

4. Interferences and spectrometers

4.1 Introduction

As early as 1864, James Clerk Maxwell predicted the existence of electromagnetic waves which propagate with speed of light. Visible light, x-rays, γ -rays, radio waves etc. belong to the spectrum of electromagnetic radiation. Their wavelengths spread out over a huge range.

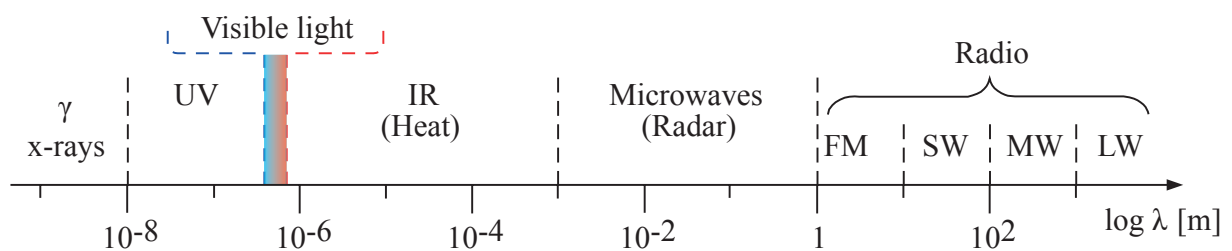


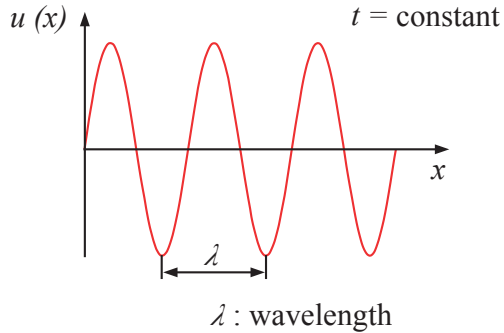
Figure 4.1: Spectrum of electromagnetic waves.

By eye, only a small section of this spectrum is perceivable: it ranges from wavelengths of 400 nm (violet light) to 700 nm (red light). The colour of light is determined by its wavelength. In general, light sources do not send out monochromatic light. The spectrum, i.e. the occurring wavelength and their relative intensities, are characteristic for a certain light source. If the spectrum is known, one can read into the composition of the source (spectral analysis). As an example, this method is applied in chemistry and astrophysics.

4.1.1 Interference and diffraction

The wave nature of light can be demonstrated with interference experiments. Interference is understood as the superposition of two or more waves which, accordingly to their relative phase, can destruct or reinforce each other under certain circumstances. The superposition can be easily visualized by the time image of two harmonic waves $u_1(x, t)$ and $u_2(x, t)$.

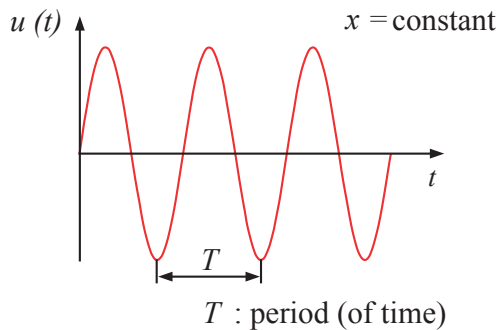
A harmonic wave $u(x, t) = u_0 \sin(kx - \omega t)$ can be represented as a stationary and a time image.



Stationary image: At a **fixed instant of time** the excitation u is being observed as a function of x (“photography” of the wave):

$$k = \frac{2\pi}{\lambda}$$

$k =$ wave number



Time image: At a **fixed position** the excitation u is being observed as a function of time:

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}$$

Figure 4.2: Spatial and time image of a harmonic wave.

For the superposition of two harmonic waves $u_1(x, t)$ and $u_2(x, t)$ the following time image (for $x = \text{const.}$) graphs arise:

