# 2. Geometrical optics

## 2.1 Introduction

The description of the propagation of visible light ($\lambda$ = 400-700 nm) in different materials is important for optical applications. Since a full description is a very complex mathematical endeavor (keywords are propagation of gaussian beams or paraxial beams, for instance) we usually consider two limiting cases:

**Wave optics:** The size of the objects involved in the problem is comparable to the wave length of light. In this case, the focus lies on the wave-like nature and the corresponding phenomena such as interference and diffraction (cp. In/Sp experiment).

**Geometrical optics:** All geometrical dimensions appearing in the problem are large in comparison to the wave length. Under this condition, the light propagation can be described through rays of light. The two fundaments of geometrical optics are the reflection and the refraction laws.

In this experiment, we will concern ourselves exclusively with geometrical optics.

Geometrical optics is the basis for the understanding of numerous devices and appliances like glasses, microscopes, binoculars or cameras, which we use in our every day life to extend our vision. A fascinating example of an astoundingly versatile optical apparatus is the eye: it has not only a variable focus, an automatic aperture, an automatic cleaning mechanism, and an automatic gate, but also reaches the limits in image resolution and sensitivity of the actually possible.

Every optical device, be it a simple magnifying glass, glasses or a microscope, consists of one or more lenses (or mirrors). In this experiment, the essential properties of simple lenses will be explored. Keywords to this experiments are:

- Significance of the focal length
- Differences between convergent and divergent lenses
- Imaging through a thin convergent lens, and
- Imaging with a model eye (example from biology).

## 2.2 Theory

### 2.2.1 Focal points, Focal lengths

If rays of light hit the bounding surface between two media with two different refraction indices $n_1$ and $n_2$ askew, they are bent on that bounding surface:

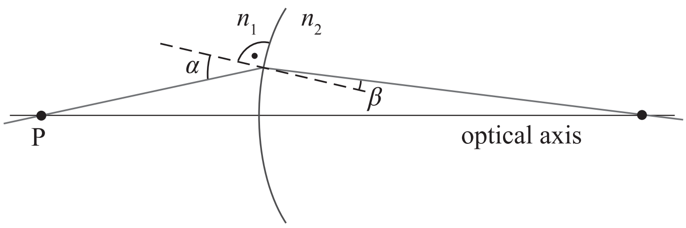



Figure 2.1: Refraction of a ray of light on a spherical bounding layer.

The refraction law states that the angle of incidence $\alpha$ and the emergent angle $\beta$ are related as follows:

$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1} \tag{2.1}$$

If rays of light travel parallel to the optical axis and hit a spherical bounding surface, the refracted light is focused in the image side focal point $F_2$. If the rays of light appear out of the object side focal point $F_1$, they then propagate parallel to the optical axis after refraction on the spherical bounding surface:

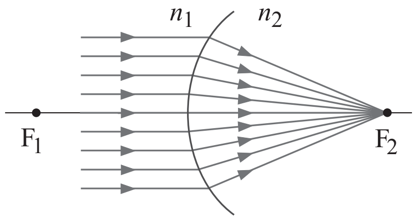



Figure 2.2: Incoming light rays parallel to the optical axis.

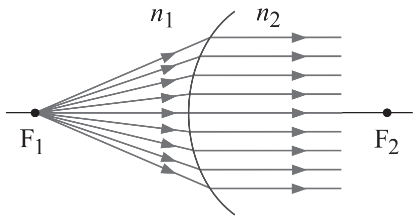



Figure 2.3: Light rays emerging from focal point $F_1$.

A **lens** consists of two spherical bounding surfaces. This system of bounding surfaces has an object side and an image side focal point as well (Figures 2.4 and 2.5).

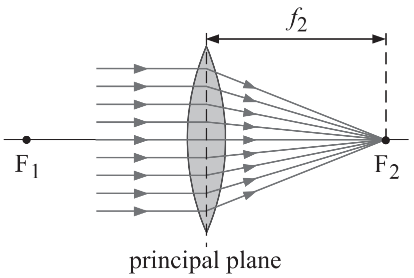



Figure 2.4: Incoming light rays parallel to the optical axis.

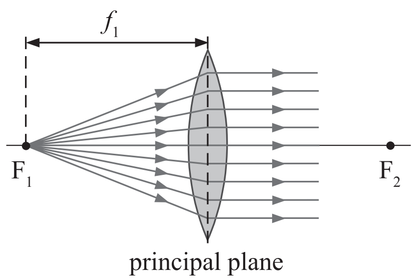



Figure 2.5: Light rays emerging from focal point $F_1$.

