8. Wavelength measurement with a grating spectrometer

8.1 Introduction

A very exact examination of the chemical composition of a substance can be undertaken by analysing the electromagnetic radiation it emits or absorbs when light shines through it. The electrons of an atom can only occupy states of certain, discrete energies, whereas the spectrum of allowed states is characteristic for a given kind of atoms or molecules. The wavelength of electromagnetic radiation which is being emitted or absorbed when a transition from one to another state happens is determined by the energy difference of the initial and final state and is therefore quantised as well (quantization = only discrete values). This leads to the rise of a line spectrum which is characteristic for the sort of examined atoms or molecules. Often even slight traces of a substance are sufficient to generate verifiable spectral lines.

In this lab course, the line spectrum of a helium lamp shall be examined in the range of visible light and the wavelength of the occurring spectral lines shall be determined with the help of interference on a fine graticule.

A second method for the decomposition of visible light into its spectral colors, namely the refraction of light inside of a cuneiform prism, is being observed in the lab course Spk. Interference phenomena with visible light are also examined in the lab courses I and M.

8.2 Theoretical part

The emergence of an interference pattern at a single slit is being discussed extensively in the theoretical part of the lab course I. When a plain wave hits a slit perpendicularly, the Huygens-Fresnel principle tells us, that every point in the slit can be seen as the source of a spherical wave and all these spherical waves are in phase with each other in the slit plane. If the slit width s is in the same order of magnitude as the wavelength $\lambda = 2\pi/k$ of the light, the superposition of those spherical waves leads to the raise of an interference pattern. At a point P, which is located at a distance $r_0 \gg s$ from the slit at an angle of θ to the symmetric axis, the cumulative wave from the single slit adds up to

$$u_{\text{slit}} = \frac{\sin\left(\frac{k\cdot s}{2}\cdot\sin\theta\right)}{\frac{k\cdot s}{2}\cdot\sin\theta}\cdot\cos\left(\frac{k\cdot s}{2}\cdot\sin\theta+\omega t-kr_0\right) \tag{8.1}$$

at an intensity of

$$I_{\text{slit}}(\theta) \propto \left\{\frac{\sin\left(\frac{k\cdot s}{2}\cdot\sin\theta\right)}{\frac{k\cdot s}{2}\cdot\sin\theta}\right\}^2 \tag{8.2}$$

The overall picture for a graticule which consists of N parallel slits of width s results from the superposition of the wave pattern of all single slits. If the distance r_0 of the respective point to the grating is very large compared to the widening of the grating, one can assume the observation angle θ to be the same for all slits of the grating (see Fig. 8.1). If d is the distance of two neighbouring slits, then the waves of those two slits are displaced against each other in the point P by a phase of $k\cdot d\cdot\sin\theta$.

Therefore, the over-all wave in P becomes

$$u_{\text{grating}}(\theta) = \sum_{n=0}^{N}\left\{\frac{\sin\left(\frac{k\cdot s}{2}\cdot\sin\theta\right)}{\frac{k\cdot s}{2}\cdot\sin\theta}\cdot\cos\left(\frac{k\cdot s}{2}\cdot\sin\theta+\omega t-kr_0-k\cdot n\cdot d\cdot\sin\theta\right)\right\} \tag{8.3}$$

and after the accomplishment of the summation and squaring it follows after some nasty calculation (see lecture script) that the intensity of the wave amounts to

$$I_{\text{grating}}(\theta) \propto \left\{\frac{\sin\left(\frac{k\cdot s}{2}\cdot\sin\theta\right)}{\frac{k\cdot s}{2}\cdot\sin\theta}\right\}^2\cdot\left\{\frac{\sin\left(\frac{N\cdot k\cdot d}{2}\cdot\sin\theta\right)}{\sin\left(\frac{k\cdot d}{2}\cdot\sin\theta\right)}\right\}^2 \tag{8.4}$$

The intensity behaviour is schematically depicted in Fig. 8.2. So-called major maxima appear for

$$\sin\theta = n\cdot\frac{\lambda}{d} \qquad \text{where} \quad n = 0, 1, 2, \ldots \tag{8.5}$$

and in-between two major maxima lie $N-a$ intensity minima and $N-2$ sub maxima. The amplitude of the major maxima is modulated with the interference pattern of a single slit. In

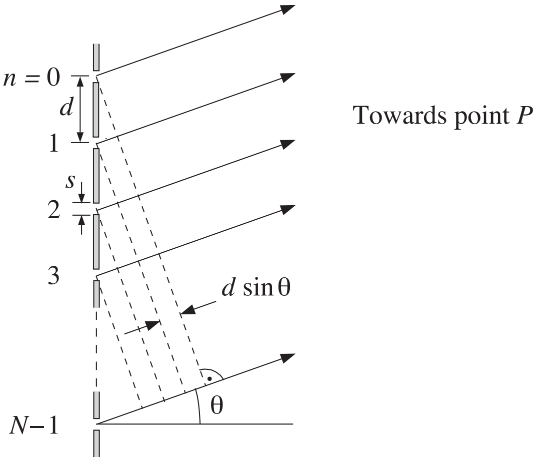

Figure 8.1: Interference at the grating.

the figure the square root of the intensity is being shown in order to enhance the visibility of the strongly suppressed sub maxima next to the major maxima.