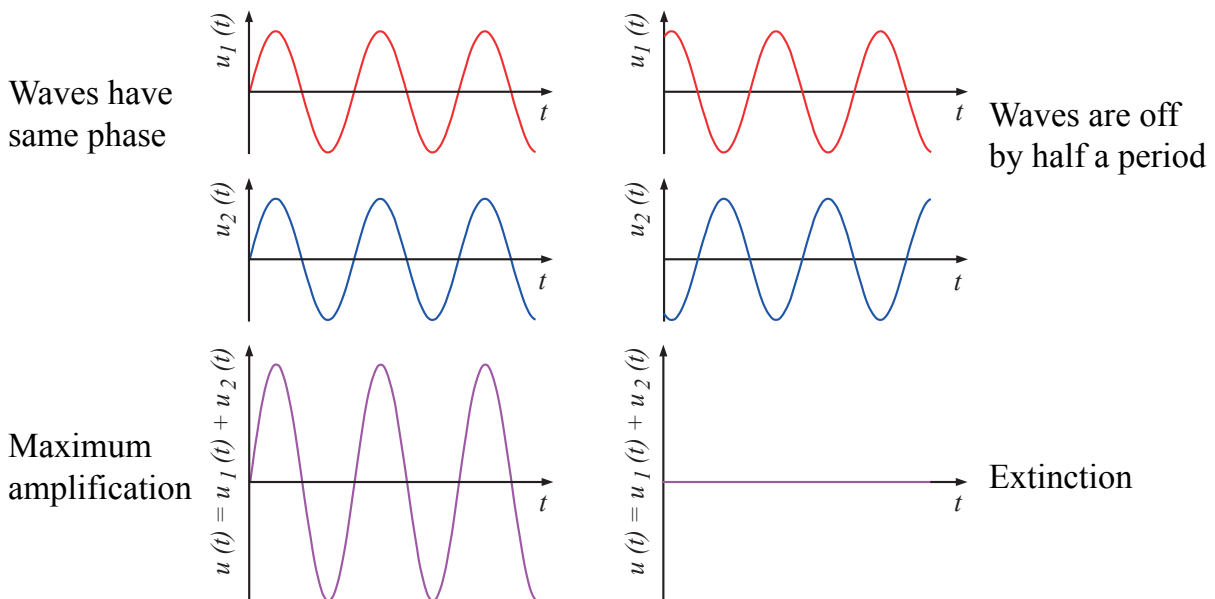


Figure 4.3: Superposition of two harmonic waves in the time image.

The superposition of the two waves $u_1(x, t)$ and $u_2(x, t)$ can be depicted in the stationary image ($t = \text{const.}$) as well.

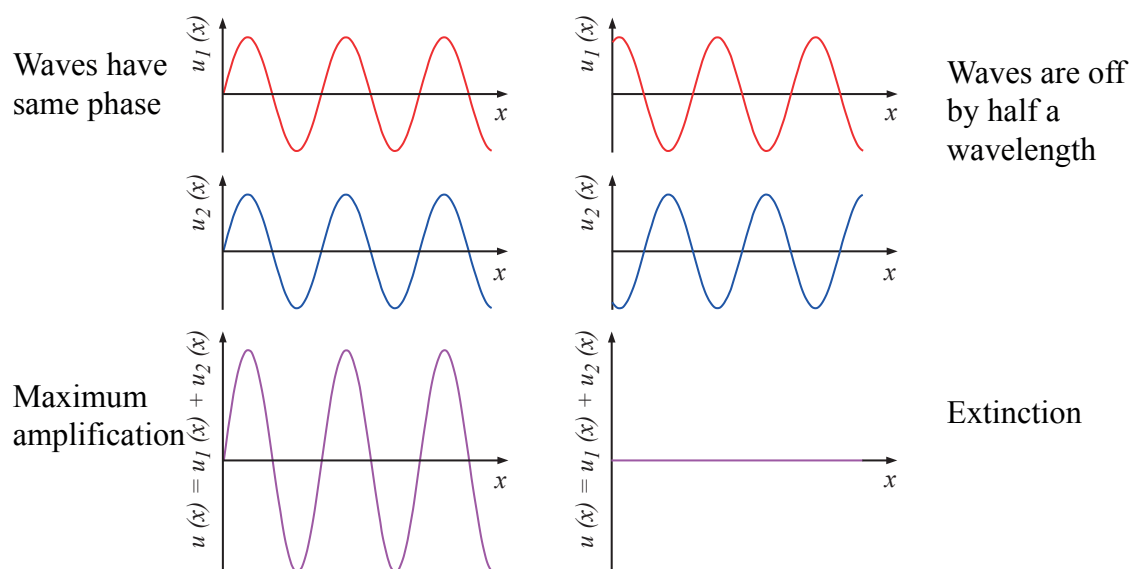


Figure 4.4: Superposition of two harmonic waves in the stationary image.

In everyday life interference phenomena are rarely observed. They can only be seen if the superimposed waves are **coherent**. Two waves are called coherent if they have a fixed phase relation to each other over time, i.e. their path difference does not change over time at a fixed observing position. However, this condition is never fulfilled for two waves that are sent out by two different light source. Inside of a usual light source (lasers represent an exception), single atoms are excited, e.g. via collisions, completely independent from each other. Afterwards, for a short period of time, the gained energy is being released again by sending out short bursts of electromagnetic light. The observed light makes for the superposition of all of those bursts. In the case of a second light source, the atoms radiate completely independent of each other as well. Therefore, the phase relation changes every single time such a burst is being sent out. The coherence time approx. corresponds to the time a single atom needs to emit a new burst, namely around 10^{-8} s. After this time, the phase relation changes abruptly, the interference image averages out for our eye.