It is also the case here that incoming light rays parallel to the optical axis are being focused in the image side focal point $F_2$, while rays of light emerging from the object side focal point $F_1$ run parallel to the optical axis after the lens. The focal length of thin lenses describes the distance between the principle plane of the lens and the focal points. For thin lenses is therefore always $f_1 = f_2 = f$.

The focal length of a lens depends on the two radii of curvature $r_1, r_2$ and the refraction index $n_1$ (environment) and $n_2$ (lens material). The formula is:

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \tag{2.2}$$

The following algebraic sign rule is convention: if the center point $M_i$ lies to the right of the joint face $i$, then the radius of curvature is $r_i > 0$; if the center point $M_i$ lies to the left of the joint face $i$, then the radius of curvature is $r_i < 0$.

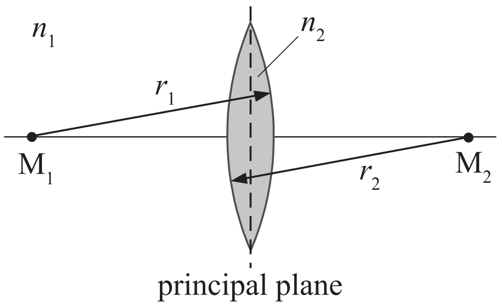



Figure 2.6: Convergent lens with $r_2 < 0$ and $r_1 > 0$.

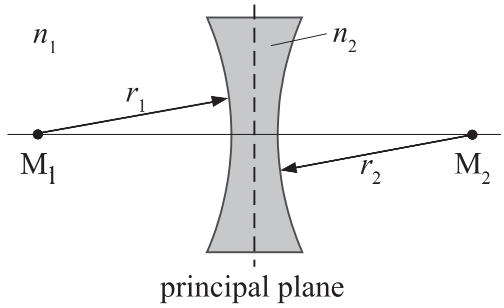



Figure 2.7: Divergent lens with $r_1 < 0$ and $r_2 > 0$.

For **convergent lenses** (Figures 2.4, 2.5, and 2.6), the focal lengths are **positive** and the rays of light are converging. For a **divergent lens** (Figures 2.7, 2.8, and 2.9), the rays of light are diverging. The center point $M_1$ lies on the left side of the joint face and $M_2$ on the right side. Thus, the radii of curvature $r_1$ and $r_2$ change sign and the focal lengths $f_1$ and $f_2$ become **negative**. Here, the rays of light do not run through the focal points anymore. If one draws their extensions, these "virtual" rays in turn intersect in the focal point.

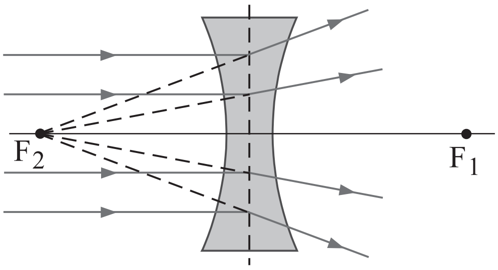



Figure 2.8: Incoming light rays parallel to the optical axis.

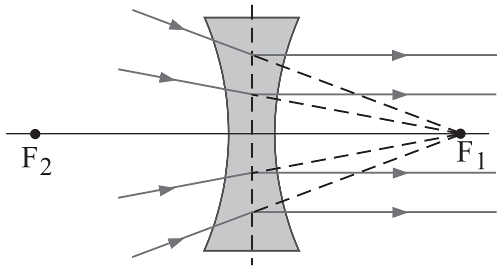



Figure 2.9: Outgoing light rays parallel to the optical axis.

### 2.2.2 Imaging through a thin convergent lens

In optical imaging through a thin convergent lens, rays emerging from an object point P are being focused by the imaging system in exactly one image point P′. Every object point is assigned exactly one image point.

