4. Collisions and momentum

4.1 Introduction

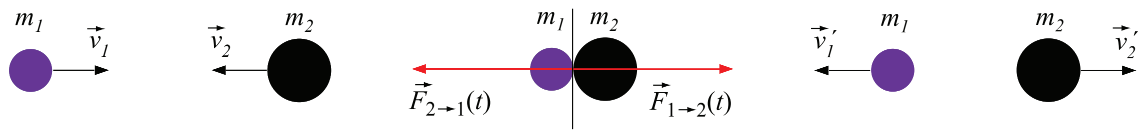
Collisions occur when two or more bodies interact for a short time. Examples include a ball bouncing back from a wall, the crash of a car, a jump. At each collision, energy, momentum, and angular momentum are exchanged,. The basic physics behind many phenomena in physics, chemistry, and biology, are based on collisions:

- Thermodynamic equilibrium between atoms in gasses, liquids, and solids, is reached by collisions.
- Heat exchange between two bodies happens through collision between atoms in the interface between both bodies.
- Collisions between molecules lead to chemical reactions.
- Absorption of light particles (photons) can be understood as a collision of photons with electrons or nuclei.
- The action of ionizing particles like electrons, protons, α-particles, or π-mesons, are based on collisions.
- Isotopes can be activated by collisions to become radioactive.

In this experiment, you will investigate the physics of collisions using a jump on a plate as an example (see Fig. 4.1). We will focus on the following topics: impulse, time interval of the interaction, and hard and soft landing on a plate.

4.2 Theory

During a collision of two masses m_1 and m_2, time-dependent forces act between the two bodies. These forces $\vec{F}(t)$ lead to an acceleration and, thereby, to a change of the momenta $\vec{p_i} = m_i\vec{v_i}$ of both masses. According to the law of momentum conservation, the total momentum $\vec{p}_{\text{tot}} = \vec{p}_1 + \vec{p}_2$ is not changed provided that no external forces are acting.



Before impact:	**During the collision**	**After impact**
momenta:	momentum change: $\Delta\vec{p}_1,\ \Delta\vec{p}_2$	momenta:
$\vec{p}_1 = m_1\vec{v}_1$	conservation of momentum:	$\vec{p}_1{}' = m_1\vec{v}_1{}'$
$\vec{p}_2 = m_2\vec{v}_2$	$\Delta(\vec{p}_1 + \vec{p}_2) = 0$	$\vec{p}_2{}' = m_2\vec{v}_2{}'$
	force acting on 1: $\vec{F}_{2\to1}(t)$	
	force acting on 2: $\vec{F}_{1\to2}(t)$	
	Newton's third law: $\vec{F}_{1\to2} = -\vec{F}_{2\to1}$	

Figure 4.1: Collision between two masses..

In general, the functional form and time-dependence of the force $\vec{F}(t)$ are unknown. We may assume, however, that the interaction occurs during a limited time interval, the so-called time of collision τ.

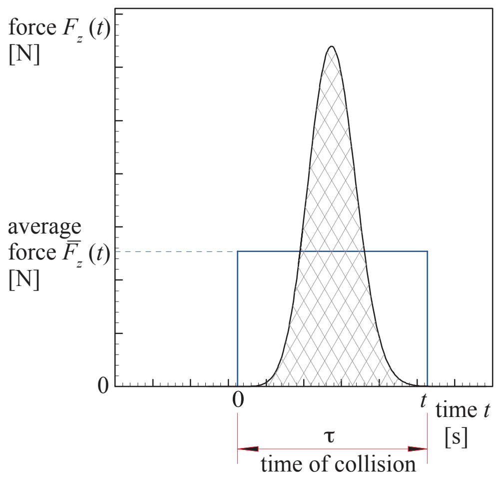


Figure 4.2: Example of an impulse force acting in z-direction.