Hence, to generate observable interferences, one needs **coherent** waves. This can be achieved by a trick: a single physical light source is being split artificially into two or more virtual light sources. The different interference assemblies only differ by the way the split-up of the light takes place.

Goal of this experiment

In this experiment four different assemblies to observe interference phenomena are going to be examined: double slit, single slit, reflection grating and diffraction on particles. As a monochromatic light source a He-Ne laser (red light) is being employed. In the second part of the experiment, a simple grating spectrometer is being assembled and used to measure the visible part of the spectrum

of a mercury lamp.

Key words for this lab course are:

- (electromagnetic) waves, their description and properties,
- coherence as a requirement for interference,
- interference and diffraction,
- the spectrum of electromagnetic waves,
- spectrometers and
- measurement of wavelengths.

4.2 Theory

4.2.1 Double slit (Young's double-slit interferometer)

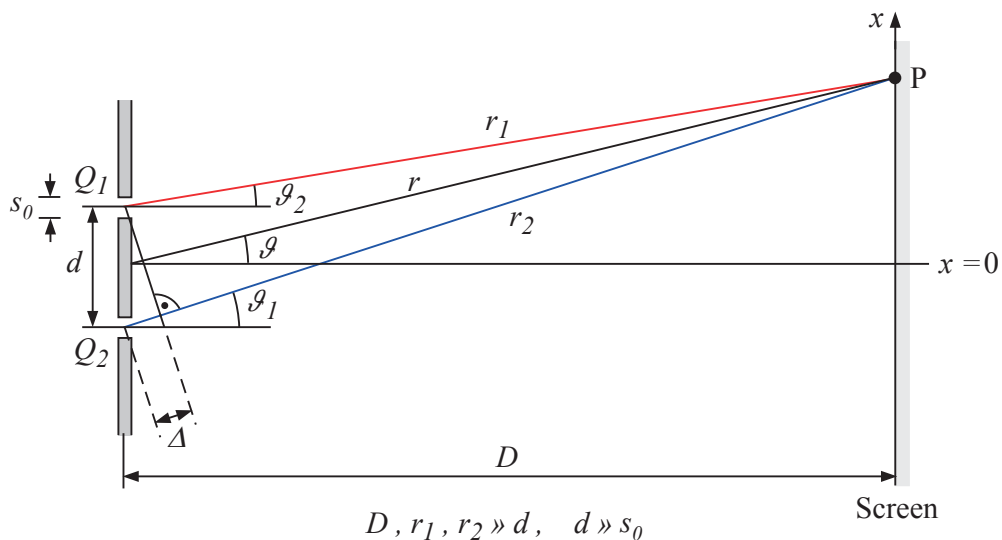


Figure 4.5: Double slit, Young's assembly. For $D \gg d$ one has $\vartheta \simeq \vartheta_1 \simeq \vartheta_2$.

Coming from the left, parallel, monochromatic light hits a double slit. Following the Huygens-Fresnel principle (see lecture), the two slits represent two sources Q_1 and Q_2 which emit coherent spherical waves. These two waves can amplify or weaken one other in the point P , depending on their relative phase.

You shall investigate the intensity distribution $I(x)$ on a screen which is located at a distance D from the double slit. According to Fig. 4.4 the waves amplify each other when being in phase or shifted by an integer multiple of their wavelength.

$$\Delta_{\text{Ampl.}} = |r_1 - r_2| = m \lambda \quad m = 0, 1, 2, \dots = \text{ordinal number of the maximum}$$

In case of a phase shift of $(m - 1/2)\lambda$, the two waves cancel out (Fig. 4.4):

$$\Delta_{\text{Cancel}} = |r_1 - r_2| = \left(m - \frac{1}{2}\right) \lambda, \quad m = 1, 2, \dots = \text{ordinary number of the minimum}$$

For the path difference one has $\Delta = |r_1 - r_2| \approx d \cdot \sin \vartheta$ (Fig. 4.5). Therefore the maxima arise at:

$$\sin \vartheta_{\text{max}} = m \frac{\lambda}{d} \quad (4.1)$$

and the Minima at:

$$\sin \vartheta_{\text{min}} = \left(m - \frac{1}{2}\right) \frac{\lambda}{d} \quad (4.2)$$