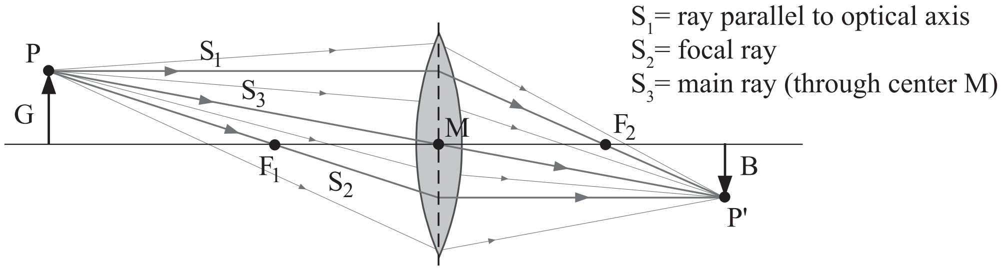



Figure 2.10: Imaging and image construction using a thin convergent lens.

For the geometrical construction of the image point, two rays are necessary. Usually, two of the three rays $S_1$, $S_2$, and $S_3$ (Figure 2.10):

- Parallel ray (S1): incoming light ray runs parallel to the optical axis
- Focal ray (S2): incoming light ray runs through the focal point
- Principal ray (S3): incoming light ray runs through the middle of the lens (is not being deflected)

#### Lens equation, Magnification

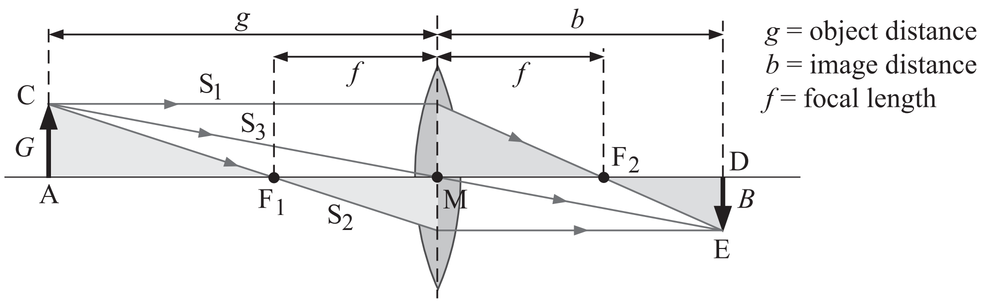



Figure 2.11: Object distance, image distance, and focal length.

From the similarity of the shaded triangle (Figure 2.11), a relation between object distance $g$, image distance $b$, and focal length $f$ can be found:

$$\frac{f}{G} = \frac{b-f}{B} \qquad \text{and} \qquad \frac{G}{g-f} = \frac{B}{f}$$

Multiplying these two equations:

$$\frac{f}{g-f} = \frac{b-f}{f} \qquad \Rightarrow \qquad gb = bf + gf$$

The result is the so-called lens equation or imaging equation, which relates the object distance $g$ and the image distance $b$ with the focal length $f$:

$$\frac{1}{g} + \frac{1}{b} = \frac{1}{f} \tag{2.3}$$

The imaging scale or rather the magnification $m = B/G$ comes from the similarity of the triangles $ACM$ and $DEM$ (Figure 2.11):

$$m = \frac{B}{G} = -\frac{b}{g} \tag{2.4}$$

If $m$ is negative, the image is upside down. If $m$ is positive, the image is upright.

**Question 1:** Where do you have to put the screen, if you want to project an object standing 25 cm from a convergent lens of focal length $f = 20$ cm sharply onto the screen? How big is the magnification $m$?

### The convergent lens as a magnifying glass

If you put an object between the focal point and the lens, meaning $g < f$, then the produced image is not a real image. As only the backwards extended rays intersect in one point (Figure 2.12). The image is virtual and upright, the magnification $m > 1$ and the image length $b$ is negative (equation 2.3).

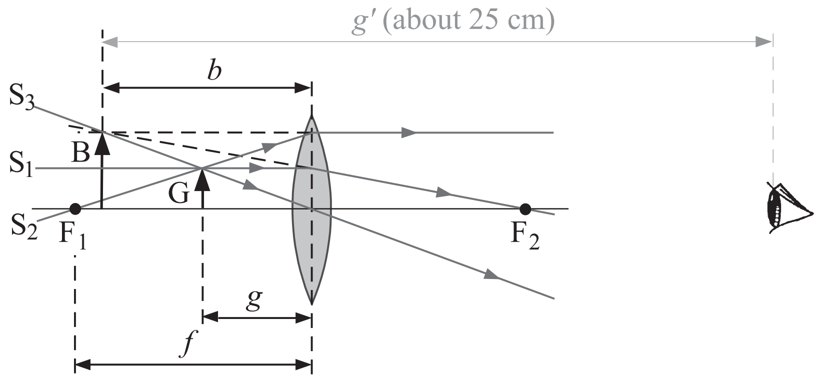



Figure 2.12: Image construction of a magnifying glass.

