

9. Microwaves

9.1 Introduction

Electromagnetic waves with wavelengths of the order of 1 mm to 1 m, or equivalently, with frequencies from 0.3 GHz to 0.3 THz, are commonly known as *microwaves*, sometimes also *centimetre waves*.¹ The characterising name for these waves is motivated purely by technological reasons; the physics is the same for all electromagnetic waves, and the same fundamental theoretical relations apply, the *Maxwell equations*.

Microwaves are of great technical importance for many different applications, such as radar, communication links (satellite and earthbound transponders), as carrier waves for cellular telephone transmission, used in heating equipment (laboratory and kitchen) to name but a few. Common sources of microwaves use either vacuum tube technology (e.g. a *klystron*), or solid-state technology, *microwave diodes*, as in this lab, where the active element is a *Gunn diode*. The wave may be continuous, or consist of short repeated bursts of energy.

Just as for the visible range of the electromagnetic spectrum (light) (cf. labs I, M, Spm), microwaves are subject to reflection, absorption, polarization, scattering, diffraction, and interference.

In the present three-part laboratory, absorption and reflection of microwaves will be investigated, the wavelength of the microwave source determined using a Michelson interferometer, and the diffraction and interference patterns for single and double slits measured.

Safety consideration

Although the transmitted radiation is of low power, do not expose your head and eyes to the radiation, in particular not close to the opening of the transmitter horn. Remember that metallic surfaces reflect almost perfectly microwave radiation. Never remove the horn wave-guide from the transmitter whilst connected to the power supply. Do not connect the receiver to a voltage supply; the terminals are intended for an audio amplifier.

¹Microwaves are also defined as the waves in the frequency range for which the electronic circuitry is not designed with coils and condensers, but with wave-guide and transmission line technology. The upper frequency limit also marks the onset of atmospheric absorption.

9.2 Theory

a) Electromagnetic waves

An electromagnetic wave is composed of two interconnected, perpendicularly oscillating electric and magnetic fields, \vec{E} and \vec{B} respectively, as shown in Fig. 9.1 for the case of a continuous *plane-polarized wave*. The two fields are completely described by Maxwell's Equations.

The direction of the \vec{E} vector is defined as the direction of polarisation (here the y -direction). An electromagnetic wave is also characterised by its direction of propagation, in the figure indicated by the vector \vec{k} , the *wave vector*, along the x -axis. These three vectors are perpendicular at all times.

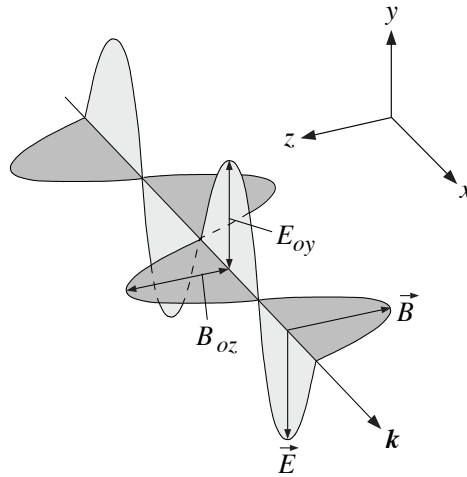


Figure 9.1: The geometry of the fields and the wave vector, \vec{k} for a plane electromagnetic wave.

In a general case a propagating electromagnetic wave may become quite intricate to describe. Here we limit ourselves to the simple special case of a planar travelling wave solution to the wave equations. For the given geometry in Fig. 9.1 the electromagnetic field of a plane wave, of wavelength λ , travelling in the positive x -direction, may be written:

$$\vec{E}(x, t) = \vec{E}_{oy} \cdot \sin(kx - \omega t) \quad (9.1)$$

where \vec{E}_{oy} is a constant, the maximum amplitude. The entity k is known as the *angular wave number*, often abbreviated only *wave number*, the magnitude of the wave vector. The *angular frequency* $\omega = 2\pi \cdot \nu$, where ν is the frequency.

These entities are related through:

$$k = |\vec{k}| = \frac{\omega}{c} = \frac{2\pi}{\lambda} \quad (9.2)$$

The *electromagnetic* wave number introduced above, should not be confused with the *spectroscopic*

wave number² $\tilde{\nu} = 1/\lambda$, as used in atomic physics, and sometimes also designated k .