If the screen is far away from the double slit ($D \gg d, x$), one has

$$\frac{x}{D} = \tan \vartheta \approx \sin \vartheta$$

and the brightness maxima appear on the screen at:

$$x_{\text{max}} = m \frac{D\lambda}{d}$$

The brightness minima appear at:

$$x_{\text{min}} = \left(m - \frac{1}{2}\right) \frac{D\lambda}{d} \quad (4.3)$$

The distance between two successive maxima or minima ($\Delta m = 1$) is:

$$\Delta x = \frac{D\lambda}{d} \quad (4.4)$$

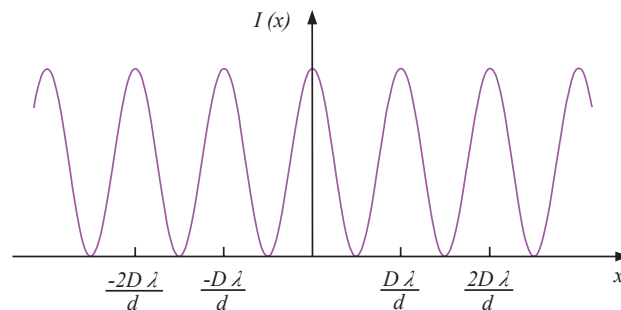


Figure 4.6: Intensity pattern for the double slit with $d \gg s_0$.

Hence, an intensity pattern of bright and dark strips emerges on the screen. If both slits are very narrow ($d \gg s_0$), all of the maxima possess the same brightness (Fig. 4.6).

Question 1: Assume the red laser would have been replaced by a Na-lamp (yellow light). How would the interference image change? Sketch both interference patterns.

Question 2: Equation 4.1 and equation 4.2 give the angles ϑ at which the maxima and minima appear. What is the highest order m_{\max} , that can be observed for a given slit distance d ?

Question 3: How does the interference pattern on the screen look like when one chooses $d \gg \lambda$?

4.2.2 Interference on the single slit

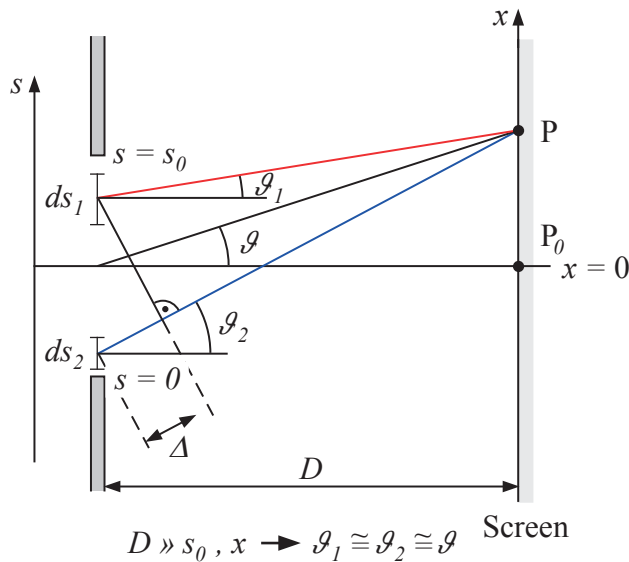


Figure 4.7: Interference on the single slit.

After the Huygens principle, each point which is hit by the incoming wave in the center of the slit represents the origin of a secondary spherical wave. For $\vartheta = 0$ and $D \gg s_0$ all of the incoming waves in P_0 are in phase and an interference maxima of 0-th order arises. For the determination of the maxima and minima positions for $\vartheta \neq 0$, one splits the slit into small intervals ds . The total excitation of the wave at point P is compounded by the contributions of all intervals ds_i . One sums, or integrates all of those contributions while incorporating the path difference Δ .

The intensity is always proportional to the square of the excitation (see lecture). Hence, the intensity as a function of ϑ becomes:

$$I(\vartheta) \approx \frac{\sin^2\left(\frac{ks_0}{2} \sin \vartheta\right)}{\left(\frac{ks_0}{2}\right)^2 \sin^2 \vartheta} \quad \text{where } k = \frac{2\pi}{\lambda} \quad (4.5)$$