The image cannot be projected onto a screen. If observed by eye however, the incoming divergent rays are refracted by the lens system of the eye and thus focused in one point on the retina. The virtual image points are projected there as real (think in this context of mirrors). The virtual image becomes an object for the eye, which can easily focus on a distance of 25 cm.

## 2.2.3 Imaging through a thin divergent lens

For the geometrical construction of the image point, again at least two or three rays $S_1$, $S_2$, and $S_3$ are necessary.
The image is always **virtual** and **upright**, the image distance always negative, the magnifiction $m \leq 1$.

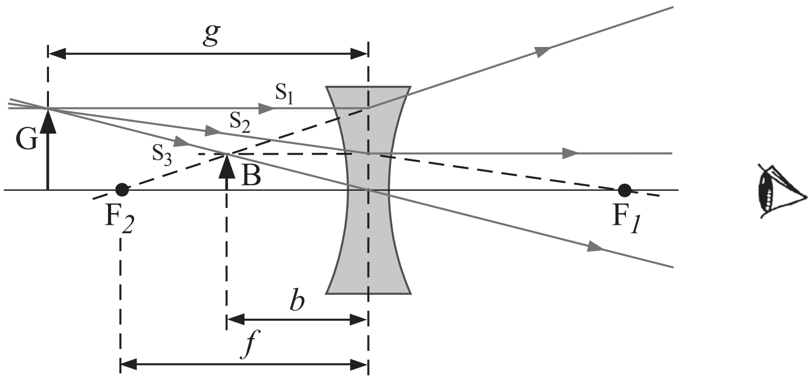



Figure 2.13: Image construction for a divergent lens.

### 2.2.4 Algebraic sign conventions

Using the imaging equation (2.3), the following rules apply:

- The light comes in from the left.
- Object distances are positive, if the object sits to the left of the lens.
- Image distances are positive, if the image is to the right of the lens.

**Question 2:** Do you know imaging systems other than lenses? If so, which ones? Draw a sketch of such a system.

For glasses, their optical refraction power instead of their focal length is specified, where $f$ is to be inserted with meters as a unit.

$$P = 1/f$$

The thus calculated optical refraction power is assigned the unit “diopter” (dpt.), e.g.:

$$P = 2\,\text{dpt.} \qquad \Leftrightarrow \qquad f = 50\,\text{cm}$$

### 2.2.5 The eye

**Structure of the human eye**

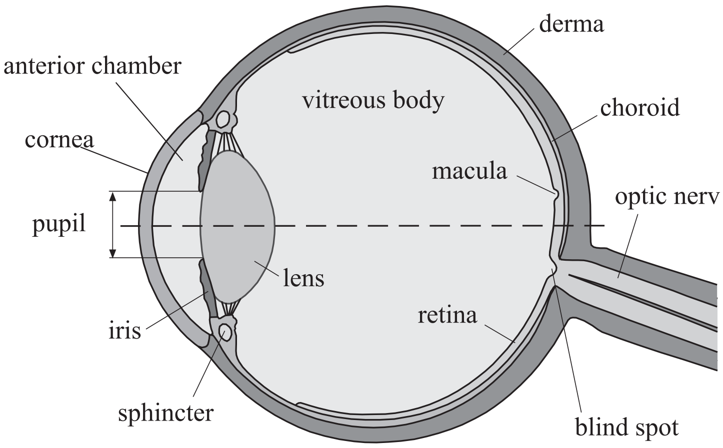



Figure 2.14: Structure of the human eye.

The total **optical refraction power** of the relaxed eye is 58.9 dpt. This corresponds to a focal length of $f \approx 17\,\mathrm{mm}$ (cp. table 2.1). The cornea is responsible for the major part (43 dpt.), the lens allows through variation of curvature the focussing for different distances. The shape of the eye is sustained by the intraocular pressure.

