lens are determined with the illustrated arrangements of Fig. 2.8. The required parallel light can be created using an aperture and a converging lens, as shown in Fig. 2.9. The aperture, with a hole that is as small as possible, is illuminated with a lamp and positioned exactly in the focal point of the converging lens. The outgoing light to the right of the lens is then paraxial.

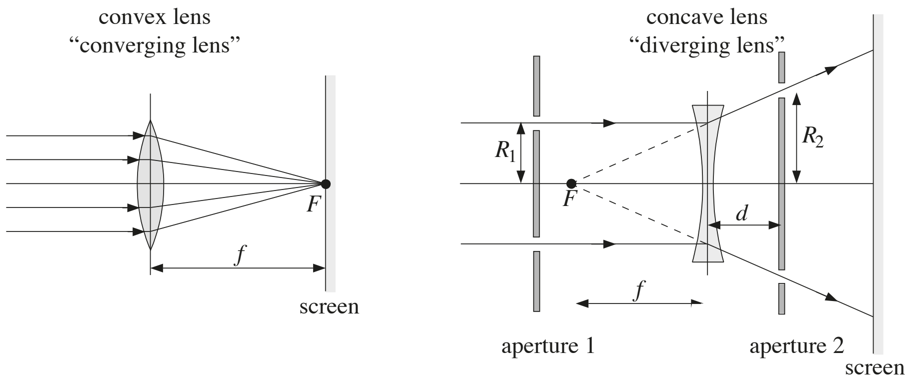

Figure 2.8: Determination of the focal length of thin converging and diverging lenses using paraxial light.

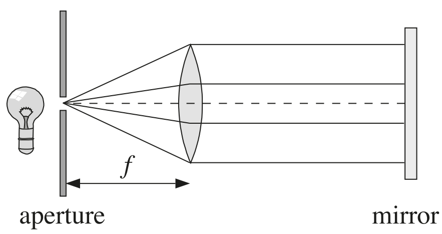

Figure 2.9: Creation of parallel light using a converging lens.

- Assemble the arrangement for the creation of praxial light as shown in Fig. 2.9. Verify the parallel alignment of the outgoing light by reprojecting it using a mirror and observing the image of the reflected light on the aperture. Move the lens until the diameter of the reflected light spot is minimal. Understand why the light rays to the right of the lens are now parallel.

- Remove the mirror now and insert the converging lens and the screen in the optical path as shown on the left side of Fig. 2.8. Move the screen until the diameter of the light spot is minimal on the screen itself. Determine the focal length of the lens from the distance between the lens and the screen and estimate the error.
- Remove the converging lens now and insert the diverging lens and the two circular apertures in the optical path as shown on the right of Fig. 2.8. Move the second circular aperture such that it allows light from the first circular aperture to pass until you see a circle of light on the screen. Measure the two radii R_1 and R_2 as well as the distance d between the lens and the second circular aperture and calculate the focal length of the lens using the intercept theorem. Estimate the error.

b) Imaging of an object using a lens with known focal length

An object is to be reproduced initially with three times the magnification on the screen using a lens with known focal length. Then it is to be reproduced with the same lens so that its image appears sharply exactly 1 m away from the plane of the lens. In both cases the correct positions of object, lens, and screen are to be calculated using Eq. 2.9 and 2.10 and then verified in the experiment.

- Calculate the distances between object, lens, and screen that allow for a three times magnification for the given focal length of the lens.
- Assemble the arrangement with the calculated distances. Verify that the image on the screen is sharp. Measure the object size and the image size and calculate the actual reproduction scale.
- Calculate the distance between object and lens such that a sharp image is created a distance of 1 m away from the lens.
- Assemble the arrangement with the calculated distances. Verify that the image on the screen is sharp.

c) Determination of the focal points and the principal planes of a lens system

The determination of the focal length and the two principal planes of a lens system using parallel light is demonstrated using a simple thick lens. Reminder: The position of the image side principal plane is the plane in which incoming paraxial rays are refracted such that they intersect in the image side focal point (see Fig. 2.4). Their positions are to be determined using the illustrated arrangement in Fig. 2.10. The required parallel light is created, just as in the first part of the experiment, using an illuminated aperture and a thin converging lens.

- Assemble the in Fig. 2.9 depicted arrrangement for the creation of parallel light again and verify the parallel alignment of the outgoing light.

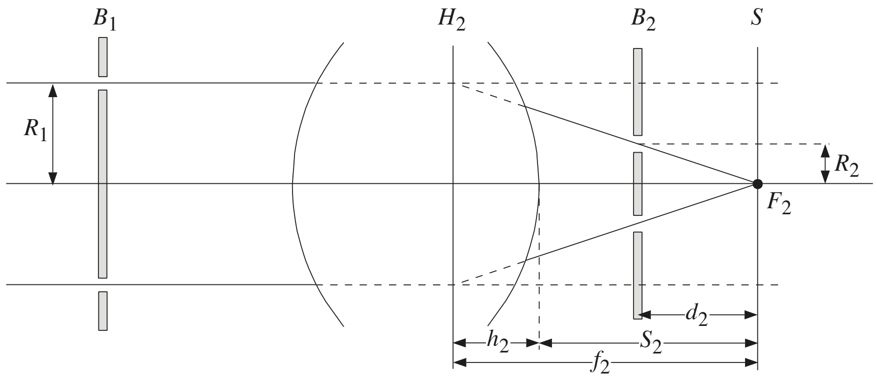

Figure 2.10: Determination of the image side principal plane of a lens system using parallel light.

- Remove the mirror and insert the thick lens and the screen in the optical path. Move the screen in the image side focal point of the lens system (observe the light spot on the screen and move the screen until the diameter of the light spot is minimal).
- Insert in addition, as shown in Fig. 2.10, the two circular apertures B_1 and B_2 in the optical path. Move the circular aperture B_2 such that it allows the light from the first circular aperture to pass and that you can see the light spot on the screen. Measure the two radii R_1 and R_2 as well as the distance d_2 between the screen and the aperture B_2, and calculate the focal lengths f_2 using the intercept theorem. Measure the distance S_2 between the screen and the end of the lens system and the distance h_2 between the end of the lens system and the image side principal plane. What is the significance of the leading sign of h_2?
- Insert now the thick lens in reverse (meaning with the backside in front) in the optical path, repeat the measurement, and determine the position of the second principal plane. Verify that the two focal lengths are actually equal.
- Calculate the distance between the two principal planes and draw the result of your measurements true to scale.