The change in momentum is given by the impulse, i.e. the integral of the force over time:

$$\Delta\vec{p} = \int_0^\tau \vec{F}(t)\,dt \tag{4.1}$$

If we assume a one-dimensional collision with the force acting along z, Eq. (4.1) can be simplified yielding

$$\Delta p_z = \int_0^\tau F_z(t)\,dt. \tag{4.2}$$

Thus, the change in momentum produced by a force $F_z(t)$ is given by the integral (area) between the curve $F_z(t)$ and the time axis (see Fig. 4.2).

The average impulse force is then given by

$$\overline{F}_z = \frac{1}{\tau}\int_0^\tau F_z(t)\,dt. \tag{4.3}$$

The area of the rectangle defined by the average impulse force and the time of collision $\bar{F}_z \cdot \tau$ equals the integral of the true force $F_z(t)$ and thereby the change in momentum of each of the colliding bodies.

Landing on a plate.

A jump from height z_0 onto a plate can be divided into two processes:

1. A free fall until the feet touch the plate.
2. A slow-down (deceleration) which is caused by the impact onto the plate. This corresponds to a collision between the person jumping and the plate.

We obtain the velocity of the moving body just before the impact v_0 from energy conservation during the free fall:

$$v_0 = \sqrt{2g\,(z_0 - z_1)}, \tag{4.4}$$

where z_0 denotes the height of the center of mass with respect to the plate before the jump, z_1 the height at the moment of impact, and, thereby, $z_0 - z_1$ height of the fall. Note: in the remainder of this manual, height will refer to the vertical position of the center of gravity with respect to the plate surface.

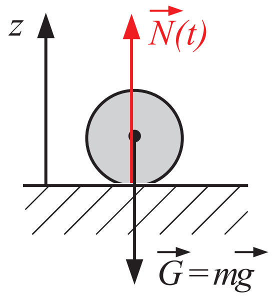


Figure 4.3: Forces occuring during impact.

The forces acting during the collision are depicted in Fig. 4.3. Using Newton's laws we obtain the following equation of motion during impact:

$$\frac{dp}{dt} = ma = N(t) - mg \tag{4.5}$$

The plate used in this experiment acts as a scale of a balance, which measures the component of the forces normal to the plate $N(t)$:

$$N(t) = ma + mg \tag{4.6}$$

The jump is best described by graphical representation of the motion as shown in Fig. 4.4. From the height of the center of gravity $z(t)$, the velocity $v(t)$ and the acceleration $a(t)$ we can calculate the forces acting during the jump: the time interval $t \le 0$ corresponds to the free fall, the impulse forces acting during the interval of the impact $0 \le t \le \tau$.

Problem 1: Which function describes the time-dependence $z(t)$ for $t \le 0$?

Integration of Eq. (4.5) yields

$$\Delta p = p(\tau) - p(0) = \int_0^{\tau} (N - mg)\, dt, \tag{4.7}$$

where

$$p(\tau) = 0 \tag{4.8}$$
$$p(0) = -m\,v_0 = -m\,\sqrt{2g\,(z_0 - z_1)} \tag{4.9}$$

We will verify this relationship experimentally.