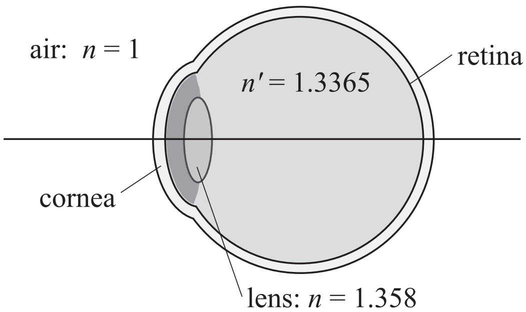


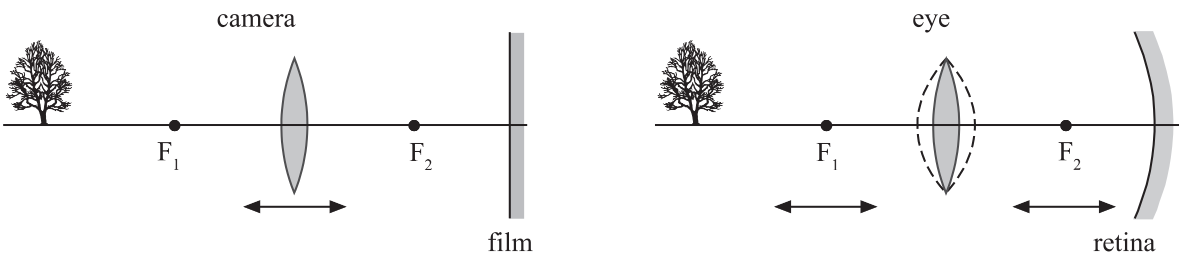

Figure 2.15: Schematic of the structure of the human eye.

Table 2.1: Values for the human eye.

| | |
|---|---|
| Thickness of the cornea | 0,8 mm |
| Curvature $r$ of the cornea | 7,83 mm |
| Refraction index of the aqueous fluid and vitreous $n'$ | 1,3365 |
| Refraction index of the ocular lens $n_L$ | 1,358 |
| Diameter eye pupil | 2 bis 8 mm |
| Frontal focal length $f$ with relaxed eye | 17,055 mm |
| Rear focal length $f'$ with relaxed eye | 22,8 mm |
| Near point distance during middle age | 25 cm |

### Accommodation

"Accommodation" means "focussing on different distances". For better explanation of the process, we will compare between camera and eye.




- Focussing through movement of the lens
- Focal length remains unchanged.

- Focussing through variation of the focal length of the lens
- Position of the lens remains unchanged.

Figure 2.16: Comparison between camera and eye.

### Accommodation of the eye

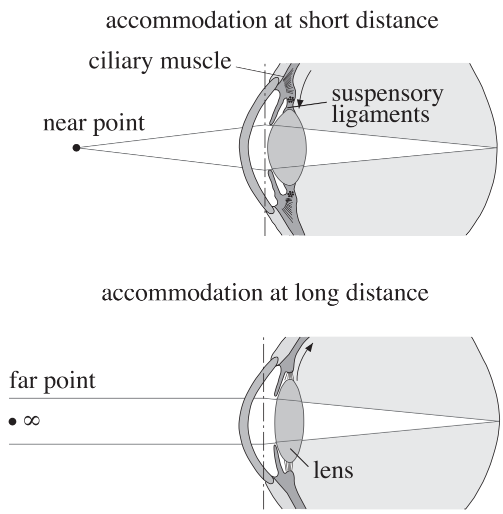



Figure 2.17: Accommodation of the eye.

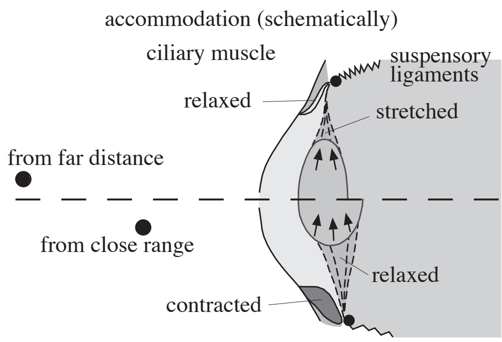



Figure 2.18: Schematic of the accommodation of the eye.

The variation of the optical refraction power of the lens is achieved through changing its surface curvature. Because of its inherent elasticity, the lens strives towards a stronger curvature, which is antagonised by the traction of the zonule. The ciliary muscle regulates this traction: muscle relaxes: far range accommodation, muscle contracts: close range accommodation.

In maximal far range accommodation, the so-called **far point**, in maximal close range accommodation the so-called **near point** is the focus point and appears sharp.