Linear polarisation (or plane polarisation) requires that the \vec{E} -field vector is always oscillating in the same plane.

The intensity I of any wave is defined as the energy transmitted through a surface perpendicular to the direction of propagation (the wave vector). For an electromagnetic wave this is the time average of the product of the magnitude of the two fields. According to Maxwell's equations $B \propto E$, which is equivalent to saying that the intensity is proportional to the *time average* of the square of the strength of the electric field. We write this as:

$$I \propto \overline{E^2(t)} \quad (9.3)$$

b) Diffraction and Interference

The theory of wave diffraction in a single slit and wave interference in a double slit is thoroughly treated in the theory section of the manual for lab I. Only the main results will be briefly summarised in the following. One should recall here that the phenomena of diffraction and interference appear at openings and slits with dimensions that are of the order of the wavelength of the radiation.

Single slit diffraction

A wave with wavelength λ , perpendicularly incident on a single slit of width s , gives rise to a diffraction pattern far behind the slit, (*Fraunhofer diffraction*), e.g. on a distant screen, with minima (dark straight fringes parallel to the slit) at angles θ determined by the relation:

$$\sin \theta = n \cdot \frac{\lambda}{s} \quad \text{where the integer } n = \pm 1, \pm 2, \pm 3 \dots \quad (9.4)$$

Angle θ is measured from the normal of the plane containing the slit. The respective minima are said to be of *order* n .

For a perpendicularly incident wave, single slit diffraction intensity maxima is found at angles approximately described by the relation:

$$\sin \theta = \left(n + \frac{1}{2}\right) \cdot \frac{\lambda}{s} \quad \text{where } n = 0, \pm 1, \pm 2, \pm 3 \dots \quad (9.5)$$

This is a nontrivial result.

It is worth noting, that the angles for both maxima and minima depends inversely on the the slit width, so that the essential features of the pattern becomes larger for decreasing slit width.

As shown in the right part of Fig. 9.2 the intensity of the maxima decreases strongly with angle. The ordinate shows the square root of the intensity I only for the sake of convenience.

²By tradition the spectroscopic wave number is sometimes given in units of reciprocal centimeters, cm^{-1} or keyser (sometimes even retemitnec),

Double slit diffraction (Young's experiment)

The electrically generated microwaves have good spatial coherence, and therefore the wave incident on a *double slit* will give rise to interfering waves. The intensity variation is determined by interference between waves from the two slits, and diffraction in each of the two single slits. With slit centre *distance* d , a perpendicularly incident wave with wavelength λ , gives rise to a complicated intensity variation. Minima, i.e. dark fringes parallel to the slits, appear at angles θ , determined by the relation:

$$\sin \theta = \left(n + \frac{1}{2}\right) \cdot \frac{\lambda}{d} \quad \text{where } n = 1, 2, 3, \dots \quad (9.6)$$

Intensity maxima occur at angles θ according to the relation:

$$\sin \theta = n \cdot \frac{\lambda}{d} \quad \text{with } n = 1, 2, 3, \dots \quad (9.7)$$

These results are obtained straightforwardly, but they only describe the angular positions of minima and maxima. The full expressions for the intensity variations are quite a bit more complicated. For the hypothetical case of infinitely narrow slits (there would actually be no intensity) the maxima would all be equally intense. The square root of the intensity distribution for the double slit with finite slit widths is shown on the right in Fig. 9.2, showing the intensity of single slit diffraction as an envelope to the double slit interference maxima.

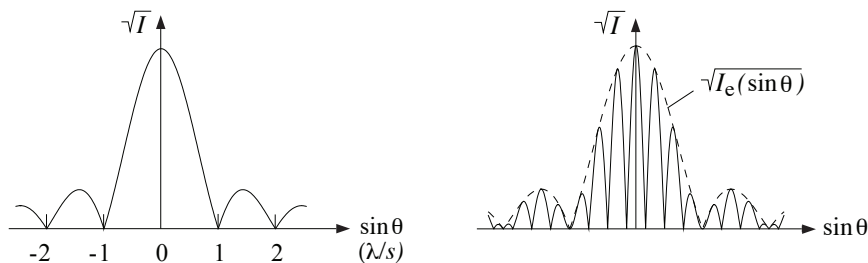


Figure 9.2: Intensity distribution for diffraction in a single slit (left) and a double slit (right).

c) The Michelson interferometer

The Michelson interferometer is treated in some detail in the theory section of the manual for lab M, and in this manual section 9.3. Although different in mechanical design, the principles of the set-up employed here are the same, and therefore only the main features are summarised. Compare Fig. 9.5 and the corresponding figure in lab M.