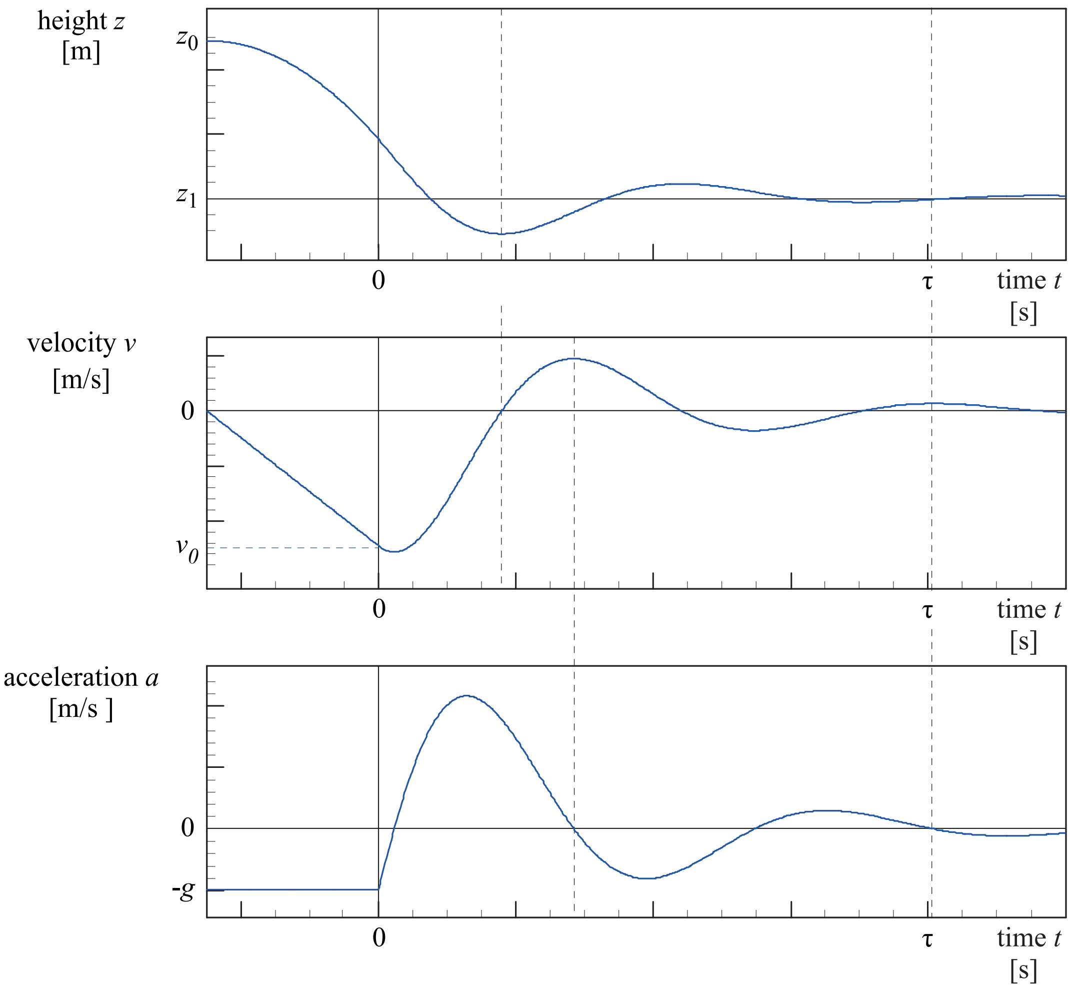


Figure 4.4: Height, velocity, and acceleration of the center of gravity during the jump. In the beginning, the feet are roughly 30 cm above the plate surface, at $t = 0$ they touch the plate with a speed of about 2.5 m/s. The person jumping absorbs the shock and takes his/her final position.

4.3 Experimental

4.3.1 Setup

At the bottom face the plate is equipped with piezo-electric crystals (see Fig. 4.5). With load, the crystals are compressed. The compression leads to a charging of the surfaces of the crystals (see Appendix for details). These charges are measured as voltage, charge and voltage being proportional to the deformation of the crystals and thereby to the forces acting onto the crystals.

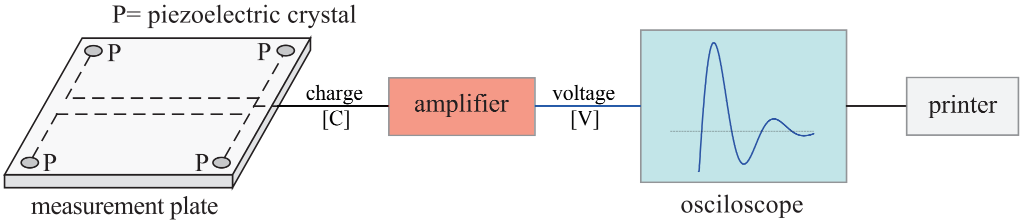


Figure 4.5: Experimental setup.