The **accommodation range** is an expression for the distance between the maximal and the minimal range of focussing.

The **accommodative power** is the difference of the optical refraction powers.

With higher age, the elasticity of the lens deteriorates and thus the accommodative power decreases (Fig. 2.19).

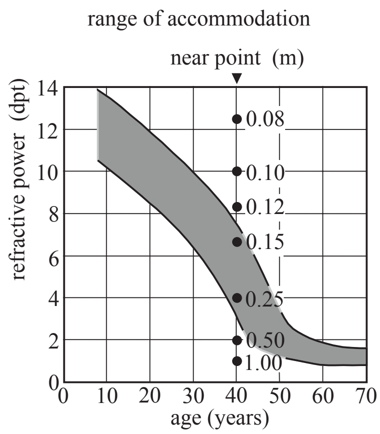



Figure 2.19: Accommodation range and accommodative power of the human eye.

#### Myopia and hypermetropia

**Exercise:** Try to understand the 3 forms of ametropia using the following figures.

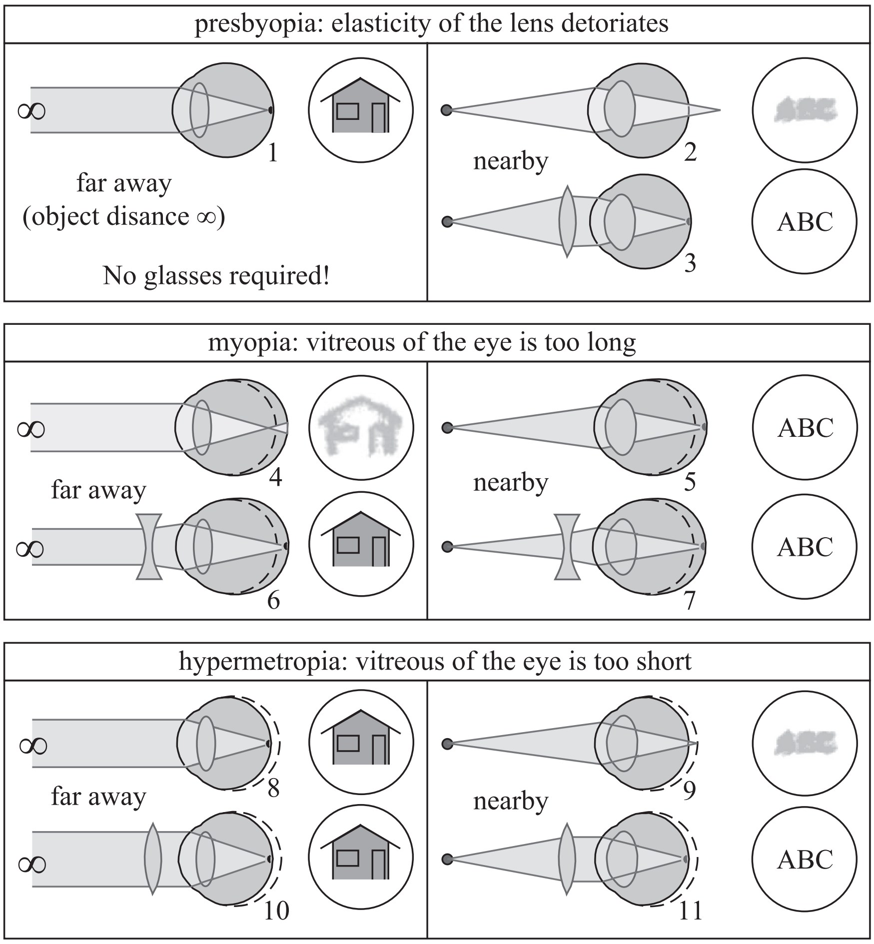



Figure 2.20: Myopia and hypermetropia in humans.

## 2.3 Experimental Part

### 2.3.1 Problem description

- Qualitative study of the properties of divergent and convergent lenses
- Creation of a parallel light beam
- Determination of the focal length of a convergent lens
- Imaging with a thin convergent lens
- Study of a model eye

### 2.3.2 Properties of divergent and convergent lenses

- Look at an object lying on the table through a divergent lens. Is the image upright or upside down? Is the magnification $m < 1$ or $m > 1$?
- Now use a convergent lens ($f = 10$ cm) as a magnifying glass, by looking e.g. at writing.
- With the same lens, try to create a **real** image without using a screen.
- At what object distance does the approximate transition between virtual and real image lie?
- Note down your observations.