For the location of the minima one has:

$$\begin{aligned} \sin\left(\frac{ks_0}{2} \sin \vartheta_{\min}\right) &= 0 && \text{(where } \vartheta_{\min} \neq 0) \\ \frac{ks_0}{2} \sin \vartheta_{\min} &= \frac{\pi s_0}{\lambda} \sin \vartheta_{\min} = m\pi \\ \sin \vartheta_{\min} &= m \frac{\lambda}{s_0} && \text{where } m = 1, 2, 3, \dots \text{ ordinary number of the minimum} \end{aligned} \quad (4.6)$$

and for the location of the maxima one has:

$$\begin{aligned} \sin\left(\frac{ks_0}{2} \sin \vartheta_{\max}\right) &= 1 \\ \frac{ks_0}{2} \sin \vartheta_{\max} &= \frac{\pi s_0}{\lambda} \sin \vartheta_{\max} = \left(m + \frac{1}{2}\right) \pi \\ \sin \vartheta_{\max} &= \left(m + \frac{1}{2}\right) \frac{\lambda}{s_0} \quad \text{where } m = 1, 2, \dots \text{ ordinary number of the maximum} \end{aligned} \quad (4.7)$$

Question 4: How wide does the slit has to be at the minimum in case one wants to observe the maximum of first order ($m = 1$)?

From equation (4.5) we learn that the brightness of the intensity maxima decreases with increasing angle ϑ_{\max} . The arising intensity distribution is being shown in Fig. 4.8.

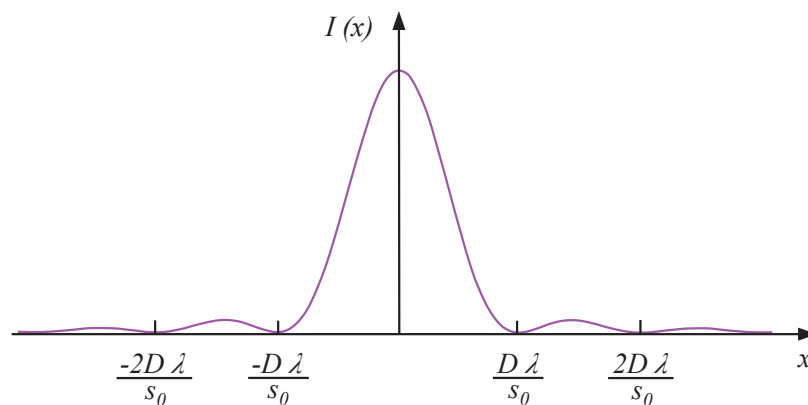


Figure 4.8: Intensity distribution on the single slit.

Frage 5: Based on equations (4.6), (4.7) and Fig. 4.8, what can be seen on the screen if

- $s_0 < s_{\min}$ (from question 4)
- $s_0 \gg \lambda$.

If the screen is located far away from the single slit ($D \gg x$), then:

$$\frac{x}{D} = \tan \vartheta \approx \sin \vartheta$$

and the brightness maxima on the screen appear at:

$$x_{\max} = 0 \quad \text{and} \quad x_{\max} = \left(m + \frac{1}{2}\right) \frac{D\lambda}{s_0} \quad m = 1, 2, 3, \dots$$

and the minima at:

$$x_{\min} = m \frac{D\lambda}{s_0} \quad m = 1, 2, 3, \dots$$

The distance between two consecutive maxima or minima ($\Delta m = 1, m \neq 0$) is:

$$\Delta x = \frac{D\lambda}{s_0} \quad (4.8)$$

4.2.3 Interference on the grating

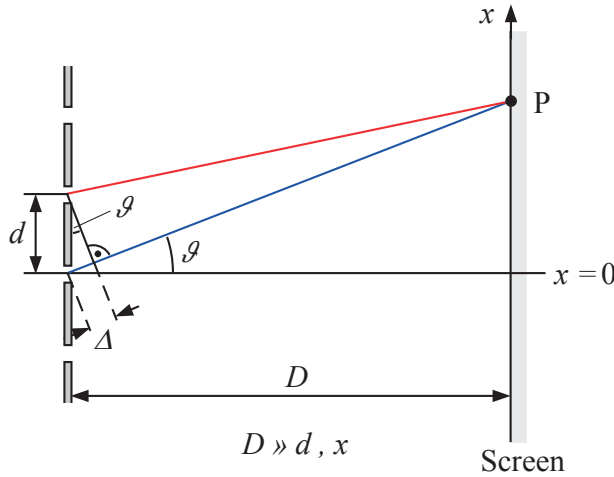


Figure 4.9: Interference on the grating.

A grating consists of a great amount of equidistant slits whose intervals are comparable to the wavelength of the light. If parallel light falls on such a grating, each slit point becomes the center of a secondary spheric wave (Huygens principle). These secondary waves superimpose and, based on their relative phase, reinforce or weaken each other.

Note: In the following we only consider the the location of the so-called main maxima (see lecture). Furthermore it is assumed that the slit width s is substantially narrower than the grating constant d , i.e. each slit is being considered as a infinitely small light source.