The signals from the four crystals are summed up and measured using a very sensitive charge amplifier (electrometer). The amplifier output is measured by means of an oscilloscope, the latter being connected to a printer.

4.3.2 Realization of the experiment

1. The assistant will demonstrate the use and functioning of the amplifier and of the oscilloscope.
2. Use a scale of 1 V/div for the channel of the oscilloscope. (The settings for the charge amplifier are given on a sheet of paper with the proof copy).
3. Calibrate the voltage signal by measuring the output voltage as function of the load for several masses between 5 kg and 25 kg. Trace a calibration curve of voltage vs. force.
4. Make several jumps from a height of about 50 cm. Try hard and soft landings, but **do not exaggerate and always absorb the impact with the muscles!** Otherwise one may harm oneself!
5. Print a good trace for hard and soft landing.

Qualitative observations

Sketch and explain the traces observed on the oscilloscope screen for the following excercises:

- Make some squats on the plate.
- March on the plate.
- Tune the oscilloscope to higher sensitivity; try to stand still and to watch the effect of your breathing (or even heart beat).

4.3.3 Analysis

- Determine your own mass and calculate the change in momentum during impact according to:

$$\Delta p = p(\tau) - p(0) = -p(0) = mv_0.$$

- Choose one of the printed curves $N(t)$. Estimate the area below the curve after subtraction of your weight mg, like shown in Figs. 4.6 and 4.7.

- Verify Eq. 4.7 by comparing the impulse $\int_0^\tau F dt$ with the change in momentum mv_0 calculated above.

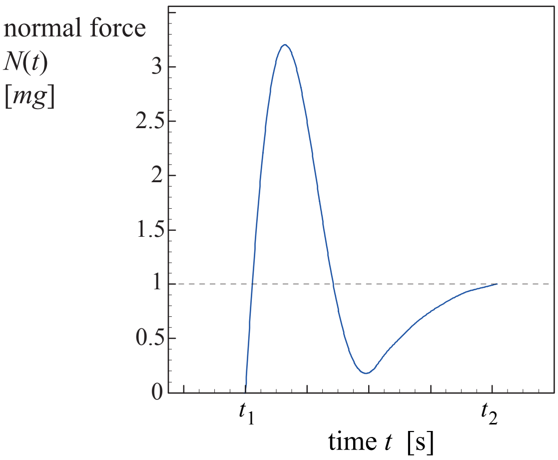


Figure 4.6: Trace as recorded by the oscilloscope.

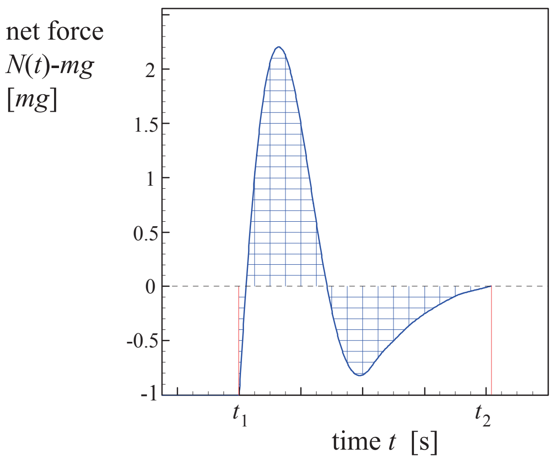


Figure 4.7: Measured trace after subtraction of the weight. Integration can be performed e.g. by counting squares on the graph paper.

Respond to the following questions:

Problem 2: Determine the times of collision for hard and soft landings?

Problem 3: Compare the values with the corresponding peak forces $N(t)$?

Problem 4: Compare hard and soft landings: what differences can you find?

4.4 Appendix

4.4.1 Piezo crystals

In 1880, Jacques and Pierre Curie discovered that in certain materials positive and negative charges appear on the surfaces when the crystal was subject to compressive and tensile stress. This effect is called piezoelectricity. Later on it was shown that the sign of the charge changes upon changing load, i.e. between compressive and tensile stress. The surface charge is proportional to the mechanical load.