### 2.3.3 Creation of a parallel light beam

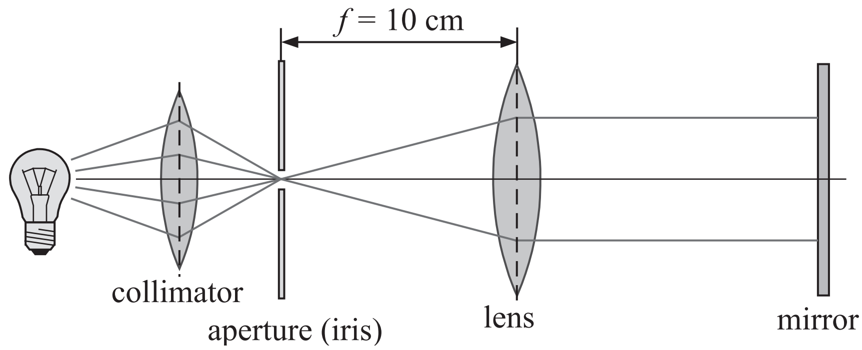



Figure 2.21: Experimental set-up for parallel light production.

A light source is projected onto an aperture using a collimator. The leaving, divergent light beam is parallel if the aperture lies exactly in the focal axis of the lens. In this experiment, the focal length $f$ is specified, but since the position of the lens axis is not accurately known, $f$ is not sufficiently accurate per se. Thus, it is necessary to perform the so-called **autocollimation technique** (Figure 2.21):

- Let the incoming light onto the lens reflect on a mirror, so that a light spot is created on the frame of the aperture.
- Move the lens between the aperture and the mirror until the diameter of the light spot is the same as the diameter of the aperture itself.

The aperture is now projected on itself sharply, it thus stands in the focal axis of the lens. Therefore the rays of light between the lens and the mirror are parallel to the optical axis.

### 2.3.4 Determination of the focal length of a convergent lens (lens 1)

- Create a parallel light beam using the above mentioned technique (Figure 2.21).
- Insert a convergent lens, whose focal length you want to determine, into the parallel light beam.
- To accurately measure the position of the focal point of the convergent lens, only light rays close to the optical axis should be used. Hence insert an annular aperture in front of the lens.
- Observe the light spot created by the lens on the screen and move the screen until the diameter of the light spot is minimal.
- Measure the focal length $f$ with the ruler. Estimate the measurement error on $f$.

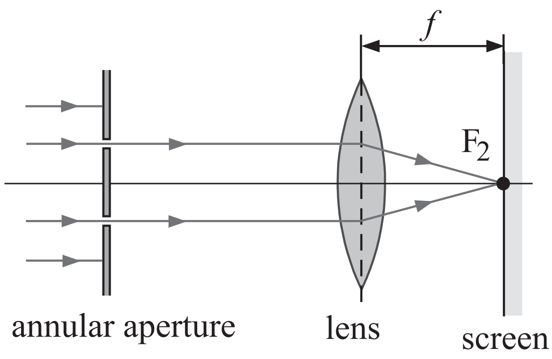



Figure 2.22: Experimental set-up for focal length determination of a convergent lens.

### 2.3.5 Real imaging on a screen

- Project the object (a slide) sharply onto the screen using a convergent lens (Figure 2.23).
- Measure the object distance and the image distance and calculate the focal length of the lens using the imaging equation 2.3. Compare your results with the specified focal length of the lens.

