WS1

11. Alternating currents and impedances

11.1 Introduction

Besides ohmic resistors, capacitances and inductances play an important role in alternating current (AC) circuits. In this experiment, we investigate their behaviour in AC circuits and their influence on the current-voltage characteristics as a function of the applied AC frequency. This will be done on the basis of simple integrated circuits with one or two elements.

In this first experiment we will essentially us harmonic AC voltages and currents:

$$V(t) = V_0 \cos \omega t \qquad \text{and} \qquad I(t) = I_0 \cos(\omega t - \phi)$$

where
V_0 = voltage amplitude
I_0 = current amplitude
ω = $2\pi\nu = 2\pi/T$ = angular frequency
ϕ = phase shift between applied voltage and current

The time dependence of current and voltage will be recorded by means of a cathode-ray oscilloscope. You will have the opportunity to familiarize yourself with the working principles and the operation of an oscilloscope.

Oscilloscopes are an important tool in research and industry, e.g. for troubleshooting and adjustments of electrical devices of any kind (e.g. radios, televisions, microwaves, and radars). In medicine, they are used for surveillance of biological functions which manifest themselves in form of electrical signals. Devices like an EEG (electroencephalography, procedure to measure and track the electric activity of the brain) or an ECG (electrocardiogram, procedure to record the electric processes in the heart) use oscilloscopes.

Keywords for this lab course are:

- electric circuits,
- measurement of voltage and current as a function of time,
- Kirchhoff's laws,
- AC resistance or **impedance**

- impedance of an ohmic resistor, inductor, and capacitor as wel as their frequency dependences.

11.2 Theory

11.2.1 Impedance of an inductor

A time-varying magnetic field can induce a voltage in an electric conductor. If the magnetic field is produced by a current flowing through this conductor one speaks about self-induction. Since the induced voltage is proportional to the time derivative of the magnetic flux, and the magnetic flux proportional to the current we end up with a linear relationship

$$V_{\text{ind}} = L\,\frac{dI}{dt}. \tag{11.1}$$

The alternating current properties of an inductor (e.g. a coil) are characterised by the inductance L. It iy the ratio between the induced The inductance is determined by the length l of the coil, its cross sectional area A and the number of windings N. For a long inductor one has:

$$L = \mu_0\,\frac{N^2\,A}{l} \qquad [L] = 1\,\frac{V\cdot s}{A} = 1\,\mathrm{H}\ (\text{Henry}) \tag{11.2}$$

Where $\mu_0 = 4\pi\cdot 10^{-7}$ V.s/(A.m) is the so-called induction constant. When inserting a magnetic core into the coil, the equation above becomes

$$L = \mu\,\mu_0\,\frac{N^2 A}{l}$$

where μ is the magnetic permeability of the core material. Since the magnetic permeability for ferromagnetic materials is $\mu \gg 1$, like e.g. for iron, one can drastically increase the inductance of the coil in that way.

A simple circuit, consisting of an AC generator, which produces a harmonic voltage $V(t) = V_0 \cos\omega t$ and an inductor of inductance L, is sketched in Fig. 11.1. The inductor is assumed to be ideal, meaning that its ohmic resistance R can be neglected.
The application of Kirchhoff's second law (voltage law) to this circuit yields the relation between the voltage $V(t)$ and the current $I(t)$:

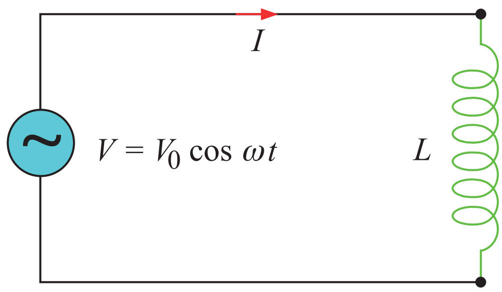


Figure 11.1: Electric circuit with an inductor with inductance L.

$$V_0\,\cos\omega t\; -\; L\,\frac{dI}{dt} = 0$$

Integrating the equation above once, yields the solution of this differential equation:

$$I(t) \;=\; \frac{V_0}{\omega\, L}\sin\omega t \;=\; \frac{V_0}{\omega L}\cos(\omega t-\phi) \qquad \text{with} \qquad \phi \;=\; +\frac{\pi}{2} \tag{11.3}$$

Hence, the current is being shifted against the voltage by a phase $\phi = \pi/2$ (see Fig. 11.2).