If the path difference of two waves which come off of two neighbouring slits and superimpose at point P is an integer multiple of the wavelength, a main maxima arises in P .

For $d \ll D$ one has:

$$\begin{aligned} \Delta &\approx d \sin \vartheta = m \lambda \\ \sin \vartheta_{\max} &= m \frac{\lambda}{d} \quad \text{where } m = 0, 1, 2, \dots = \text{ordinary number} \end{aligned} \quad (4.9)$$

The location of the main maxima depends on the wavelength. If the incoming light consists of various wavelength, the interference stripes of same order for different colours appear under different angles. If the screen is positioned far away from the grating ($D \gg d, x$ and $\vartheta \ll 1$), one has:

$$\frac{x}{D} = \tan \vartheta \approx \sin \vartheta$$

and for the maxima:

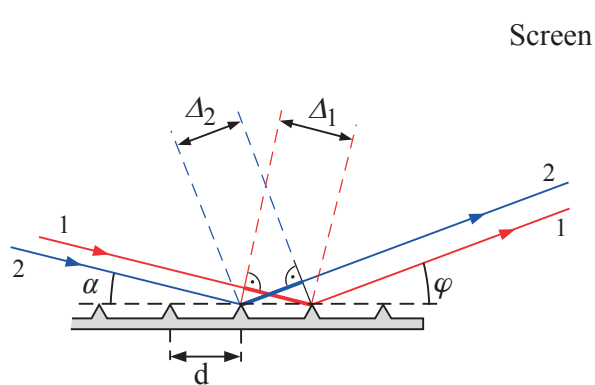
$$x_{\max} = m \frac{D\lambda}{d} \quad (4.10)$$

From the position of the maxima one can extract the wavelength using Equ. 4.10. The more wavelengths interfere the sharper the maxima become.

4.2.4 Interference on the reflection grating

For an interference maximum to appear, the path difference of two waves which come off of two neighbouring edges has to be an integer multiple of the wavelength.

$$\Delta = \Delta_1 - \Delta_2 = m \lambda$$



A metal comb with sharpened edges can be used as a reflection grating. Parallel light comes in from the left. Each edge is the center of a secondary spherical wave.

Figure 4.10: Interference on the reflection grating.

From Fig. 4.10 one can read off that

$$(i) \Delta_1 = d \cos \alpha, \quad (ii) \Delta_2 = d \cos \varphi, \quad (iii) \Delta_1 > \Delta_2 \text{ für } \varphi > \alpha$$

For a striping incident:

$$\alpha \ll 1 \quad \Rightarrow \quad \cos \alpha \approx 1 - \frac{\alpha^2}{2}$$

$$\varphi \ll 1 \quad \Rightarrow \quad \cos \varphi \approx 1 - \frac{\varphi^2}{2}$$

So:

$$\Delta = \frac{d}{2} (\varphi^2 - \alpha^2) = m \lambda$$

$$\varphi_{\max} = \sqrt{\frac{2m\lambda}{d} + \alpha^2} \quad (4.11)$$

4.2.5 Interference on a circular aperture

When replacing the single slit by a circular aperture of diameter d , instead of interference stripes, a pattern of concentric, bright and dark rings will appear.

The location of the minima follow from tedious calculations. Therefore, only the results are given at this point:

$$\sin \varphi_{1.\text{Min.}} = 1.22 \frac{\lambda}{d}$$

$$\sin \varphi_{2.\text{Min.}} = 2.23 \frac{\lambda}{d}$$

$$\sin \varphi_{3.\text{Min.}} = 3.23 \frac{\lambda}{d}$$

For $D \gg R_1, R_2, R_3$ one has $\sin \varphi \simeq \tan \varphi = R/D$ and therefore

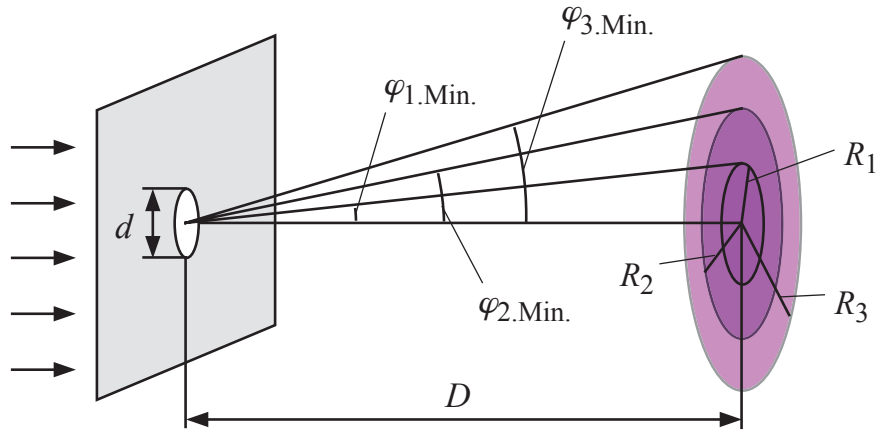


Figure 4.11: Interference on a circular aperture.