Superposition of coherent waves may cause phenomena related to interference. Assuming sufficient coherence, we restrict ourselves here to the case of two interfering waves from a single source. Depending on the relative phase of the waves, their electromagnetic fields may add constructively or destructively, thus creating an interference pattern. Since the wavelength for microwaves is about

five orders of magnitude larger than for visible light, the features of the pattern will be accordingly larger.

The interferometer is designed for investigation of interfering waves from a single source. The beam is split in two perpendicular paths by a partially reflecting mirror (beam splitter), which for microwaves may be a wire grating, as in the present lab. The interferometer arms are implemented as short optical benches for sliding or fastening the two perpendicularly mounted aluminium sheet metal screens acting as mirrors at the end of each path. The beams are reflected back onto the beam splitter, and brought to interfere such that a pattern of interference results. As opposed to the case of the Michelson interferometer for visible light, cf. lab M, there is no extra half wavelength path difference here because of the beam splitter, and therefore the condition for destructive interference (a dark fringe) on the optical axis (at the centre of the pattern), requires the optical path difference ΔW_0 of the two beams to be:

$$\Delta W_0 = \left(n + \frac{1}{2}\right) \cdot \lambda \quad \text{with } n = 0, 1, 2, \dots \quad (9.8)$$

Conversely, for constructive interference (a bright fringe) we have:

$$\Delta W_0 = n \cdot \lambda \quad \text{with } n = 0, 1, 2, \dots \quad (9.9)$$

In the present laboratory, the interferometer will be used to determine the wavelength of the microwave source. Assume first that the path difference ΔW_0 is zero. The two waves then arrive at the detector in phase, so that an intensity maximum is recorded. By moving one of the mirrors till the path difference becomes half a wavelength, the waves arrives out of phase (phase difference π) at the detector, which causes the intensity to drop to zero (under the condition that both waves have the same intensity after being split in the semi-transparent mirror). Increasing the path difference further causes the conditions for constructive and destructive interference to alternate as the path difference ΔW_0 equals an integer or half integer number of wavelengths respectively. Another way of putting this is through the relation:

$$N = \frac{\Delta W_0}{\lambda} \quad (9.10)$$

where N is a number that expresses the path difference in units of wavelength. For integer values N the condition for constructive interference. i.e. intensity maximum, is fulfilled.

9.3 Experimental

The same basic set-up will be modified to serve three different experiments. In the first, absorption and reflection of microwaves from different samples will be investigated. In the second, the set-up is arranged to become a Michelson interferometer for the determination of the wavelength of the source. Finally, the intensity distribution from diffraction in a single and a double slit is measured.

The microwave source is essentially a *Gunn diode* feeding a half-wave dipole antenna that radiates into a short wave-guide. The dipole may be seen at the bottom of the so called *horn antenna*, which is not really an antenna but effectively a tapered wave-guide with square cross-section with the purpose of increasing the intensity in the forward direction. The short piece of straight wave-guide is coupled to the horn, which must never be removed when the transmitter is operating. The radiated wave is transmitted in the direction of the horn axis; it is a plane wave, linearly polarised as indicated by the direction of the transmitting antenna dipole (and by the handle for changing the polarisation direction). The transmitter electronics is connected to an external power supply, a DC adapter, rated at 8-13 V, plugged into the mains outlet.

The receiver consists of a horn wave-guide of similar design as described above, and a dipole antenna connected to the detecting electronics. The receiver is sensitive only to waves with polarisation aligned with the dipole antenna, visible at the bottom of the horn. The detector contains only passive components, and must not be connected to a power supply: The terminals at the back are intended for an external audio amplifier. In Fig. 9.3 angles α and θ , referred to in the text, are indicated. In addition, the receiver may be rotated around the vertical shaft on which the receiver is mounted, in order to fine-tune the direction of the horn.