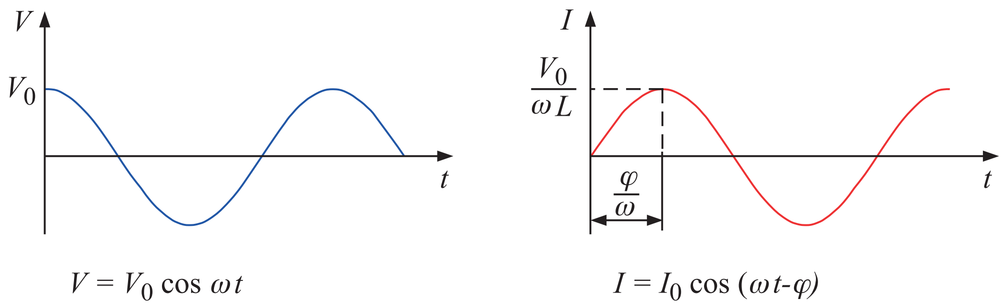


Figure 11.2: Voltage and current as a function of time for one inductor

The amplitude of the current is $I_0 = V_0/(\omega L)$. A comparison with the general definition of an electric resistor ($R = dV/dI$, see experiment KL - *voltage-current characterstics*) shows, that the quantity ωL plays the role of a resistance. One calls

$$Z_L = \omega L \tag{11.4}$$

the alternating current resistance or the **impedance** of an ideal inductor of inductance L. It is proportional to the frequency of the applied AC voltage. In general, the impedance for an element i is defined as

$$Z_i \;=\; \frac{V_0}{I_0} \;=\; \frac{\text{voltage amplitude}}{\text{current amplitude}} \tag{11.5}$$

Question 1: A 10 cm long inductor has a cross sectional area A of 4 cm^2 and 2000 windings. What is its inductance?

11.2.2 Impedance of a capacitor

Very much like the inductor, the capacitor represents an AC resistor as well.

The voltage over the capacitor is related to the charge stored in the capacitor by a proportionality factor C, called capacitance. For the circuit shown in Fig. 11.3, consisting of an AC generator and a capacitor, Kirchhoff's second law yields:

$$V_0\cos\omega t \;=\; V_C \;=\; \frac{Q}{C}$$

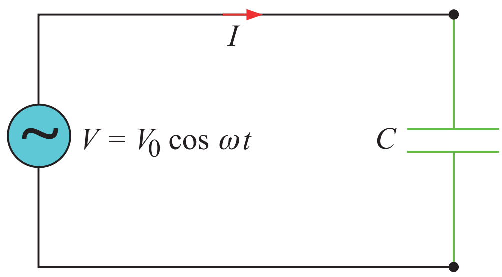


Figure 11.3: Integrated circuit with capacitor with capacitance C.

Differentiating once with respect to time yields

$$-\omega V_0 \sin \omega t = \frac{I}{C} \qquad \left(\frac{dQ}{dt} = I\right)$$

and therefore:

$$I(t) = -\omega C\, V_0 \sin \omega t = \omega C\, V_0 \cos(\omega t - \phi) \qquad \text{where} \qquad \phi = -\frac{\pi}{2} \tag{11.6}$$

So the current is shifted against the voltage by a phase $\phi = -\pi/2$ (see Fig. 11.4) and the impedance Z_C of a capacitor with capacitance C is:

$$Z_C = \frac{1}{\omega C} \tag{11.7}$$

It is inversely proportional to the frequency ω of the AC voltage.

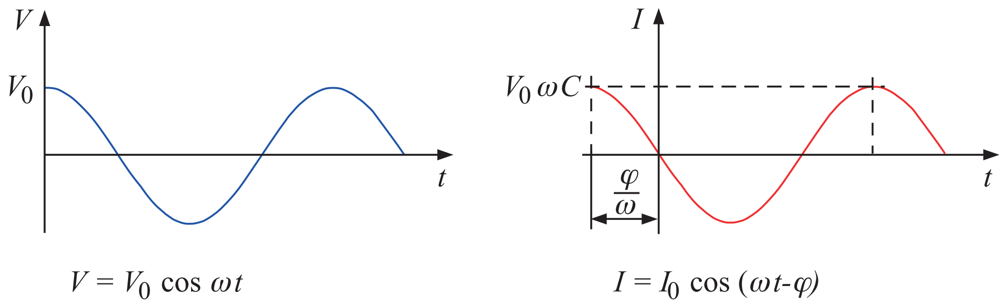


Figure 11.4: Voltage and current as functions of time for a capacitor.

11.2.3 Cathode-ray oscilloscope (CO)

In this experiment, all currents and voltages are time-dependent and will be measured by means of a (cathode-ray) oscilloscope (CO). The fundamental set-up of a CO is shown in Fig. 11.5.