$$R_1 = 1.22 \frac{\lambda D}{d} \quad (4.12)$$

$$R_2 = 2.23 \frac{\lambda D}{d} \quad (4.13)$$

$$R_3 = 3.23 \frac{\lambda D}{d} \quad (4.14)$$

When parallel light falls on a preparation of statistical independently distributed particles, again, an interference pattern consisting of concentric rings with a strong maximum in the center arises (Babinet's principle). The ring diameter now depends on the particles diameter. In this lab course we will observe the interference on lycopodium powder.

Question 6: In every interference situation, the location of the maxima depends on the wavelength of the light. What would you observe on the screen if you'd use a light source which sends out a mix of different wavelengths (e.g. white light)?

By the way, this is the same phenomenon which makes an aureole (e.g. around the moon light) come into being. There, moon light is being diffracted on fog droplets.

4.3 Experimental part I

4.3.1 Definition of task

- Qualitative observation of various interference patterns.
- Determination of the wavelength of the utilized laser out of the interference on the double slit.
- Determination of the particle diameter of a lycopodium powder out of the diameter of the interference rings.

4.3.2 Experimental set-up

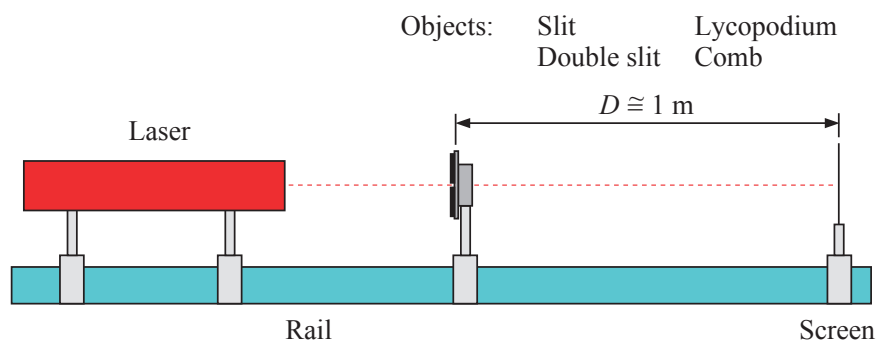


Figure 4.12: Experimental set-up.

!!! CAUTION: The laser beam must by no means hit your eyes !!!

For the measurement on the double and single slit a slide glass with slits in the following arrangement is at your disposal:

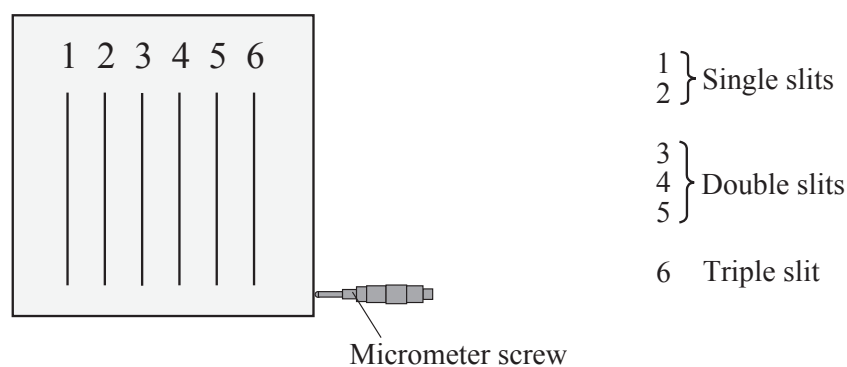


Figure 4.13: Slide glass with slits.

For this experiment, the single slit 2 as well as the double slits 4 and 5 work well. The laser is firmly mounted to the optical bench. The slide glass with the slits can be moved on a slider perpendicularly to the optical axis.

4.3.3 Test execution

Qualitative observations

- Under the guidance of the assistant, qualitatively observe the different interference patterns.

Double slit

- Use the set-up shown in Fig. 4.12 with a double slit.
- With the aid of a yardstick, measure the detachment Δx of sequential minima. The read off might appear to be difficult with the weak room lightning, you can stick a piece of paper on to the screen. Map the locations of the minima on the paper and measure the distances later on with better lightning.
- Measure the distance D between the double slit and the screen. Approximate the measuring error on D .
- Calculate the wavelength after Equ. 4.4. The slit gap d is being declared at the experimental desk.