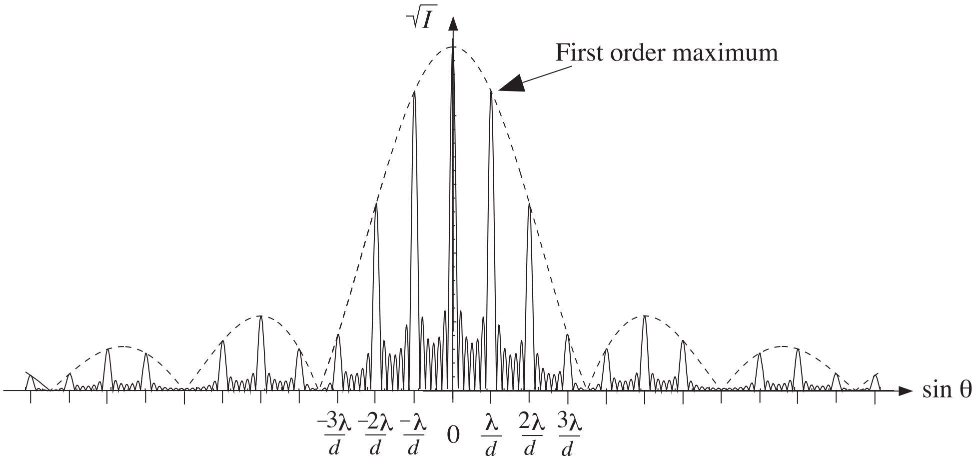

Figure 8.2: Interference pattern at the grating.

For a known slit displacement d one can determine the wavelength λ from the location of the major maxima. A grating suits better for a wavelength measurement than a single slit, since the major maxima are much better resolved than the intensity maxima of a single slit. The distance of two major maxima from a grating accounts for $\frac{\lambda}{d}$, whereas the width of a major maxima is defined by the distance $\frac{2\lambda}{Nd}$ between the two neighbouring minima. Thus, the major maxima become sharper if they're separated better from each other, which happens when we increase the number of slits N in the grating.

For a direct comparison, the interference patterns for a single slit, a double slit and a grating are shown in Fig. 8.3 one more time.

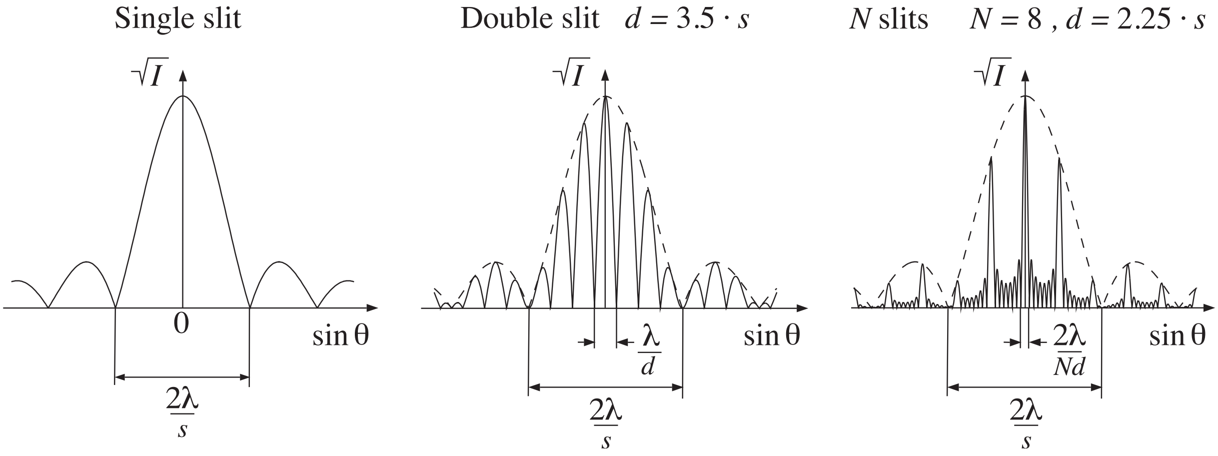

Figure 8.3: Interference pattern of a single slit, a double slit and a grating.