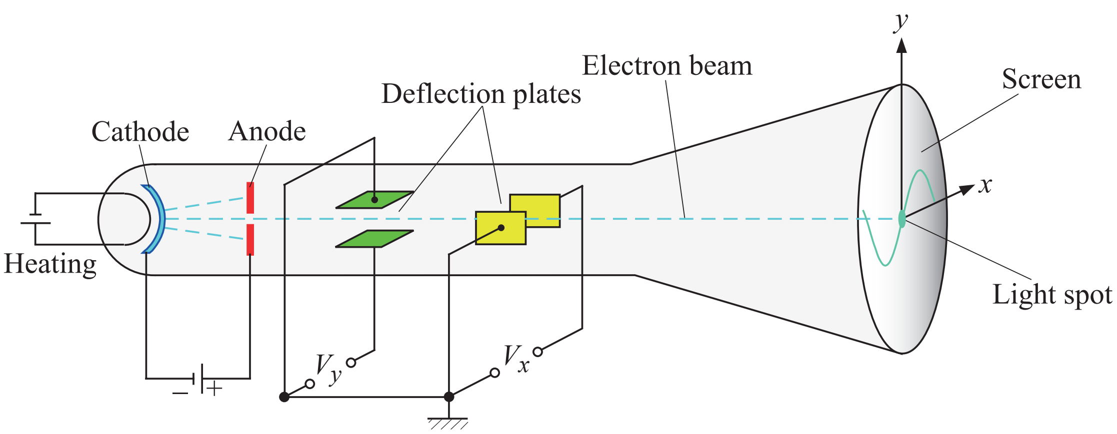


Figure 11.5: Schematic depiction of a cathode-ray oscilloscope.

Inside an evacuated glass flask electrons, which are emitted from a heated cathode (e.g. a glowing filament), are being accelerated towards an anode, which is equipped with a little hole. Electrons, which pass through the hole, form a thin electron beam. This beam successively passes two perpendicular pairs of plates before finally hitting a fluorescent screen, where it generates a short light spot. When applying a voltage to one of the pair of plates, the beam is deflected in horizontal or vertical direction, respectively. The deflection is always proportional to the applied voltage: $x \propto V_x$ and $y \propto V_y$. For the usual usage of the oscilloscope, a sawtooth voltage is being applied internally to the x-pair of plates. This particular voltage increases linearly with time until it reaches a maximum value and then quickly decreases to its starting value (see Fig.11.6).

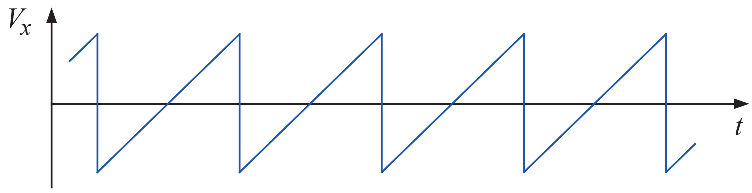


Figure 11.6: Sawtooth voltage for deflection in x-direction.

This causes the light point to repeatedly move over the screen from left to right and jumping back to its starting position after reaching the right end side of the screen. When applying an arbitrary voltage to the y-pair of plates (e.g. a harmonic AC voltage), the curve on the screen becomes a graphic representation of this voltage as a function of time. In order to measure two voltages at the same time, we use a two-channel oscilloscope. This enables us to easily represent phase shifts.

Nowadays, one usually uses digital oscilloscopes. They transform the applied voltages into digital values. Out of this data, the appropriate curves are being calculated and displayed on a screen in the same way a CO would display the external voltage signal.

11.3 Experimental part

11.3.1 Tasks

- Determination of the inductance of an inductor.
- Determination of the capacitance of a capacitor.
- Investigation of the frequency response of the AC resistance for the capacitor and the inductor.