Lycopodium powder

- Use the set-up shown in Fig. 4.12.
- Measure the distance D between the preparation and the screen. Approximate the measuring error on D .
- On the screen, measure the diameter $2R$ of the first two minima. Approximate the measuring error on those two measurements.
- For these two circles, calculate the particle diameter using equations (4.12) and (4.13).
- Measure the spore diameter directly using the microscope. Note the gauging of the microscope.
- Compare the results of both measurement methods.

4.4 Experimental part II

4.4.1 Definition of task

- Assembly of a simple spectrometer.
- Determination of the wavelengths in the visible part of the spectrum of a mercury lamp using the self-built spectrometer.

4.4.2 Assembly of the spectrometer

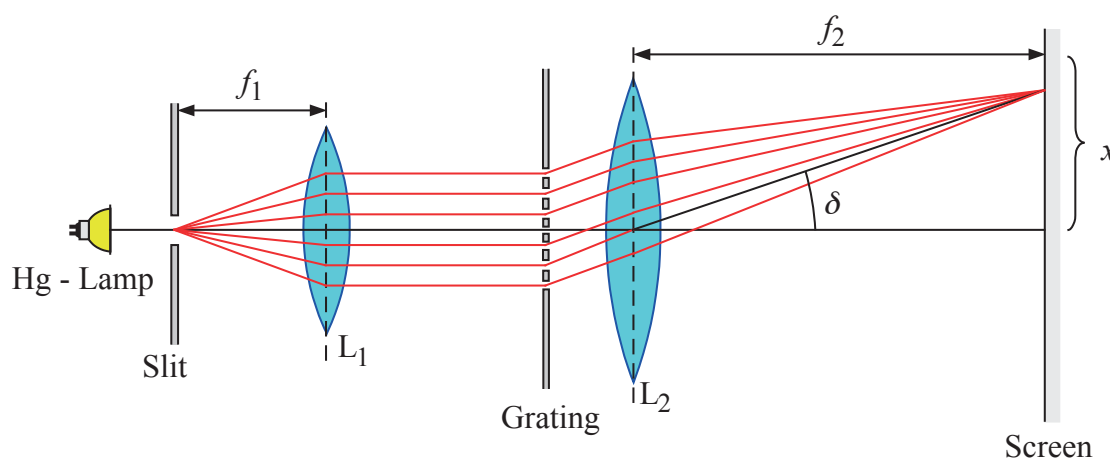


Figure 4.14: Assembly of the spectrometer.

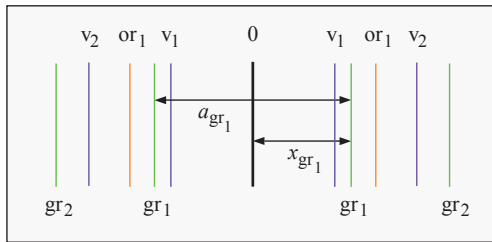
The mercury lamp illuminates a slit which is positioned in the focal plane of the lens L_1 . The lens L_2 focusses the parallel light which has been generated by the lens L_1 in its focal plane where the observation screen is being positioned. The respective focal lengths are declared on the lenses. Ensure that the grating is adjusted exactly perpendicular to the optical axis.

Note: It takes the mercury lamp about 10 minutes to shine with full intensity after switching it on.

Question 7: What is the functionality of lens L_2 ?

4.4.3 Wavelength measurement

- Observe the appearing spectrum on the screen qualitatively and watch how the relative locations of the different lines alter for higher orders. Create a draft of the spectrum.
- Stick a sheet of paper onto the screen and sketch the locations of the lines on the paper. Don't forget to always state the colour of the line. Clearly mark the maximum of 0th order (see Fig. 4.15).



v_i = violet i -th order
 gr_i = green i -th order
 or_i = orange i -th order

Figure 4.15: Lines in the measured spectrum.

- On the paper, measure for each colour the distance a of two maxima of the same order (Fig. 4.15). Approximate the measuring error on a . Summarize all measured values in a table.
- In this assembly, for small angles, one has $\tan \vartheta = x/f_2 \approx \sin \vartheta$ (cf. Fig. 4.10) and with equation (4.10) one arrives at:

$$x_{\max} = m \frac{f_2 \lambda}{d}$$

whereby in each case $x_{\max} = a/2$. Using this equation, calculate the wavelength for each of those lines.

4.4.4 Experimental report

- Answer all the question asked in this text.
- Present the demanded calculations and the results in a clear manner.
- Describe the working principle of a grating spectrometer.