8.3 Experimental part

In this experiment you shall determine the wavelengths of the spectral lines of a helium lamp in the range of visible light with the aid of a grating spectrometer. In total, one can distinguish eight distinct lines:

Colour	red	red	yellow	green	green	blue-green	blue	purple
Intensity	very weak	medium	intense	weak	intense	medium	medium	intense

Experimental set-up

The set-up is shown in Fig. 8.4.

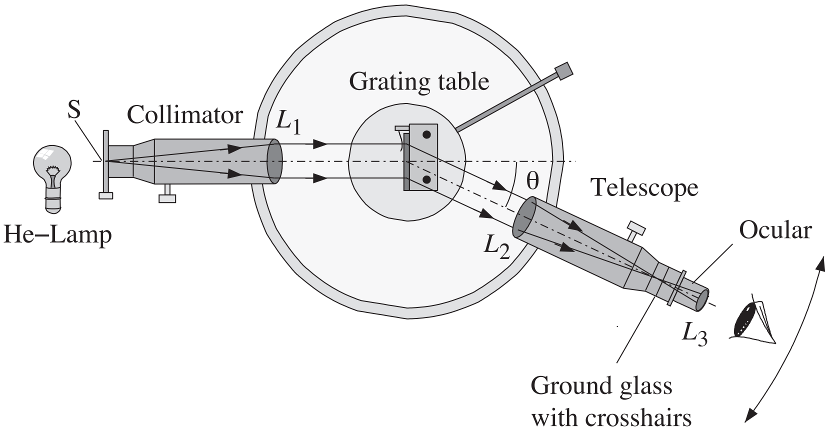

Figure 8.4: Grating spectrometer.

The collimator and the telescope serve the purpose to realise the approximations taken in the derivation of Equ. 8.5 for an incoming plane wave and a observing point which lies far away from the grating. The He-lamp lightens a narrow slit which lies in the focal plane of the collimator lens $L1$. Therefore, the light arrives just about parallel to the optical axis at the grating which corresponds to a plane wave front. The lens L_2 does focus the incoming parallel waves from the grating for its focal plane, where the interference pattern can be observed on matt screen with the help of the ocular L_3. The matt screen is provided with a reticle, which, by turning the telescope around the rotation axis of the spectrometer, is being aligned to be measured. The viewing angle θ can then be read off on a vernier gauge with an accuracy of an arc minute.

Calibration of the spectrometer

Before the actual measurement, the grating spectrometer has to be calibrated in such a way that the optical axes of the collimator and the telescope run through the rotation axis of the spectrometer and stand perpendicular to this axis.

- Replace the grating with a plane parallel mirror and the measurement ocular with a Gaussian telescope ocular. Adjust the ocular such that, both, the hair cross and its mirror image are displayed sharply at the same time.
- Align the hair cross and its mirror image by turning the grating table and inclining the table plane.
- Check, whether the telescope axis and the turning axis of the spectrometer stand perpendicular to each other by turning the grating table by 180° and again observing the hair cross and its mirror image. If necessary, align them, where you perform about half of the necessary adjustment at the telescope and the other half at the grating table. Repeat this procedure until the hair cross and its mirror image align in both settings.
- Remove the mirror and replace the Gaussian ocular with the measurement ocular.
- Lock the collimator slit in a vertical position. Adjust the collimator lens such that the image of the slit and the hair cross appear sharply in the in the ocular at the same time. Incline the collimator such that the image of the slit appears symmetrically to the hair cross.
- Attach the grating on the grating table.

The spectrometer is now ready for the measurement.

Measurement and evaluation

- For each of the eight spectral lines, measure the angle of the maximum of first order. Do it first on one side of the not deflected ray, then on the other side of the not deflected ray. Align the hair cross with the interference maximum and read off the observing angle. Also, determine the observing angle for the not deflected ray.
- Repeat the whole measurement a second time and take the average value of your measurements. Calculate the wavelength of the eight spectral lines after Equ. 8.5. The value of the grating constant d is declared at the set-up side.
- Repeat the measurement for one of the brighter lines an additional three times on both sides of the not deflected ray. From all the measurements you performed for this particular line, determine the uncertainty m_θ. Determine the uncertainty on the wavelength measurement using the propagation of uncertainty:

$$m_\lambda = \overline{\lambda} \cdot \sqrt{\left(\frac{m_d}{d}\right)^2 + \left(\frac{m_{\sin\theta}}{\sin\theta}\right)^2} \qquad \text{where} \quad m_{\sin\theta} = m_\theta \cdot \frac{d(\sin\theta)}{d\theta} = m_\theta \cdot \cos\theta \qquad (8.6)$$