11.3.2 Measurement principles

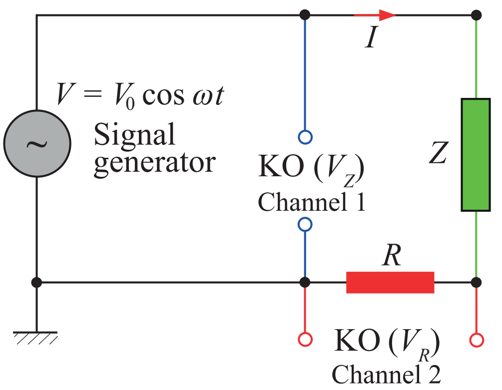


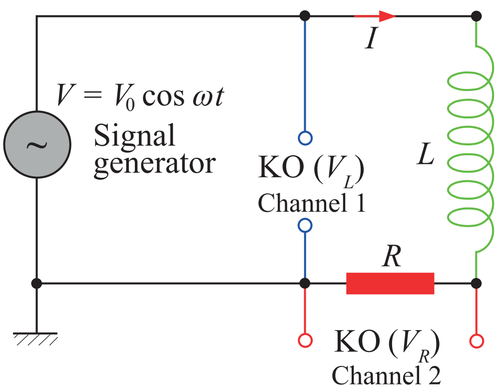
Figure 11.7: Circuit for the measurement of the current and the voltage using an oscilloscope (KO).

With the oscilloscope one can only measure voltages. Thus, a current must be measured indirectly as the potential drop over a known resistance R:

$$I = \frac{V_\mathrm{R}}{R}\,, \quad I_0 = \frac{V_\mathrm{R0}}{R} \tag{11.8}$$

If the resistance R is sufficiently smaller than the impedance Z one may neglect the voltage drop over R, and the voltage measurement represents approximately the voltage over Z: For $R \ll Z$ one has $V_Z + V_R \approx V_Z$; in this way the voltage on Z can be measured simultaneously with the current, using a two-channel oscilloscope. This is shown in Fig. 11.7.

- **The teaching assistant will explain and demonstrate the handling of the oscilloscope.**
- **Note that the outer connectors (shielding) of the coaxial inputs of the oscilloscope are internally wired and both connected to ground! Therefore, always wire the inputs in such a way, that the two outer cables lead to the same point (ground) in the circuit.**



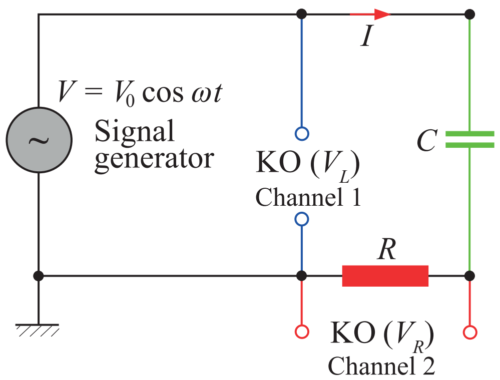
AC generator settings:

$$\begin{aligned} V_0 &= 5\mathrm{V} \\ \nu &= 1000\,\mathrm{Hz} \\ R &= 22\,\Omega \end{aligned}$$

Figure 11.8: Measurement of the inductance of a coil.

- Observe the phase shift between current and voltage on the screen by moving the two curves one on top of the other. Sketch the result.
- Using the oscilloscope, measure the amplitudes of V_L and V_R.
- Calculate the current amplitude according to Eqn. (11.8).
- Using this result, calculate Z_L according to Eqn. (11.5) and L according to Eqn. (11.4). To do so, use $\omega = 2\pi\nu$.

11.3.3 Determination the capacitance of a capacitor



AC generator settings:

$$V_0 = 5\text{V}$$
$$\nu = 500\text{Hz}$$
$$R = 22\Omega$$

Figure 11.9: Measurement of the capacitance.

- Observe the phase shift between current and voltage on the CO screen by superimposing the two curves. Sketch the result.
- Using the oscilloscope, measure the amplitudes of V_C and V_R.
- Calculate the current amplitude according to Eqn. (11.8).
- Using this result, calculate Z_C according to Eqn. (11.5) and C according to Eqn. (11.7).

11.3.4 Frequency dependence of Z_L and Z_C

- Repeat the above amplitude measurements for the inductor and the capacitor as a function of the applied AC frequency. Use the coil **with** the iron core inserted. Chose the following frequency values:
  - For Z_L: ν = 500, 1000, 2000 and 3000 Hz.
  - For Z_C: ν = 20, 50, 150, 250, 500 and 750 Hz.
- Compile the measured values in an table and calculate Z_L and Z_C for each frequency according to Eqn. (11.5).

11.3.5 Report

The report should include the following:

- Calculation of the inductance (Question 1).
- Draft of the experimental set-up.
- Draft of the observed phase shifts for the inductor and the capacitor.
- Calculation of the inductance L and capacitance C.
- Frequency dependence of Z_L and Z_C:
  - Table of the measurement values.
  - Graphical representation of the frequency dependence of $Z_L(\nu)$ and $Z_C(\nu)$ as a *linear* function. Which quantities have to be plotted in order to obtain a line?
  - Determination of the capacitance and inductance and their respective errors from the slopes of the lines.