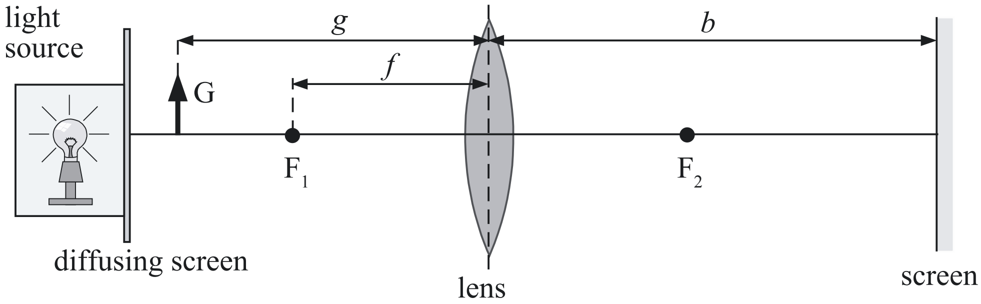
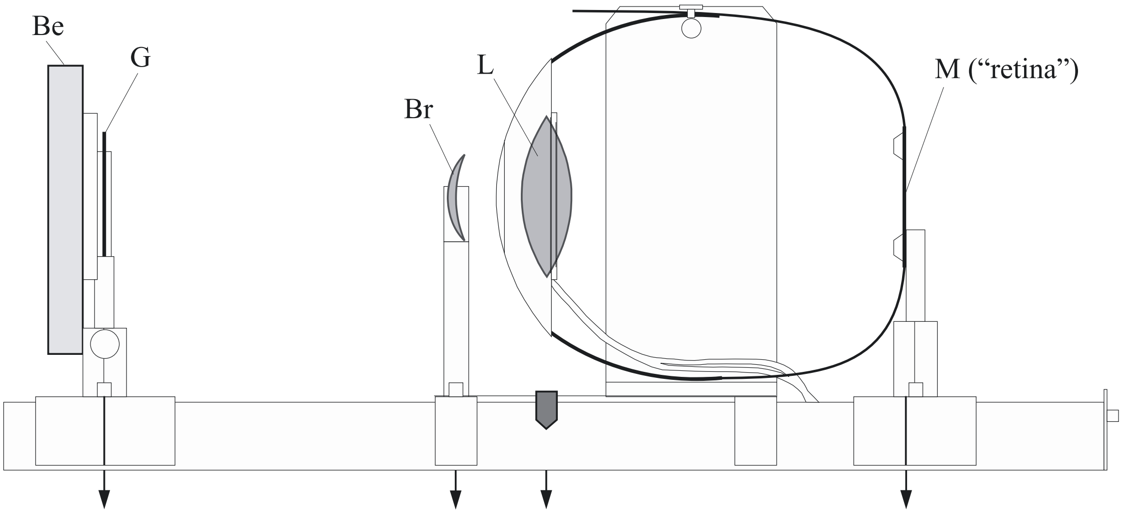



Figure 2.23: Experimental set-up for real imaging on a screen.




G = Object
M = Focussing screen ("retina") (Image plane)
L = Lens with variable focal length
Be = Illumination
Br = Glasses

Figure 2.24: Experimental set-up of the model eye.

### 2.3.6 Model eye

⇒ The focal length of the plastic lens $L$ can be varied by using the water syringe and thus changing the radius of curvature of the surfaces.

⇒ The whole eye is flexible; the focussing screen $M$ can be moved corresponding to different eye sizes. The positions (1), (2), and (3) are marked on the model eye.

#### 1. Accommodation of the normal eye

- Move the focussing screen to position (1).
- Choose an object distance of 28-30 cm. Change the amount of water in the lens until the image on the focussing screen becomes sharp and note down the syringe position.

- Measure the image distance and calculate the focal length of the lens using the imaging equation 2.3.
- Determine the minimal and the maximal object distance for which you can still create a sharp picture on the screen through variation of the water amount in the lens. This means determining the near and far point of the eye (choose the following syringe positions: 30 ml and 60 ml).
- Calculate the focal lengths of the lens for the near and far point. How large is the accommodation power and the accommodation range of the normal eye?

**2. Acommodation with hypermetropia**

- Move the focussing screen to position (2) and measure the new image distance.
- Determine again the near and far point of the eye (again, use the syringe positions of 30 ml and 60 ml)
- What is the accommodation range of this type of eye?
- Choose the appropriate glasses to improve the situation. Note down the diopter number of the glasses.
- Determine the near point of the eye with the glasses on, that means moving the object closer to the eye until the image is sharp again.
- Draw the three situations schematically as well as the respective lens curvature.

**3. Accommodation with myopia**

- Move the focussing screen to position (3) and measure the new image distance.
- Determine again the near and fat point of the eye (again, use the syringe positions of 30 ml and 60 ml)
- What is the accommodation range of this type of eye?
- Choose the appropriate glasses to improve the situation. Note down the diopter number of the glasses.
- Determine the near point of the eye with the glasses on, that means moving the object farther from the eye until the image is sharp again.
- Draw the three situations schematically as well as the respective lens curvature.