The wave received by the dipole antenna is converted to an alternating electric current of the same frequency as the transmitted wave, and rectified in the receiving antenna circuitry with a microwave silicon diode (cf. lab KL). The resulting current is measured with a sensitive amperemeter. Select the 100 μA range or occasionally the 1 mA range.

The basic set-up, common to all three experiments, is shown in Fig. 9.3. The transmitter is always kept in a fixed position, but the receiver with horn can be moved on a circular track on the base plate (angle θ). Samples or wave-optical elements may be placed in the centre mount of the base plate.

In the following we refer to the geometry in the figure, as *absorption geometry*, or $\theta = 0^\circ$. i.e. transmitter and receiver opposing each other. Accordingly the position $\theta = 90^\circ$ is referred to as *reflection geometry*. In reflection geometry, flat samples or optical elements are mounted at an angle of 45° from the optical axes of both transmitter and receiver.

A substantial amount of data will have to be recorded and a systematic procedure for taking notes is necessary.

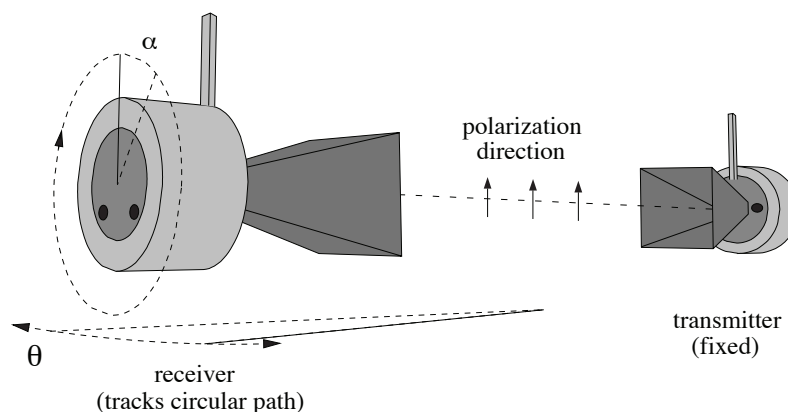


Figure 9.3: Set-up for the general investigations of microwaves.

a) Absorption and reflection.

- Align the receiver in absorption geometry. Adjust the transmitter and receiver for vertically polarised waves. Position (or hold by hand) the available material samples, one by one, in the centre mount and observe the decrease in current. From the readings of the ampere meter, calculate the attenuation factor relative to the reading without sample. Make notes for the report that relate the findings to kind and thickness of the material.
- Move the receiver in the horizontal plane (angle θ) to a position 90° from the beam. Place the material samples such that the reflective properties may be investigated. Calculate a reflection factor in the same manner as above, and collect data for the report.
- As above, investigate the absorption and reflection from different wire meshes. Characterise the meshes by some typical length and make notes for the report.
- Verify the law of reflection with an aluminium mirror in the centre mount. Turn the mirror to three different angles relative to the beam. measure that angle and determine the corresponding angle θ where the detector indicates maximum intensity.
- With the wire grating in the centre mount, position transmitter and receiver for absorption measurements ($\theta = 0^\circ$). Rotate the grating in the plane of its frame, and position the wires in turn to three positions: vertically, horizontally and 45° from the vertical (cf. Fig. 9.4). Measure the intensities, and calculate the fraction transmitted through the grating.
- Repeat the measurements for the three grating positions in reflection geometry ($\theta = 90^\circ$). Calculate the fraction reflected from the grating.
- In transmission geometry, align the receiver for horizontal polarisation ($\alpha = 90^\circ$ from the vertical position), keeping the vertical polarisation of the transmitter. Measure again the intensities for the three grating positions.
- For reflection geometry, repeat the measurements with the wire grating.

- Collect and summarise for the report, in a comprehensive manner, all experiments with the grating.
- From the data collected, determine if the grating in some position can be used as a semi-transparent mirror, transmitting half the intensity, reflecting half.