We will give a short description on piezoelectricity using quartz as an example: The quartz crystal SiO_2 consists of stacks of trigonal planes of silicon and oxygen atoms, as shown in Fig. 4.8a). The charge distribution is not homogeneous: silicon has a positive and oxygen a negative partial charge. If we exert compressive stress along the X_1 axis, the silicon atom 1 moves between the oxygen atoms 2 and 6, the oxygen atom 4 between the silicon atoms 3 and 5. As a consequence, the face A is charged negatively, the opposite face B positively (see Fig. 4.8b). Tensile stress as depicted in Fig. 4.8c) leads to opposite charging of the two faces. The same holds for the X_2 and X_3 directions. Thus, the piezoelectric effect depends on kind and direction of the stress exerted by external forces. Piezocrystals are commonly used for measurements of pressure and forces..

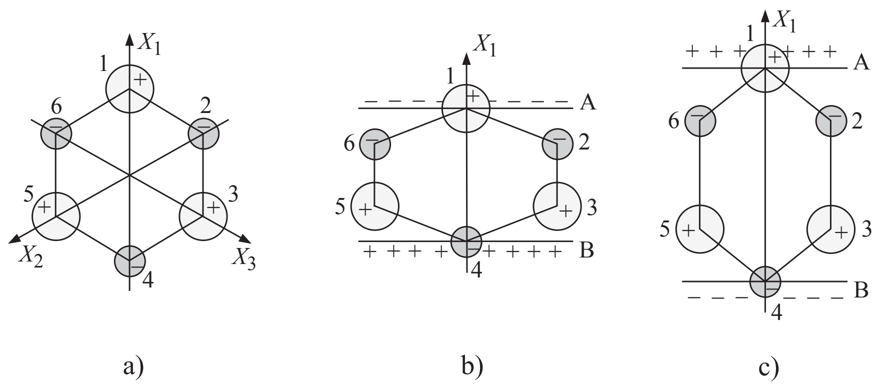


Figure 4.8: The piezoelectric effect in quartz: Si- and O-atoms are arranged in trigonal fashion in quartz crystals. (a) Crystal in mechanical equilibrium. (b) and (c): The same crystal subject to compressive and tensile stress, respectively.

4.4.2 The force sensor

The total charge on the faces is very small. Therefore, excellent electric isolation is required for the crystals (Fig. 4.9). Moreover, the charge amplifier must have a very high input impedance in order to keep the measurement current as small as possible. The output signal of the amplifier, which is proportional to the measured force $\vec{F}$, can be recorded by means of an oscilloscope for time-resolved measurements.

Arranged in a suitable way, three-dimensional deformations and even shear forces can be recorded using piezoelectric crystals. In this way, motion sequences can be recorded in biomechanics, *e.g.*

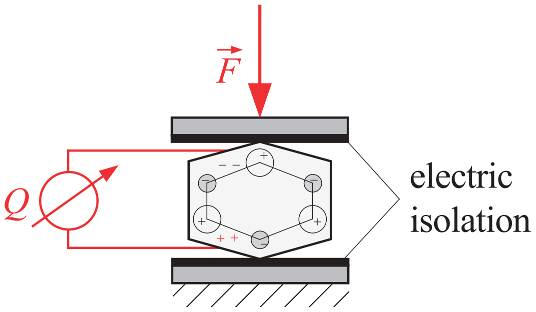


Figure 4.9: Technical realization of a force measurement using piezo-electric crystals.

the movements of a starting sprinter or of a high jumper, walking using artificial limbs or similar. Owing to the high sensitivity of charge amplifiers, very small movements of the center of gravity can be recorded in real time as you might have seen in the course of these experiments.

4.4.3 Ultrasonic applications

If a piezoelectric crystal is subject to a voltage between two opposite faces, it will be deformed, the deformation being proportional to the voltage. Using voltage signal of alternating polarity, the crystal starts to oscillate. This effect is used for generating ultrasound waves, the inverse piezoelectric effect for the detection of sound waves.