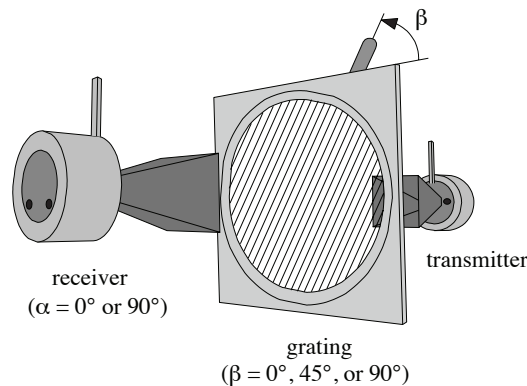


Figure 9.4: Experimental set-up with transmitter, receiver and wire grating.

b) Wavelength determination with the Michelson-interferometer

In the second part of the laboratory, the wavelength of the source will be determined using a Michelson interferometer. The set-up is shown in Fig. 9.5. As found in the previous experiments, the wire grating can serve as a partially transparent mirror for microwaves. One important parameter here, as may be inferred from the measurements with wire meshes above, is the distance between the wires. Thus the wave from the transmitter may be split in two perpendicular beam paths, one transmitted and one reflected. The arms of the interferometer are terminated with perpendicular aluminium mirrors at their far ends. The path length of one arm remains fixed, whereas the mirror of the other arm may be translated along the beam, hence a path difference incurred. The waves are reflected back to the beam splitter, and brought to interfere at the detector.

- Change the path difference between the two beams by sliding the movable mirror along the optical bench, and watch the detector signal. How can we explain the observation? (There is no standing wave here.)
- Sliding the mirror, locate two consecutive minima, note the positions. Repeat four independent measurements of the positions for the two minima. If the measurements seem consistent, calculate the mean value of their distance, which is b in Fig. 9.5.
- Finally, calculate the wave length of the radiation using Eq. 9.10. Explain how ΔW_0 is related to b . From the set of measured positions, determine the maximum error in each position, and then in b . Estimate the error of the wave length λ , using error propagation.

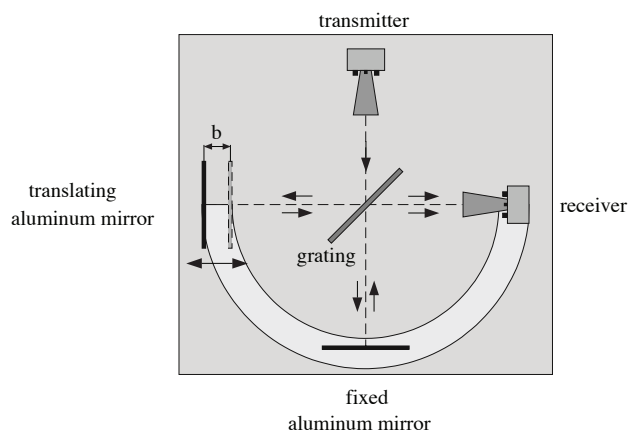


Figure 9.5: Experimental set-up for the Michelson interferometer.

c) Measuring the intensity of the diffraction pattern for single and double slits.

In the third experiment, the theoretical intensity distribution in Fig. 9.2 will be investigated. The equipment is basically the same as in Fig. 9.5, but with some modifications. Remove the mirrors and the wire grating, so that only transmitter and receiver remain, and arrange them opposing each other as in Fig. 9.3.

- Position the single slit frame in the central mount at half distance between transmitter and receiver. Confirm that there is a central maximum at $\theta = 0^\circ$; if not, adjust the horizontal direction of the receiver horn (azimuth), but keep angle $\theta = 0^\circ$. Measure the intensity as function of angle θ in steps of 5° in the range 0° to 90° in both directions. Draw a diagram of the intensity as function of $\sin \theta$ and determine the positions of maxima and minima. Take the average of the angle for both directions. Measure the width of the slit.
- Tabulate these results and compare with the theoretical predictions according to Eqs. 9.5 and 9.4. Use the value for the wavelength determined above in part b.
- Repeat the measurement for the double slit, but now with angular steps of 2° . Again draw a diagram of the intensity as function of $\sin \theta$ and determine the average of the positions for maxima and minima on both sides.
- Again, tabulate the results and compare with the theoretical predictions in Eqs. 9.7 and 9.6.
- Summarise all results for the report.